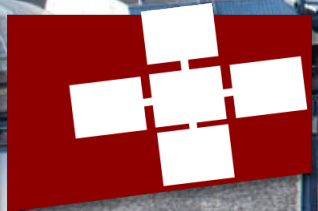


**ANDRÁS STRAUZ, FLAVIO VELLA (UNIVERSITY OF TRENTO), SALVATORE DI GIROLAMO,**

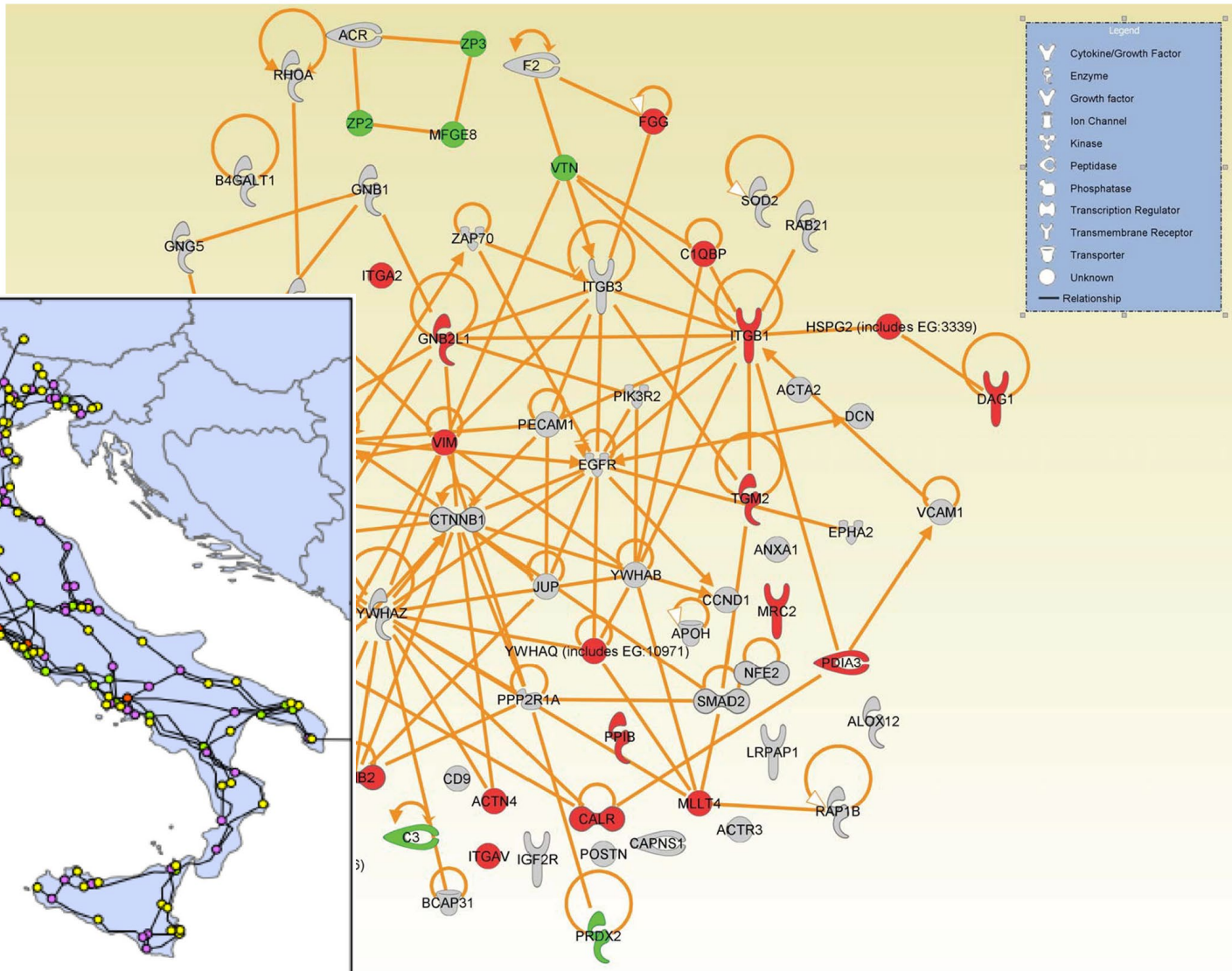
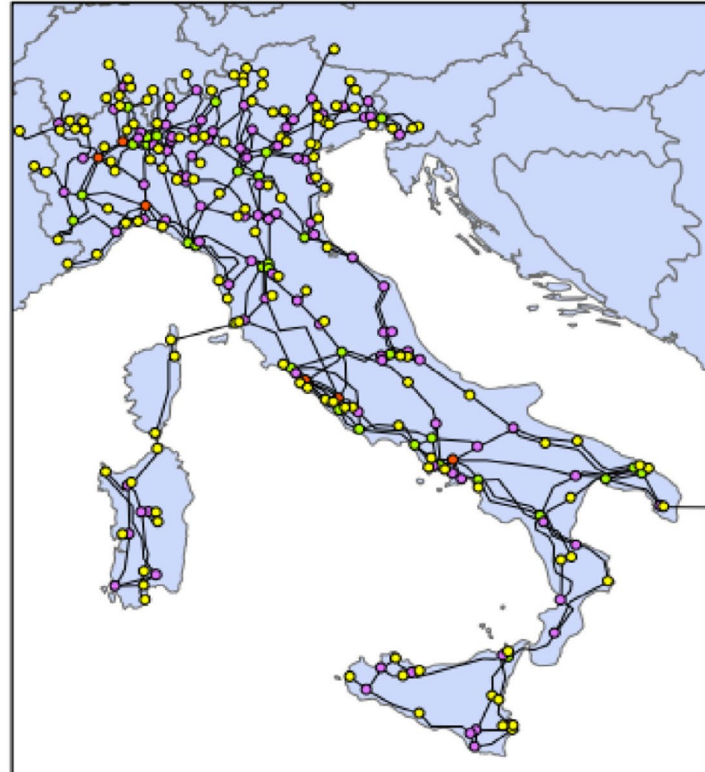
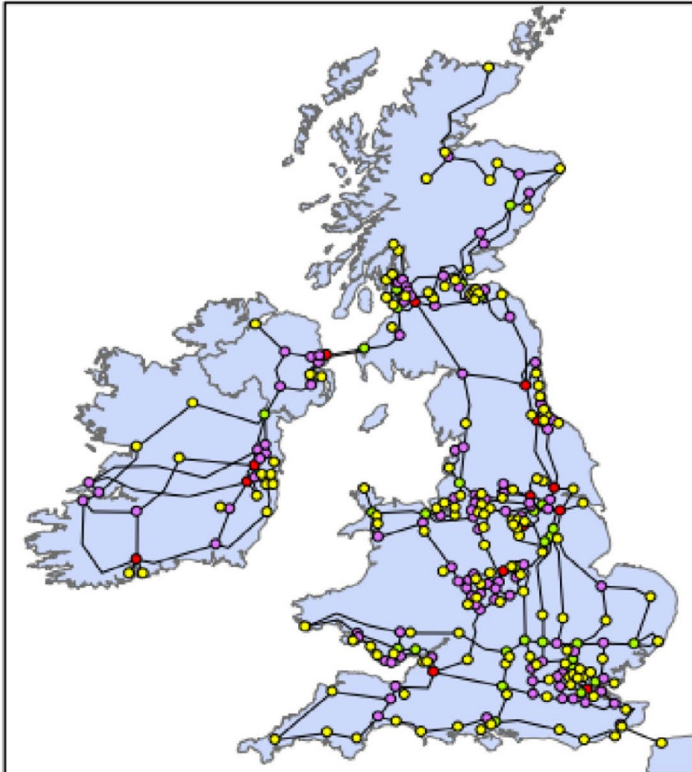
**MACIEJ BESTA, TORSTEN HOEFLER**

# **Asynchronous Distributed-Memory Triangle Counting and LCC with RMA Caching**



# Warm-up: Local Clustering Coefficient

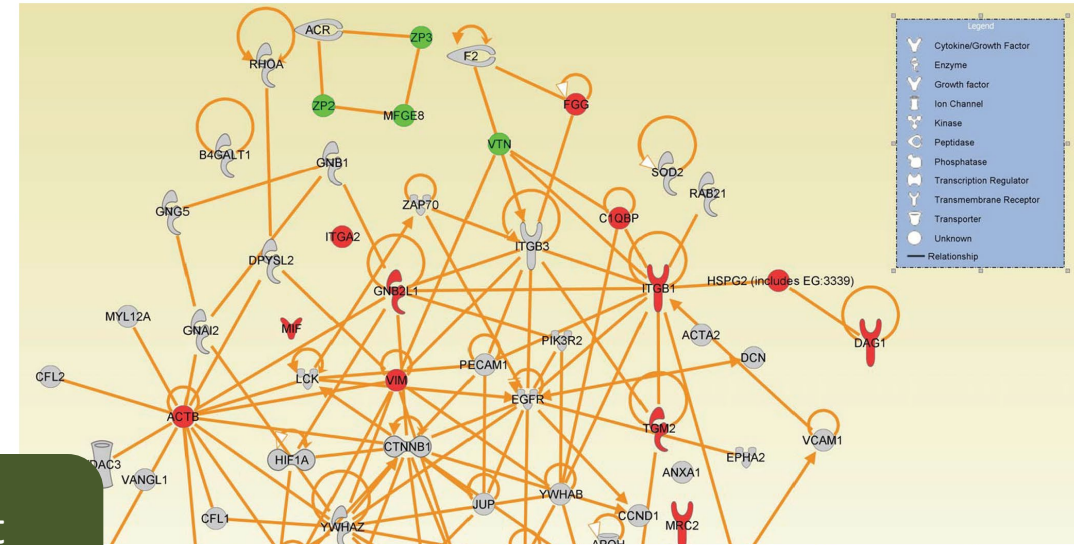
- **Graphs represent relational data very well**



Right: Peddinti, Divyaswetha & Memili, Erdoğan & Burgess: Proteomics-Based Systems Biology Modeling of Bovine Germinal Vesicle Stage Oocyte and Cumulus Cell Interaction  
 Left: Alireza Shahpari, Mohammad Khansari, Ali Moeini: Vulnerability analysis of power grid with the network science approach based on actual grid characteristics: A case study in Iran

# Warm-up: Local Clustering Coefficient

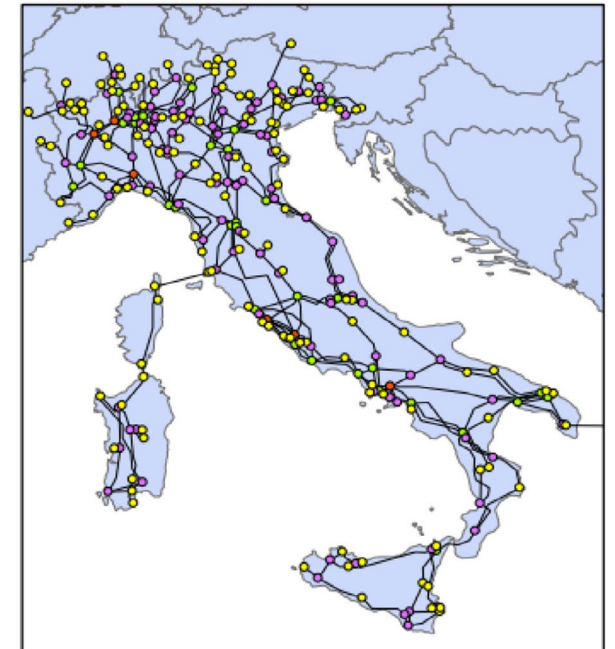
- Graphs represent relational data very well
- LCC: likelihood that neighbors of a vertex are connected



$$LCC(\text{red circle}) = \frac{\left| \forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \text{red} \\ \bullet \quad \circ \\ \text{black} \end{array} \right|}{\left| \forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \text{red} \\ \bullet \quad \circ \\ \text{black} \end{array} \right|}$$

Count triangles!

Degrees are known



# Warm-up: Local Clustering Coefficient

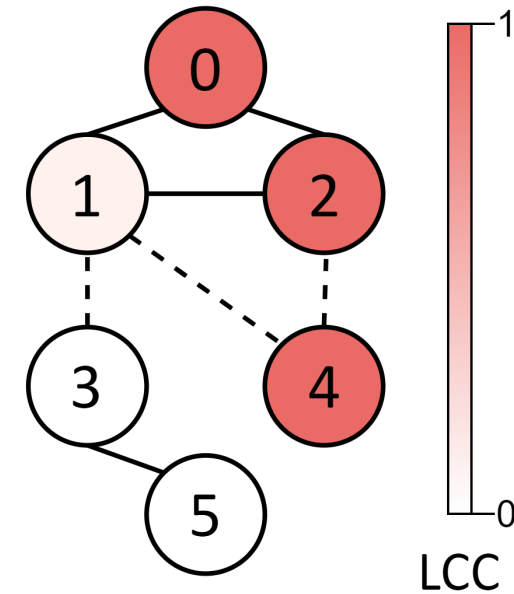
- Graphs represent relational data very well
- LCC: likelihood that neighbors of a vertex are connected

$$LCC(\text{red circle}) = \frac{\left| \forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \text{red circle} \\ \diagup \quad \diagdown \\ \bullet \quad \circ \end{array} \right|}{\left| \forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \text{red circle} \\ \diagup \quad \diagdown \\ \bullet \quad \circ \end{array} \right|}$$

Count triangles!

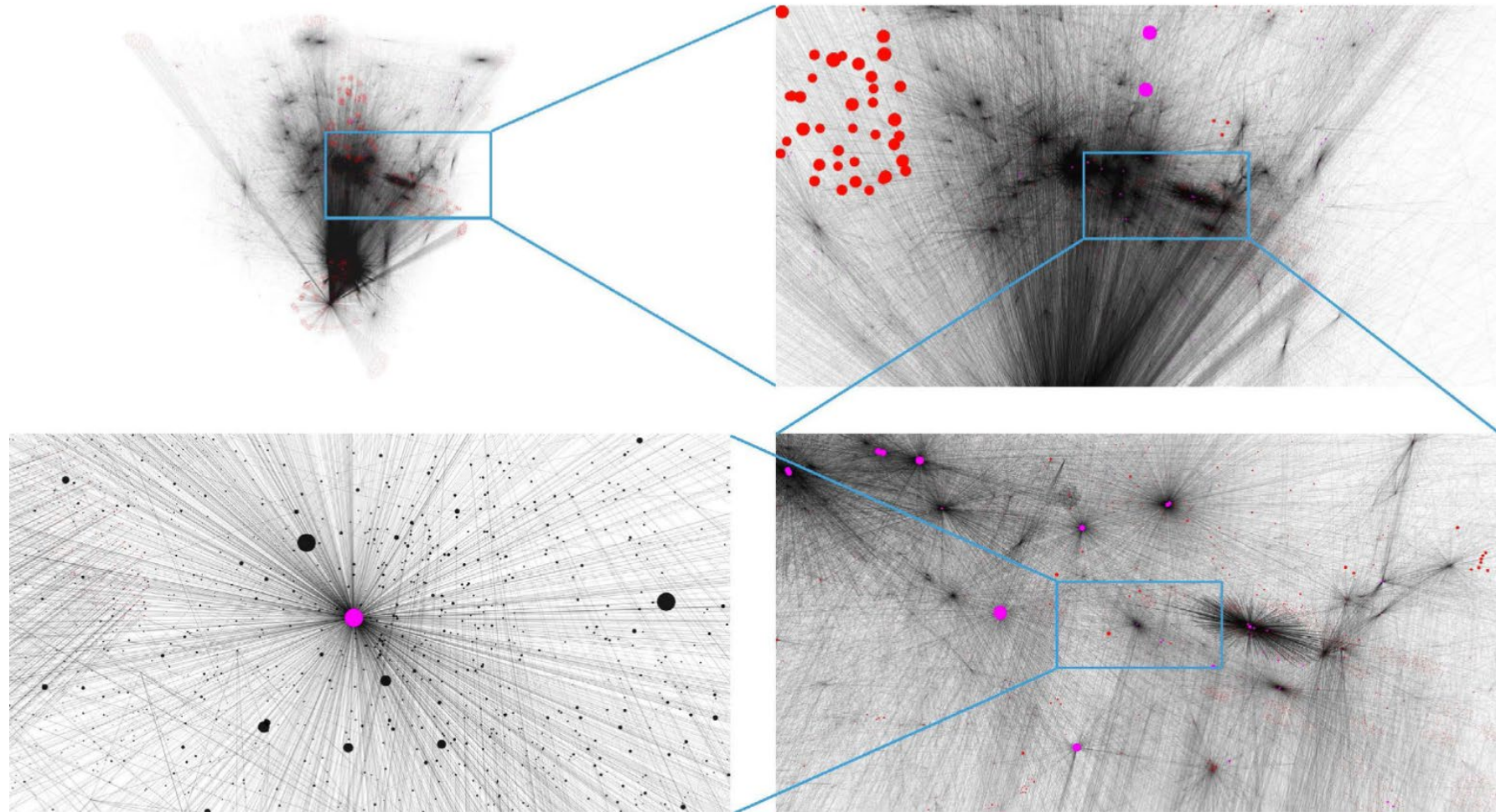
Degrees are known

- Many useful applications in link prediction problems
  - community detection, link recommendation



# Challenges: Graphs are huge and skewed

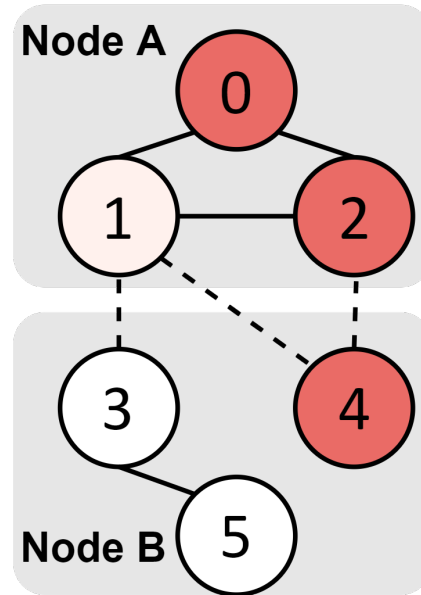
- Billions of vertices and hundreds of billions of edges
- Scale-free degree distribution



# Distributed-memory TC & LCC computing

## Current state-of-the-art:

1. **Synchronized computation**
  - Bulk Synchronous Parallel
  - MapReduce
2. **Frontier intersection**
3. **Graph partitioning**
  - Static vertex delegation



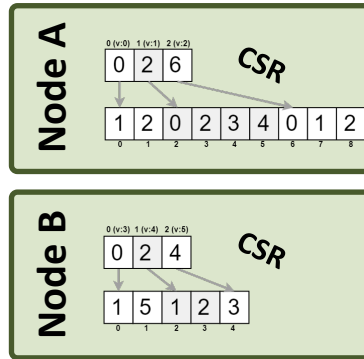
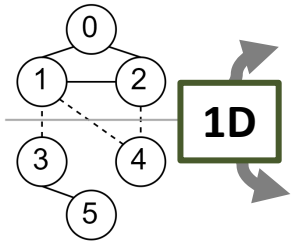
## Our work proposes:

1. **Fully asynchronous algorithm based on MPI-RMA**
2. **Hybrid strategy for local TC**
3. **Exploiting data reuse with caching**  
**Application-specific eviction policy**

In general, **4-12x faster** results for scale free graphs compared to TriC  
 Best results show **up to 100x speedup**

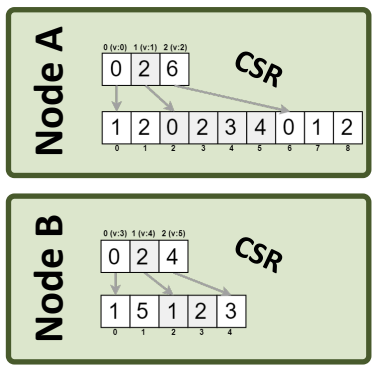
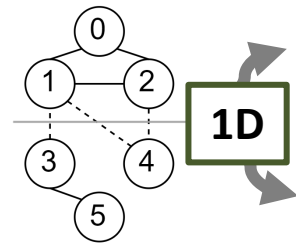
# Algorithm overview: Distribution

## 1 Distribution



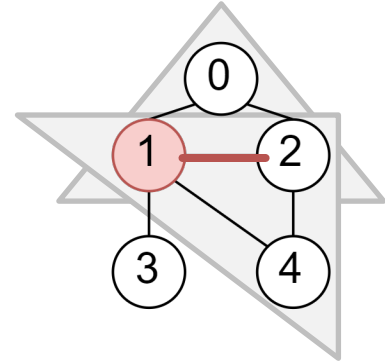
# Algorithm overview: Shared memory computation

## 1 Distribution



## 2 Local TC computation

$$LCC(\bullet) = \frac{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \circ \end{array}|}{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagdown \\ \circ \end{array}|}$$



$$\#triangles = |adj(v) \cap adj(w)|$$

**Binary search**  
 $O(|adj(v)| \log(|adj(w)|))$

**VS**  
 $\frac{|adj(v)|}{|adj(w)|} > B$

**Sorted Set Intersection**  
 $O(|adj(v)| + |adj(w)|)$

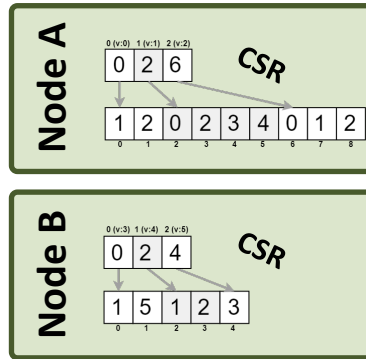
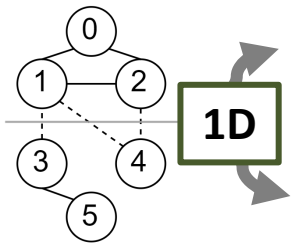
Hybrid method decreases running time by **up to 8%**

On shared memory **2.7x speedup** using 16 threads



# Algorithm overview: Shared memory computation

## 1 Distribution



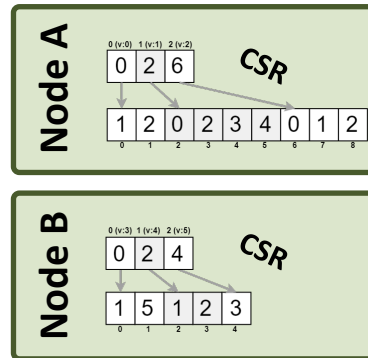
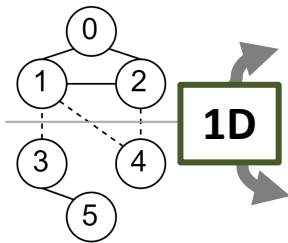
## 2 Local TC computation

$$LCC(\bullet) = \frac{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \bullet \quad \bullet \end{array}|}{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \bullet \end{array}|}$$

- Shared memory parallel
- Hybrid method

# Algorithm overview: Distributed memory algorithm

## 1 Distribution



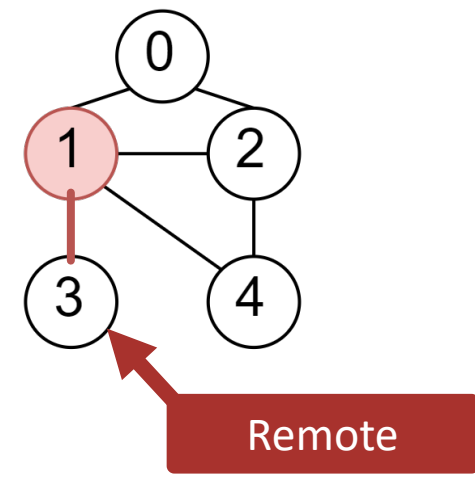
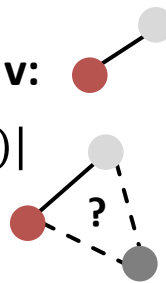
## 2 Local TC computation

$$LCC(\bullet) = \frac{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \circ \end{array}|}{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \circ \end{array}|}$$

- Shared memory parallel
- Hybrid method

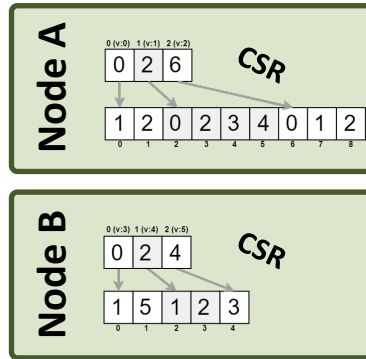
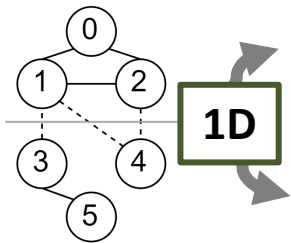
## 3 Asynchronous distributed memory algorithm

- For all local vertices  $v$ :  $\bullet$
- For all vertices  $w$  incident to  $v$ :  
 $\#triangles += |adj(v) \cap adj(w)|$



# Algorithm overview: Distributed memory algorithm

## 1 Distribution



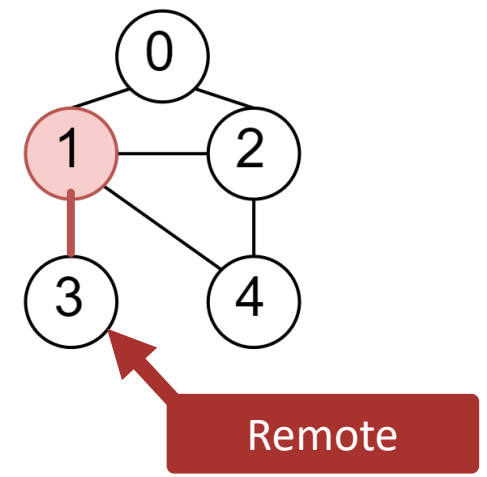
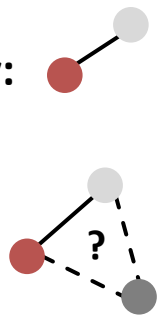
## 2 Local TC computation

$$LCC(\bullet) = \frac{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \circ \end{array}|}{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagdown \\ \circ \end{array}|}$$

- Shared memory parallel
- Hybrid method

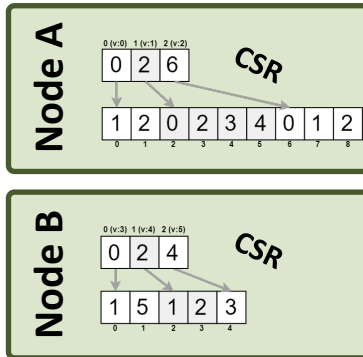
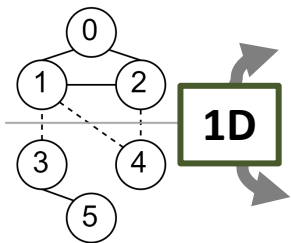
## 3 Asynchronous distributed memory algorithm

- For all local vertices  $v$ :  $\bullet$
- For all vertices  $w$  incident to  $v$ :  $\bullet$   
 If  $w$  is remote: Get  $adj(w)$   
 $\#triangles += |adj(v) \cap adj(w)|$



# Algorithm overview: MPI-RMA

## 1 Distribution



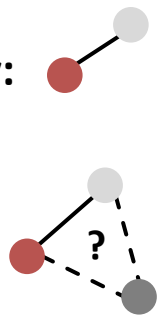
## 2 Local TC computation

$$LCC(\bullet) = \frac{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \circ \end{array}|}{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagdown \\ \circ \end{array}|}$$

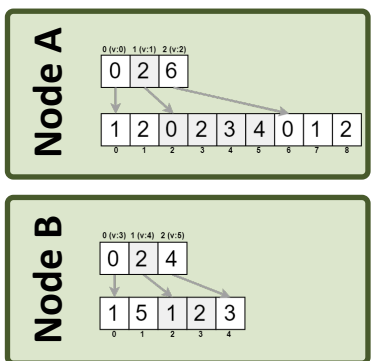
- Shared memory parallel
- Hybrid method

## 3 Asynchronous distributed memory algorithm

1. For all local vertices  $v$ : ●
2. For all vertices  $w$  incident to  $v$ : ●  
 If  $w$  is remote: Get  $adj(w)$   
 $\#triangles += |adj(v) \cap adj(w)|$

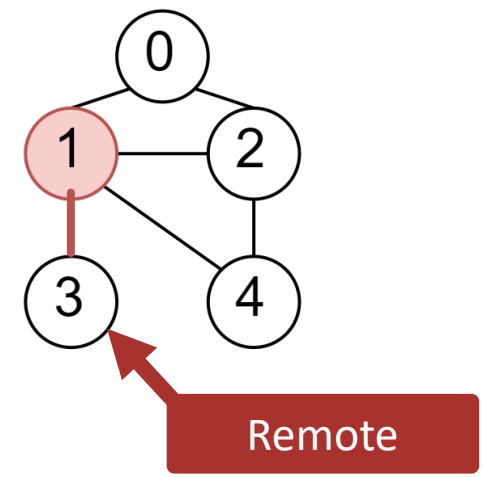


## 4 Communication



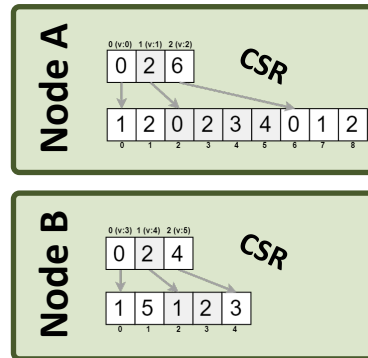
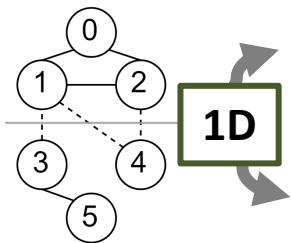
**MPI-RMA**

- Non-blocking communication
- Target process is not involved
- Hardware support
- Core elements:
  - MPI Window
  - MPI\_Get(window, target, offset, size);



# Algorithm overview: MPI-RMA

## 1 Distribution



## 2 Local TC computation

$$LCC(\bullet) = \frac{\left| \forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \circ \end{array} \right|}{\left| \forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagdown \\ \circ \end{array} \right|}$$

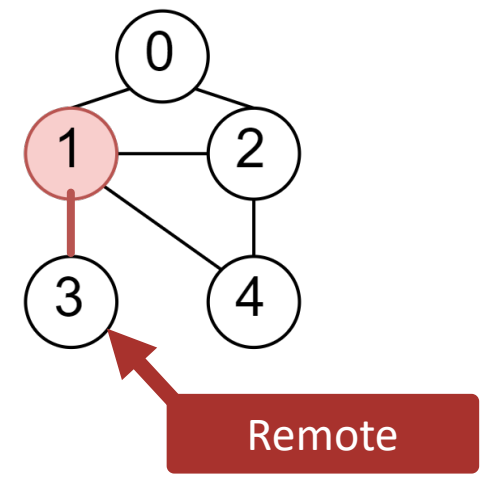
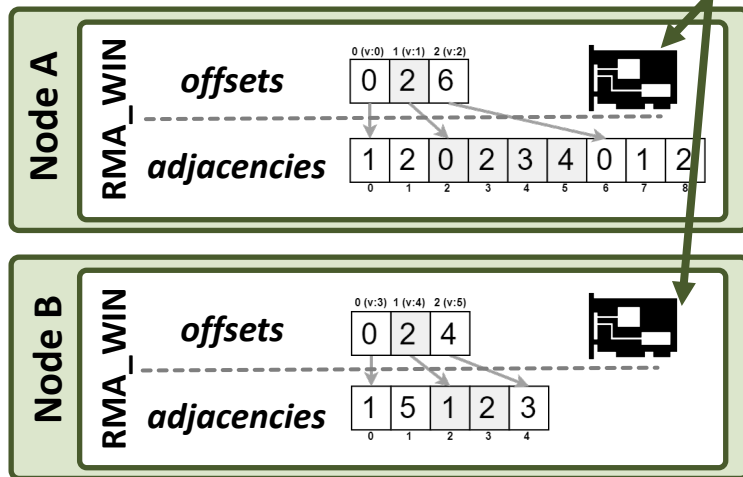
- Shared memory parallel
- Hybrid method

## 3 Asynchronous distributed memory algorithm

- For all local vertices  $v$ : ●
- For all vertices  $w$  incident to  $v$ : ●
  - If  $w$  is remote: Get  $adj(w)$
  - #triangles +=  $|adj(v) \cap adj(w)|$

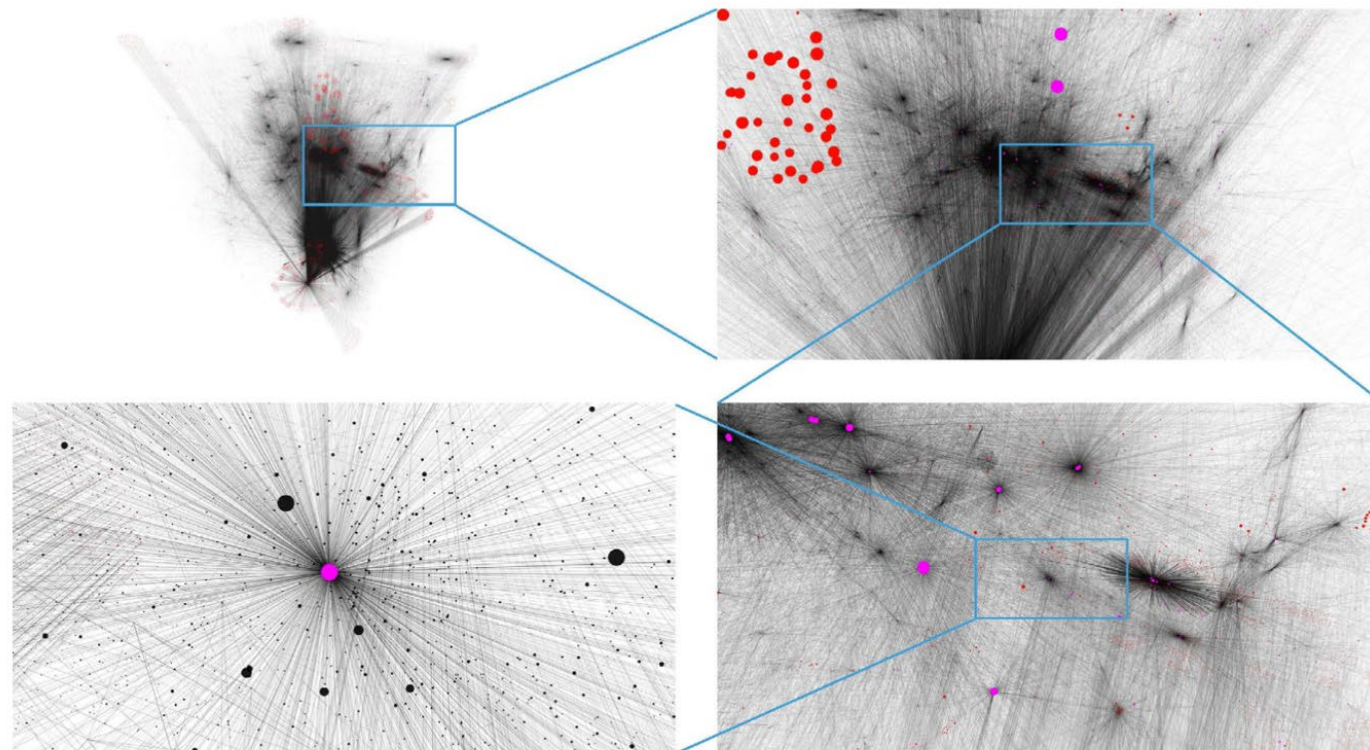
## 4 Communication

Global view of the graph



## Challenges: Graphs are huge and skewed

- Billions of vertices and hundreds of billions of edges
- Scale-free degree distribution

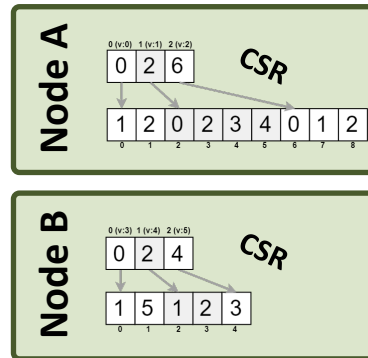
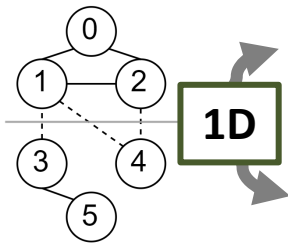


Right: Albert-László Barabási: Network Science (Chapter 4)

Exploit temporal locality by caching RMA reads

# Algorithm overview: Caching with CLaMPI

## 1 Distribution



## 2 Local TC computation

$$LCC(\bullet) = \frac{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \circ \end{array}|}{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagdown \\ \circ \end{array}|}$$

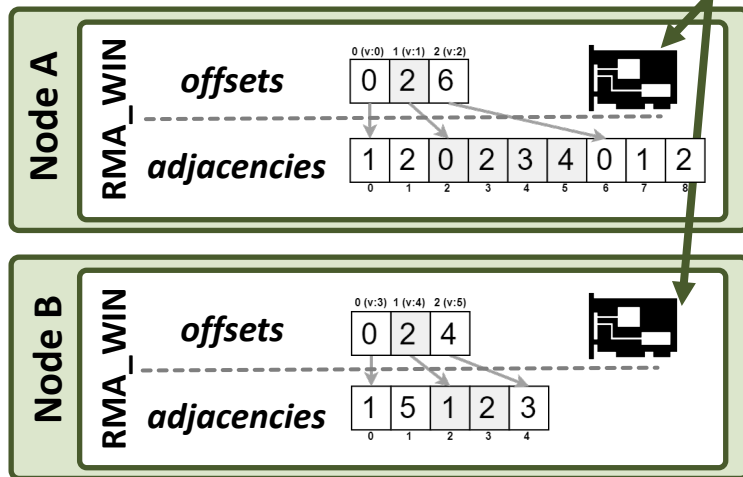
- Shared memory parallel
- Hybrid method

## 3 Asynchronous distributed memory algorithm

- For all local vertices  $v$ :  $\bullet$
- For all vertices  $w$  incident to  $v$ :  $\bullet$   
 If  $w$  is remote: Get  $adj(w)$   
 $\#triangles += |adj(v) \cap adj(w)|$

## 4 Communication

Global view of the graph



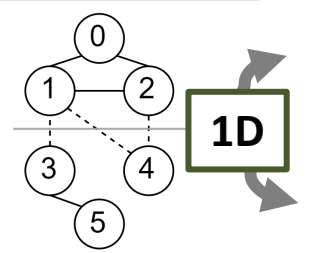
## 5 Data reuse

**CLaMPI – RMA cache**

- Transparent caching layer
- Supports variable size reads

# Algorithm overview: Caching with CLaMPI

## 1 Distribution



**Node A**

0 (v:0)	1 (v:1)	2 (v:2)	
0	2	6	CSR
1	2	0	2
2	3	4	0
3	4	0	1
4	0	1	2

**Node B**

0 (v:3)	1 (v:4)	2 (v:5)	
0	2	4	CSR
1	5	1	2
2	3	4	

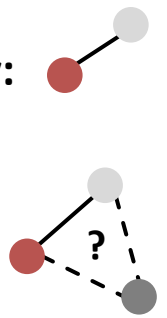
## 2 Local TC computation

$$LCC(\bullet) = \frac{\left| \forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \circ \end{array} \right|}{\left| \forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagdown \\ \circ \end{array} \right|}$$

- Shared memory parallel
- Hybrid method

## 3 Asynchronous distributed memory algorithm

- For all local vertices  $v$ :
- For all vertices  $w$  incident to  $v$ :  
*If  $w$  is remote: Get  $adj(w)$*   
 $\#triangles += |adj(v) \cap adj(w)|$



## 4 Communication

Global view of the graph

**Node A**

<b>RMA_WIN</b>	<b>offsets</b>	0 (v:0)	1 (v:1)	2 (v:2)
		0	2	6
<b>adjacencies</b>		1	2	0
		2	3	4
		0	1	2

**Node B**

<b>RMA_WIN</b>	<b>offsets</b>	0 (v:3)	1 (v:4)	2 (v:5)
		0	2	4
<b>adjacencies</b>		1	5	1
		2	3	4

## 5 Data reuse

	window	node	offset	size	data
<b>CLaMPI cache</b>	offsets	B	0	2	0 2
	adjacencies	B	0	2	1 5
	...	...	...	...	...

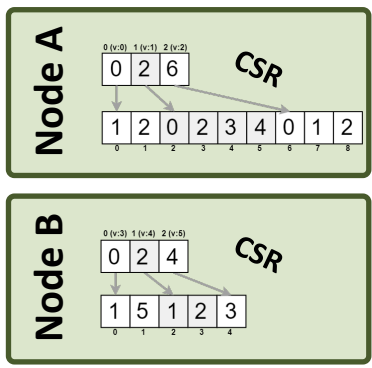
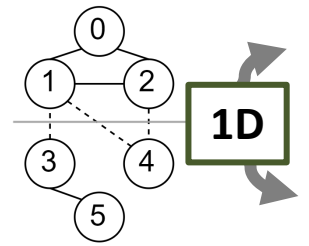
	window	node	offset	size	data
<b>CLaMPI cache</b>	offsets	A	0	2	2 6
	adjacencies	A	0	2	0 2 3 4
	...	...	...	...	...

Frequently accessed subgraph (redundant)



# Algorithm overview: Caching with CLaMPI

## 1 Distribution



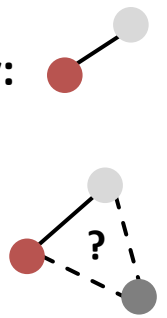
## 2 Local TC computation

$$LCC(\bullet) = \frac{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \circ \end{array}|}{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagdown \\ \circ \end{array}|}$$

- Shared memory parallel
- Hybrid method

## 3 Asynchronous distributed memory algorithm

- For all local vertices  $v$ :
- For all vertices  $w$  incident to  $v$ :  
 If  $w$  is remote: Get  $adj(w)$   
 $\#triangles += |adj(v) \cap adj(w)|$



## 4 Communication

Global view of the graph

<b>Node A</b>	<b>RMA_WIN</b>	<b>offsets</b>	0 (v:0) 1 (v:1) 2 (v:2)	0 2 6	
		<b>adjacencies</b>		1 2 0 2 3 4 0 1 2	
<b>Node B</b>	<b>RMA_WIN</b>	<b>offsets</b>	0 (v:3) 1 (v:4) 2 (v:5)	0 2 4	
		<b>adjacencies</b>		1 5 1 2 3	

<b>CLaMPI cache</b>	<b>window</b>	<b>node</b>	<b>offset</b>	<b>size</b>	<b>data</b>
	offsets	B	0	2	0 2
	adjacencies	B	0	2	1 5
	...	...	...	...	...

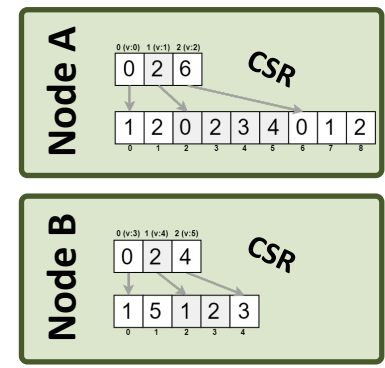
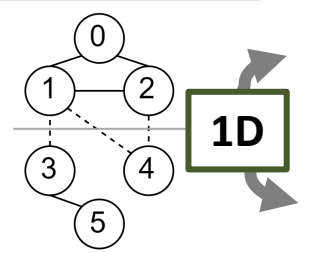
  

<b>CLaMPI cache</b>	<b>window</b>	<b>node</b>	<b>offset</b>	<b>size</b>	<b>data</b>
	offsets	A	0	2	2 6
	adjacencies	A	0	2	0 2 3 4
	...	...	...	...	...

- User defined score
  - Improvement between **14.4% and 35.6%**

# Algorithm overview

## 1 Distribution



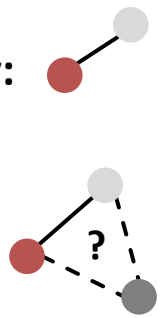
## 2 Local TC computation

$$LCC(\bullet) = \frac{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \circ \end{array}|}{|\forall (\bullet, \circ), \text{ s.t.: } \begin{array}{c} \bullet \\ \diagdown \\ \circ \end{array}|}$$

- Shared memory parallel
- Hybrid method

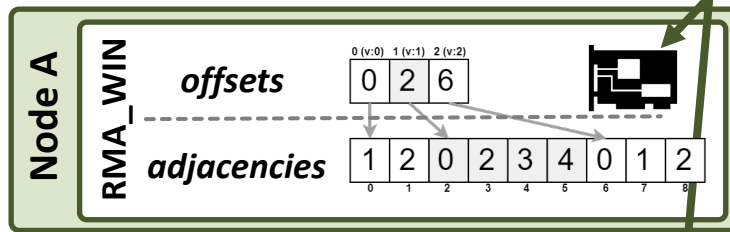
## 3 Asynchronous distributed memory algorithm

- For all local vertices  $v$ :
- For all vertices  $w$  incident to  $v$ :  
 If  $w$  is remote: Get  $adj(w)$   
 $\#triangles += |adj(v) \cap adj(w)|$

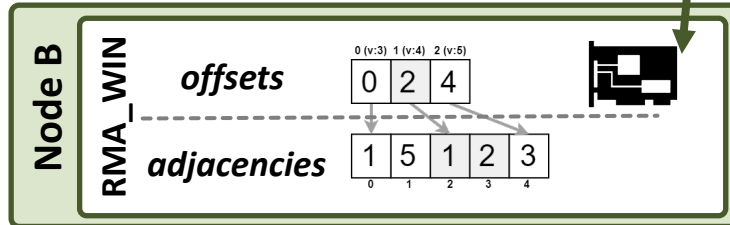


## 4 Communication

Global view of the graph



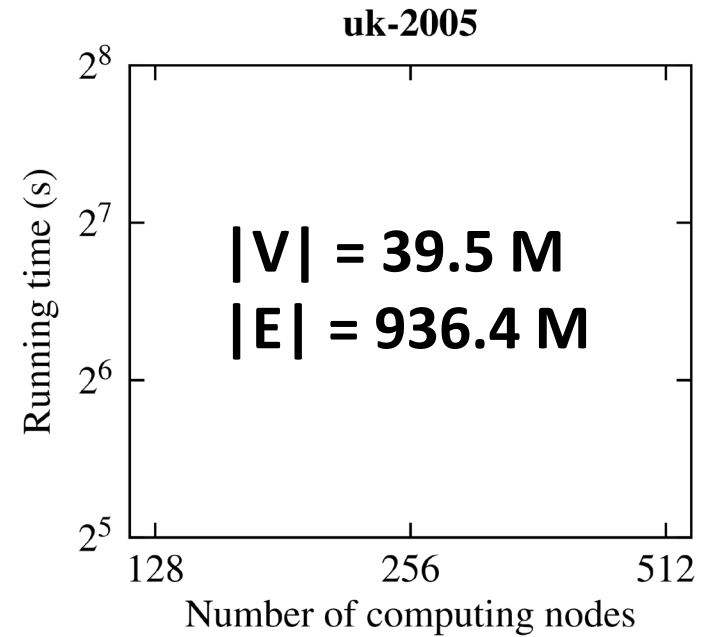
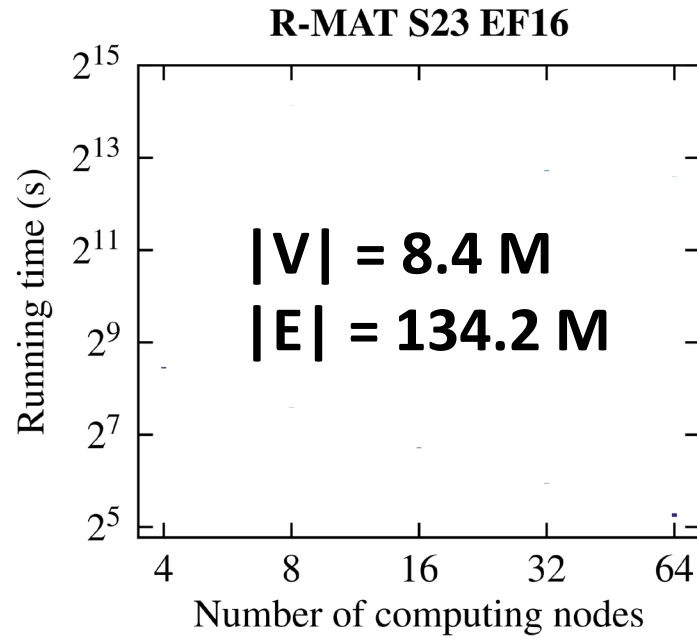
	window	node	offset	size	data
<b>CLaMPI cache</b>	offsets	B	0	2	0 2
	adjacencies	B	0	2	1 5
	...	...	...	...	...



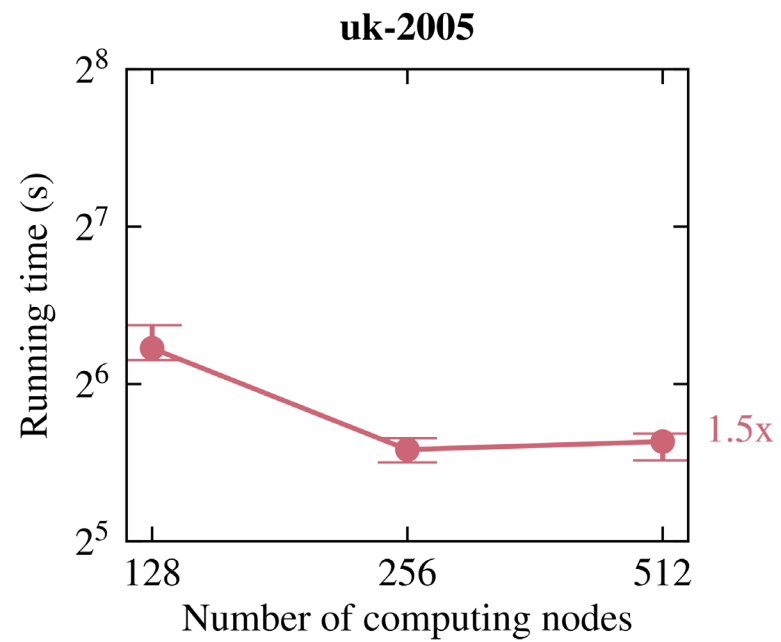
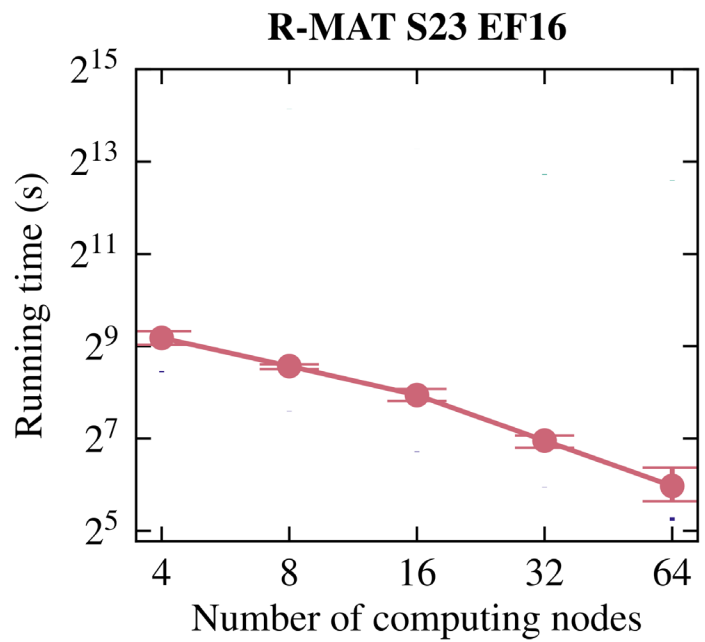
	window	node	offset	size	data
<b>CLaMPI cache</b>	offsets	A	0	2	2 6
	adjacencies	A	0	2	0 2 3 4
	...	...	...	...	...

Frequently accessed subgraph (redundant)

# Results



# Results

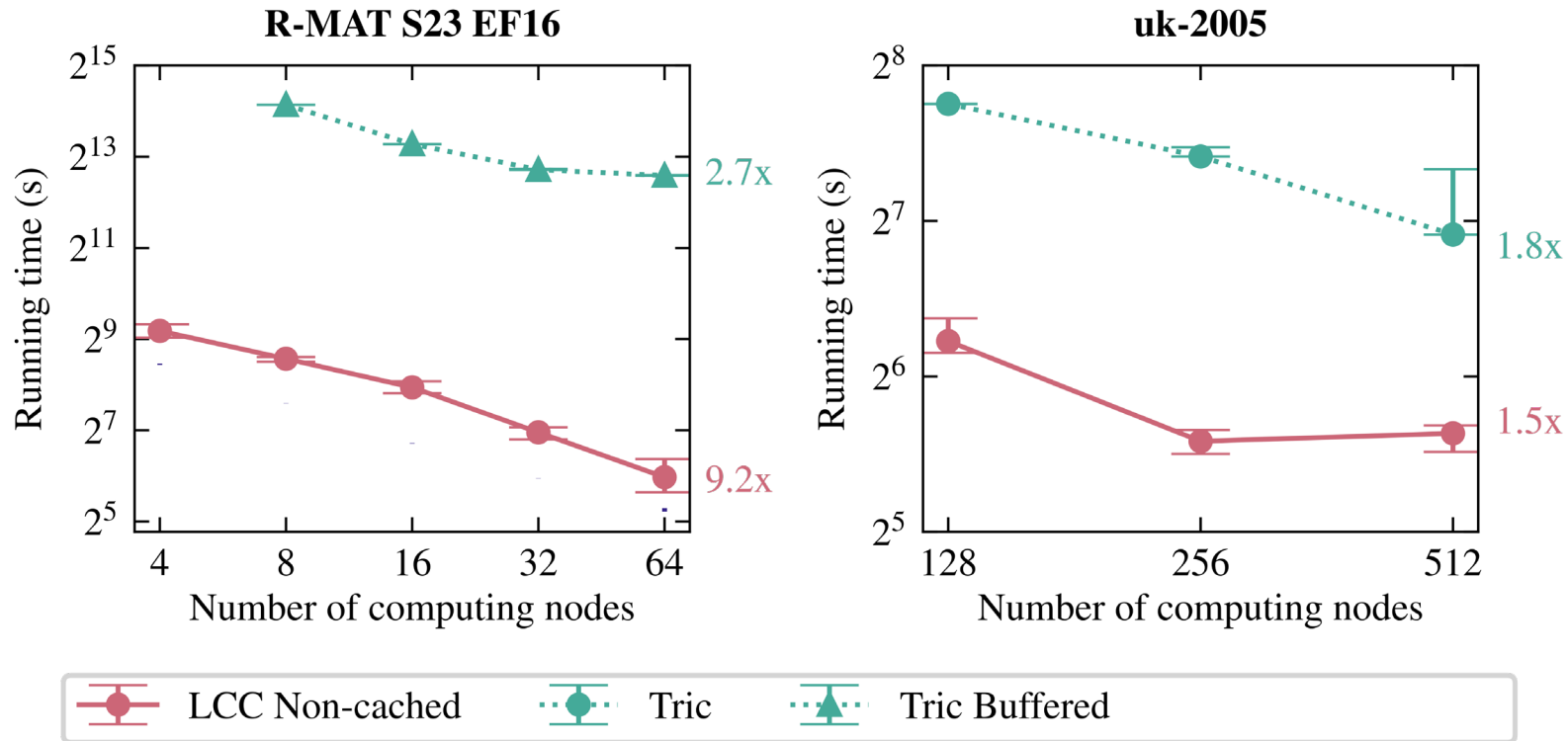


LCC Non-cached

# Results

**Better scaling,**  
especially for highly  
skewed graphs

In general, **4-12x**  
**faster** results, best  
results show **up to**  
**100x speedup**

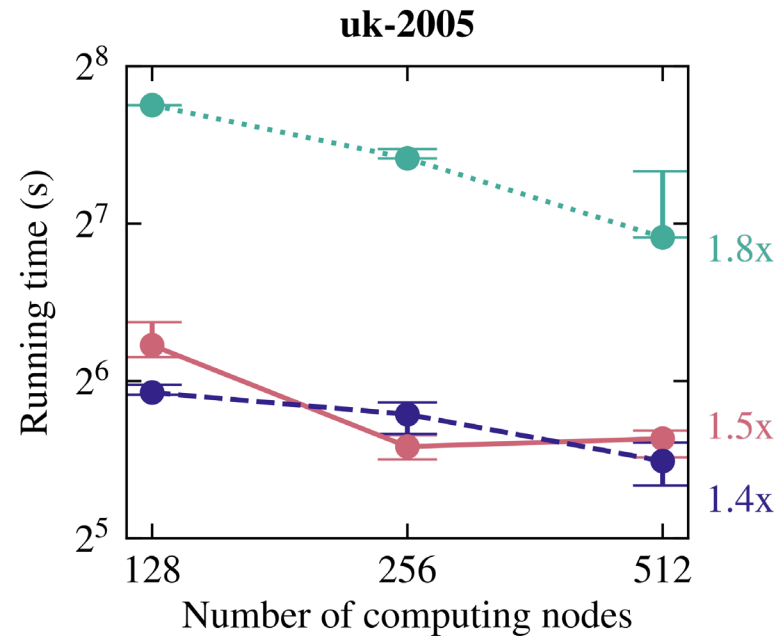
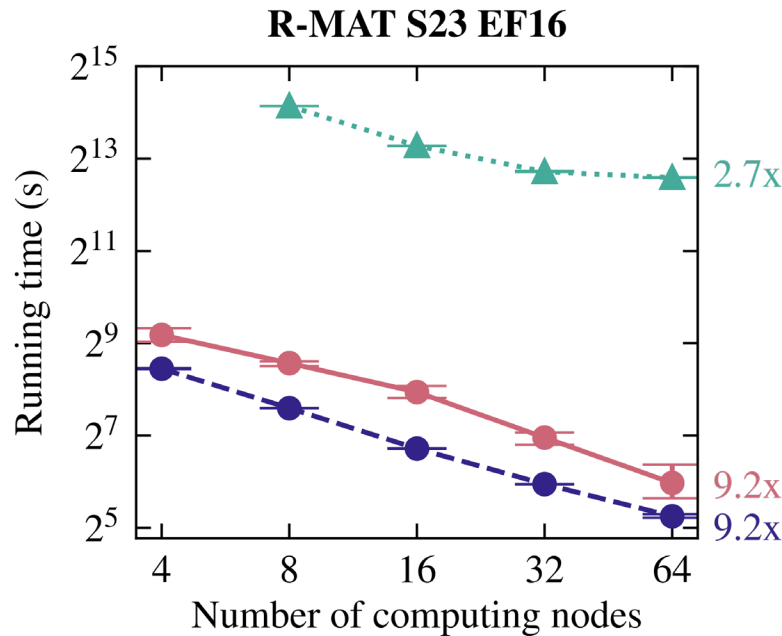


## Eliminating synchronization overheads

# Results

**Better scaling,**  
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In general, **4-12x**  
**faster** results, best  
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**100x speedup**



**Caching reduces**  
runtime with up  
to **73%**



**Eliminating synchronization overheads**



**Vertex delegation by caching**

# Results

- Asynchronous processing
- Caching performance
- Outlook
  - Different partitioning schemes
  - Caching in other graph computations
  - Application specific eviction procedure

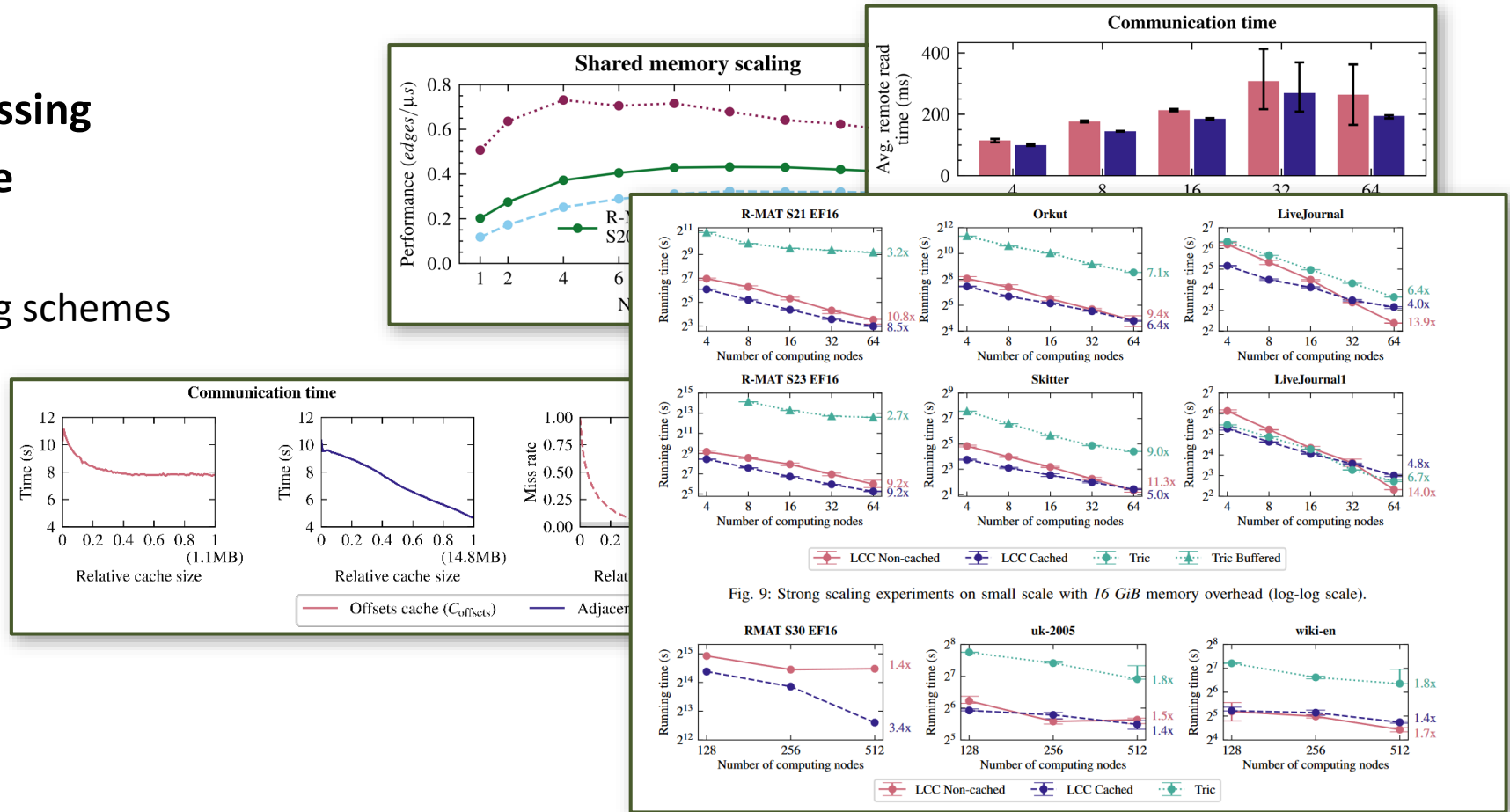


Fig. 9: Strong scaling experiments on small scale with 16 GiB memory overhead (log-log scale).

**Further results and analysis in the paper!**

# Asynchronous Distributed-Memory Triangle Counting and LCC with RMA Caching

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### Warm-up: Local Clustering Coefficient

- Graphs represent relational data very well
- LCC: likelihood that neighbors of a vertex are connected

$$LCC(v) = \frac{|\{(\omega, \theta), s.t.: \omega, \theta \text{ neighbors of } v, \omega \text{ and } \theta \text{ connected}\}|}{|\{(\omega, \theta), s.t.: \omega, \theta \text{ neighbors of } v\}|}$$

Count triangles!  
Degrees are known

### Challenges: Graphs are huge and skewed

- Billions of vertices and hundreds of billions of edges
- Scale-free degree distribution

### Distributed-memory TC & LCC computing

**Current state-of-the-art:**

- Synchronized computation
  - Bulk Synchronous Parallel
  - MapReduce
- Frontier intersection
- Graph partitioning
  - Static vertex delegation

**Our work proposes:**

- Fully asynchronous algorithm based on MPI-RMA
- Hybrid strategy for local TC
- Exploiting data reuse with caching  
Application-specific eviction policy

In general, 4-12x faster results for scale free graphs compared to TriC  
Best results show up to 100x speedup

### Algorithm overview

- Distribution**: Graph partitioned into Node A and Node B.
- Local TC computation**: Local LCC calculation on each node.
  - Shared memory parallel
  - Hybrid method
- Asynchronous distributed memory algorithm**:
  - For all local vertices  $v$ :
  - For all vertices  $w$  incident to  $v$ :  
If  $w$  is remote: Get  $adj(w)$   
#triangles +=  $|adj(v) \cap adj(w)|$
- Communication**: Global view of the graph.
- Data reuse**: Frequently accessed subgraph (redundant).

### Results

**Better scaling, especially for highly skewed graphs**

**In general, 4-12x faster results, best results show up to 100x speedup**

**Caching reduces runtime with up to 73%**

**Eliminating synchronization overheads**

**Vertex delegation by caching**

System specification: Cray XE6, Intel® Xeon® E5-2690 v3, 64 GBs per node, Cray's Aries interconnect (dragonfly topology), ICC 19.1 with -O3, cray-mpich 7.7.16 MPI

Thank you for your attention!