

# Lowering Diameter Enables Cost-Effective and High-Performance Networks

MACIEJ BESTA, ERIK HENRIKSSON, TORSTEN HOEFLER







50% [1]

[1] D. Abts et al. (2010), *Energy Proportional Datacenter Networks*, ISCA'10



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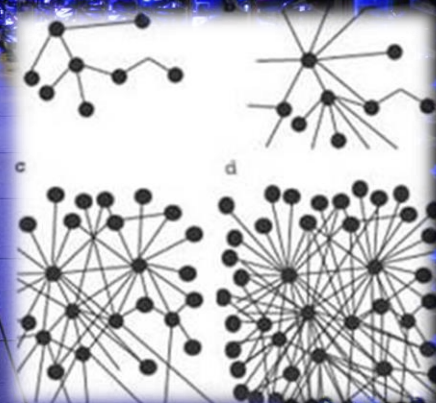
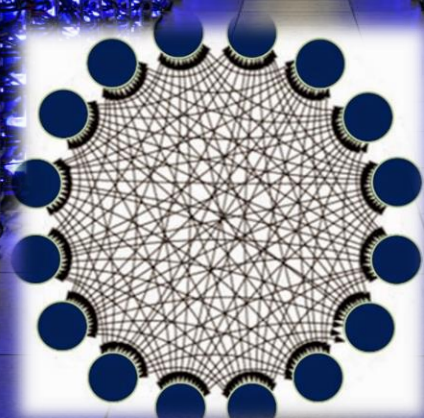
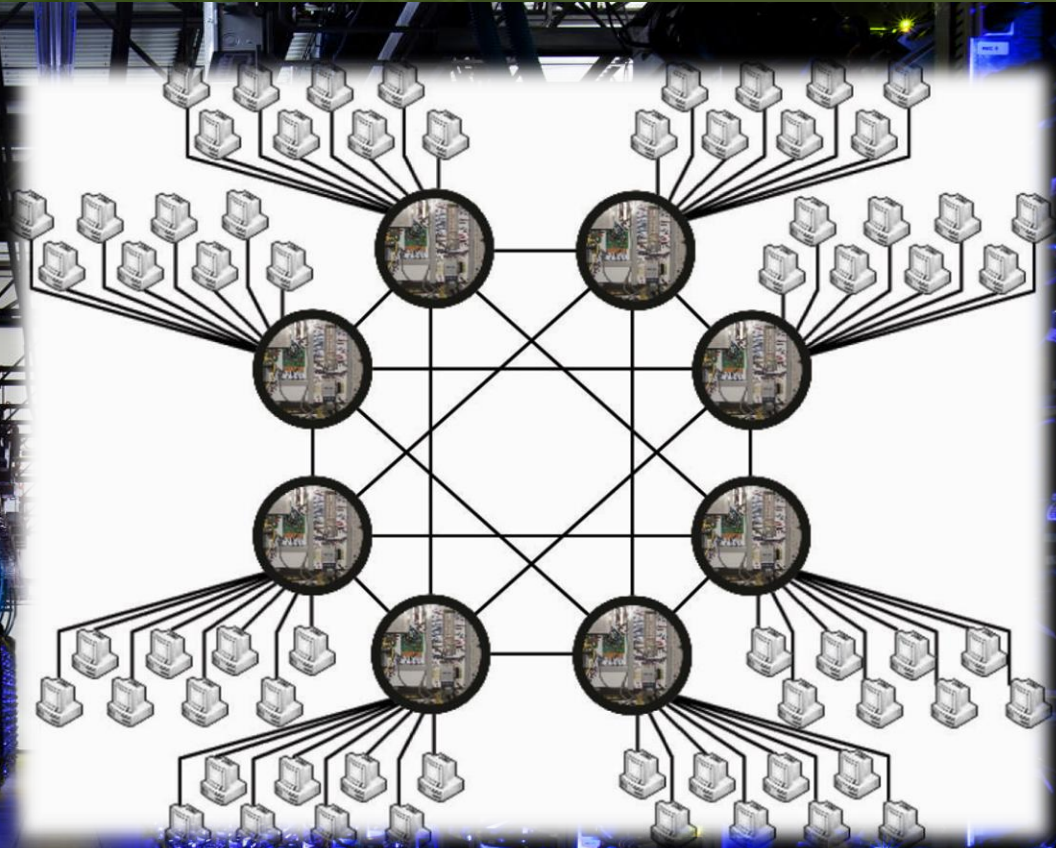
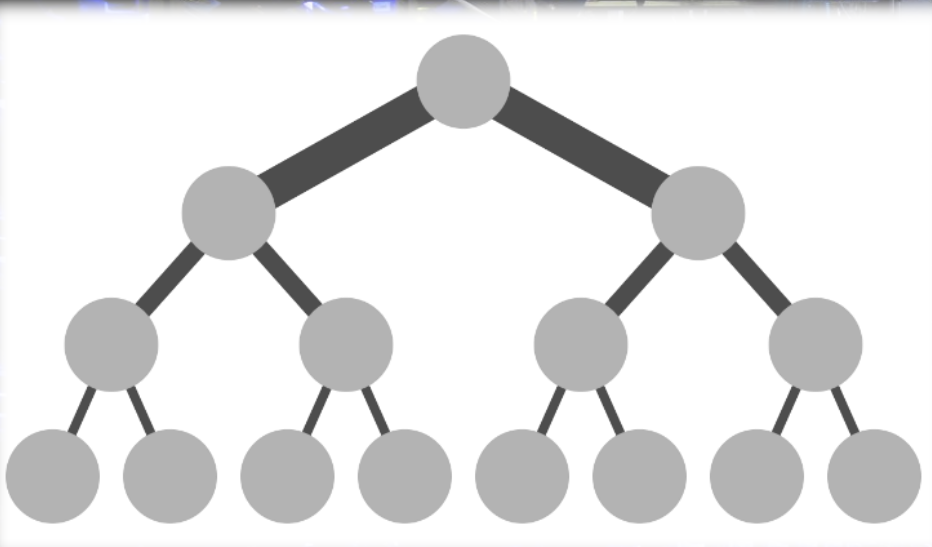


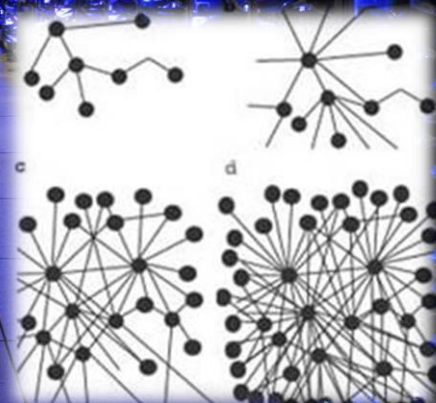
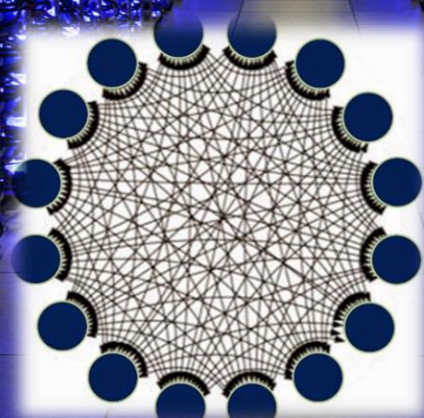
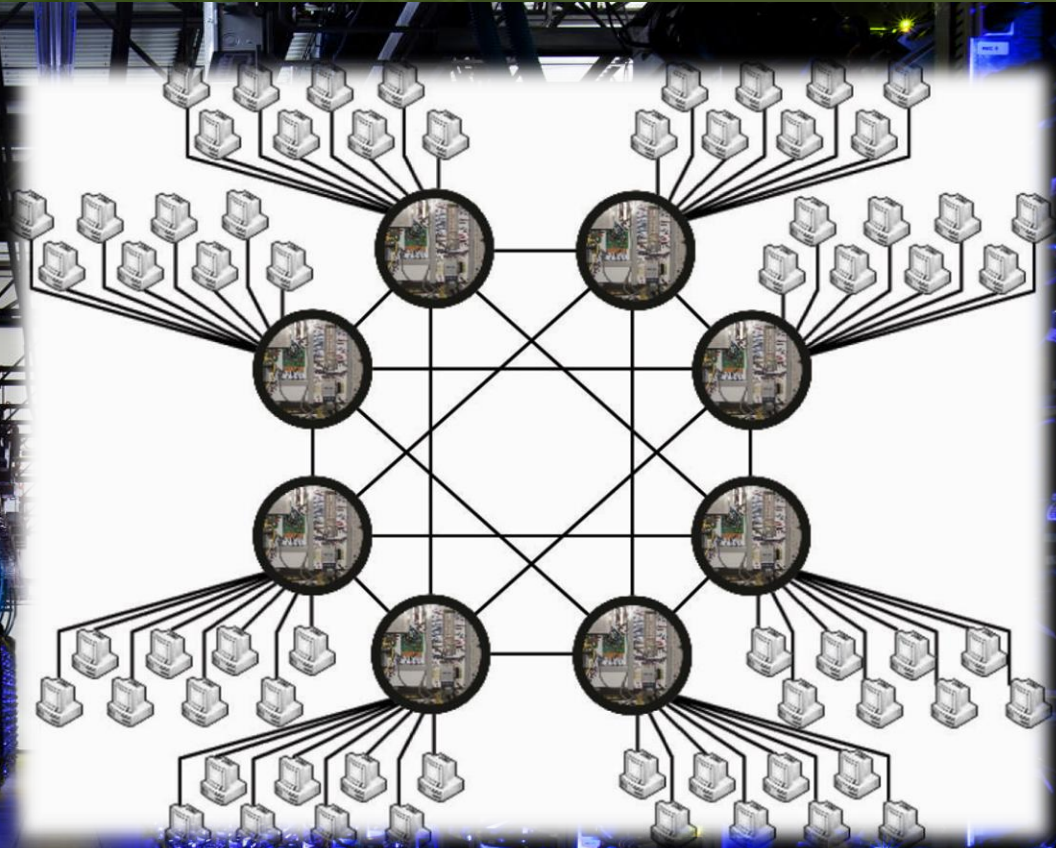
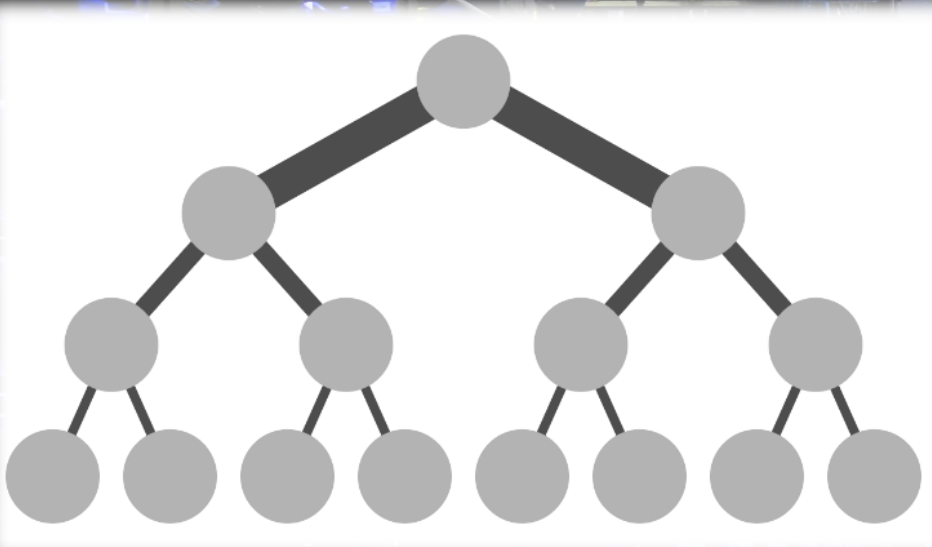
33% [2]

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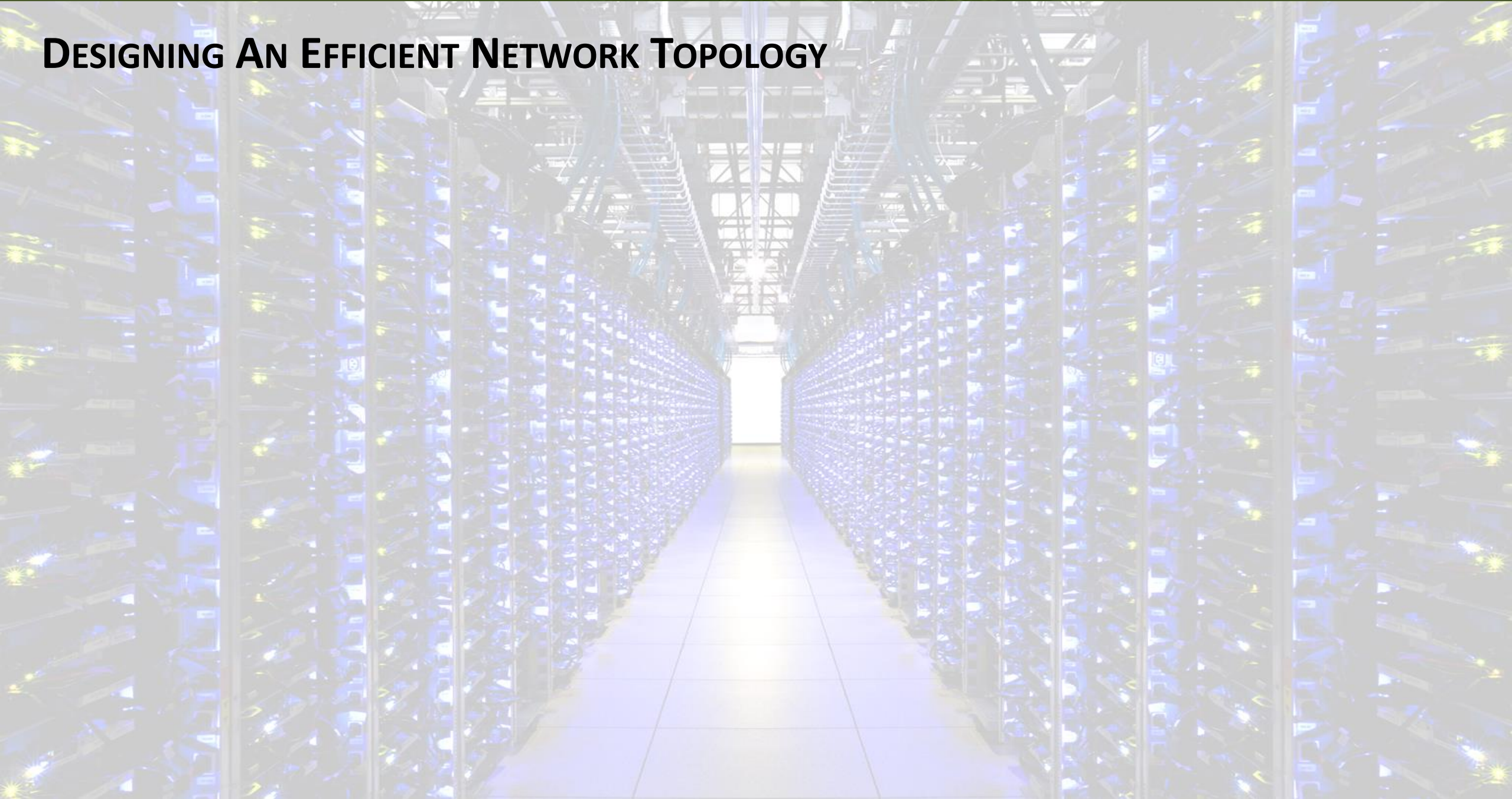
[2] J. Kim et al. (2007), *Flattened Butterfly: A Cost-Efficient Topology for High-Radix Networks*, ISCA'07







# DESIGNING AN EFFICIENT NETWORK TOPOLOGY





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Key idea



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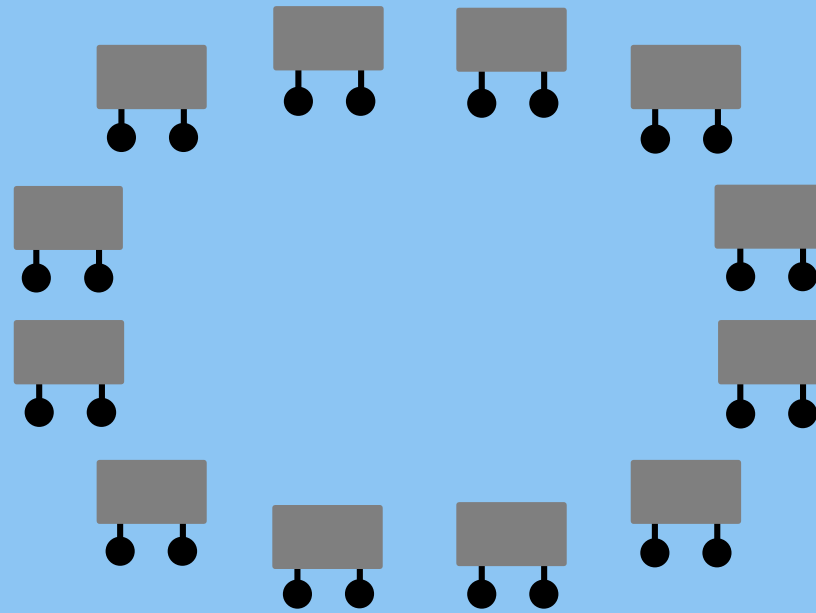
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fewer cables  
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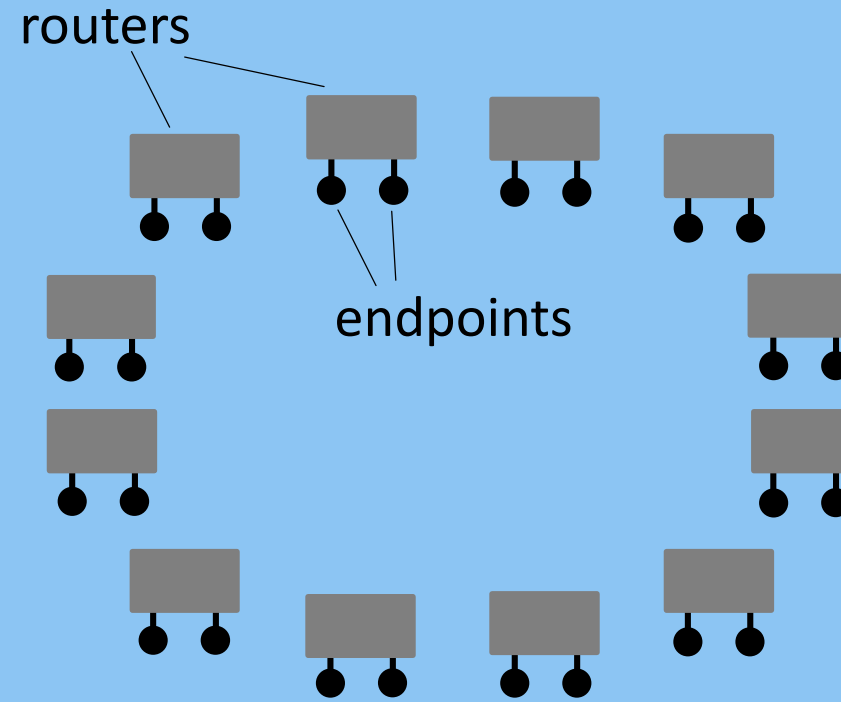


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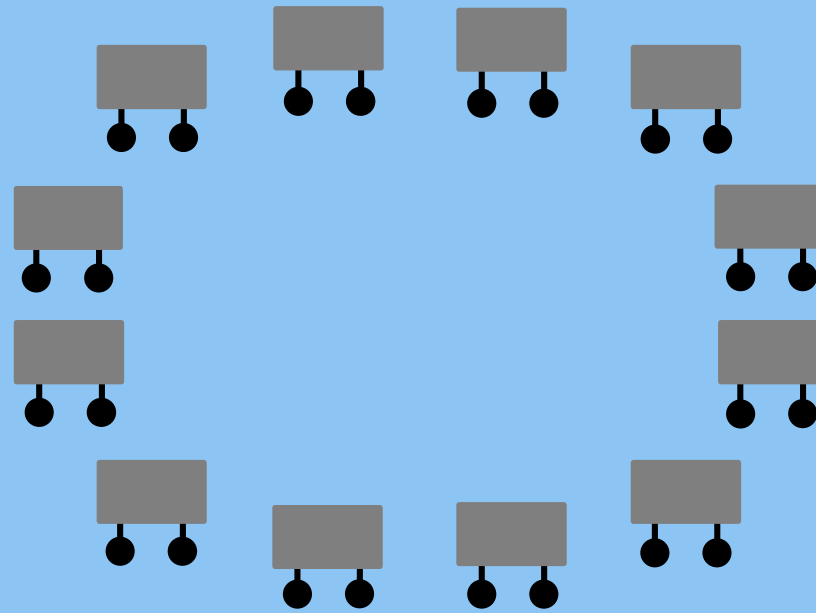


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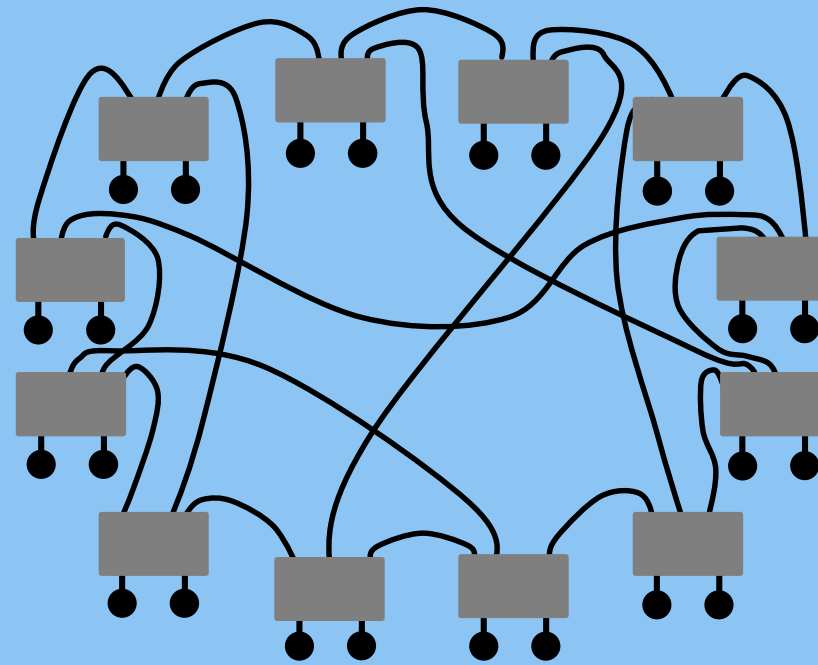


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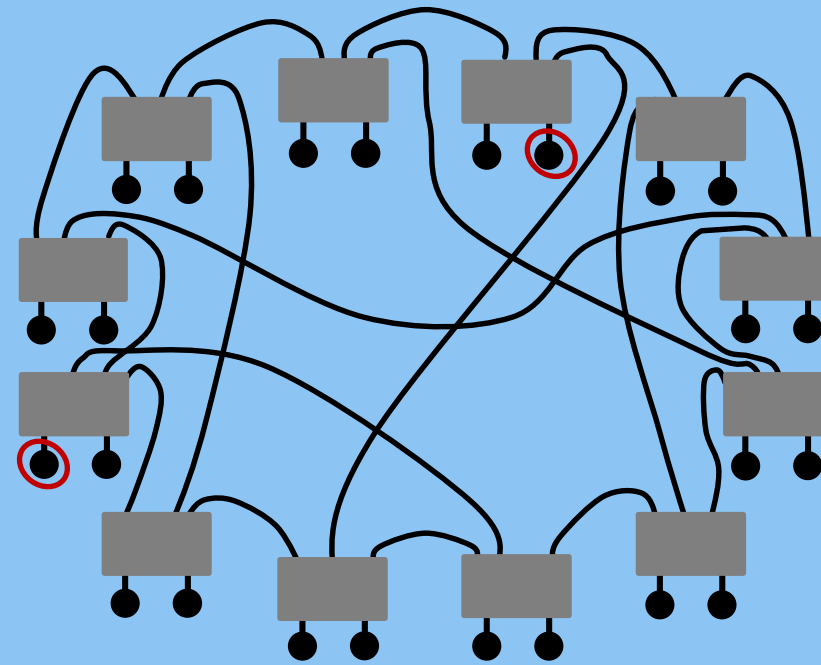


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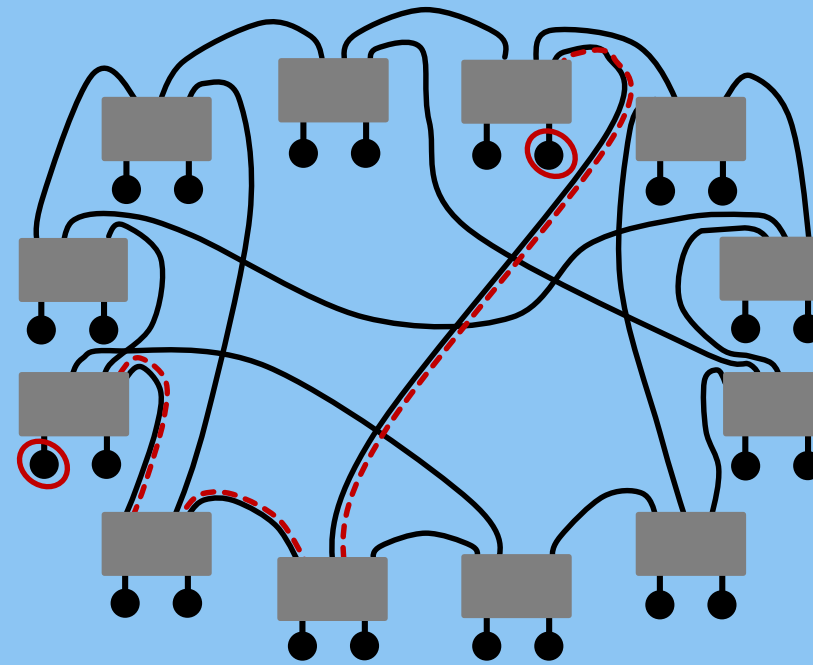


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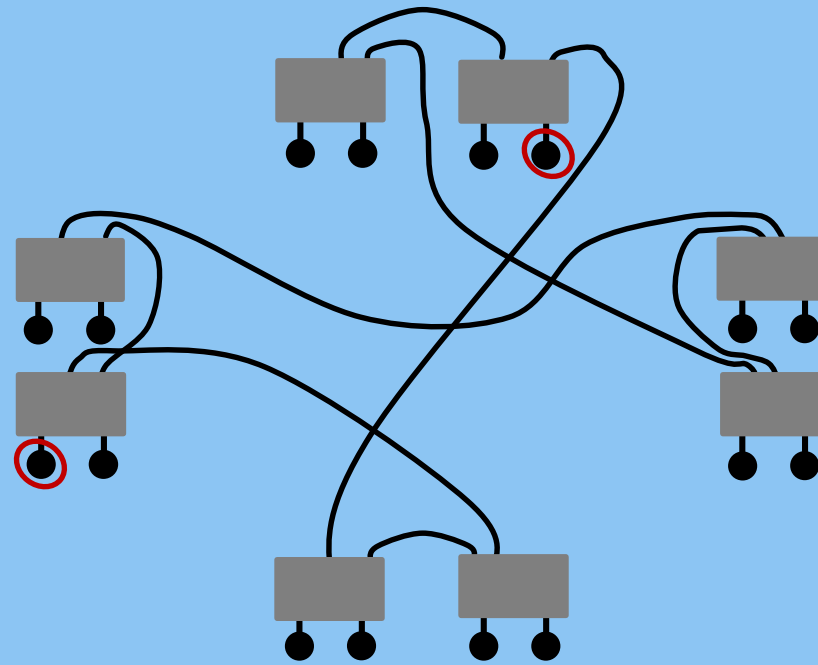


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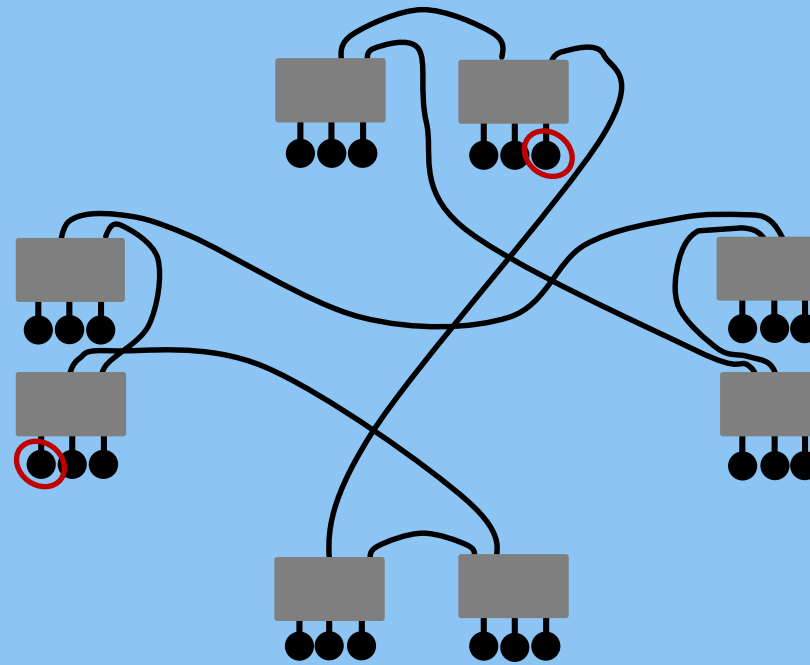


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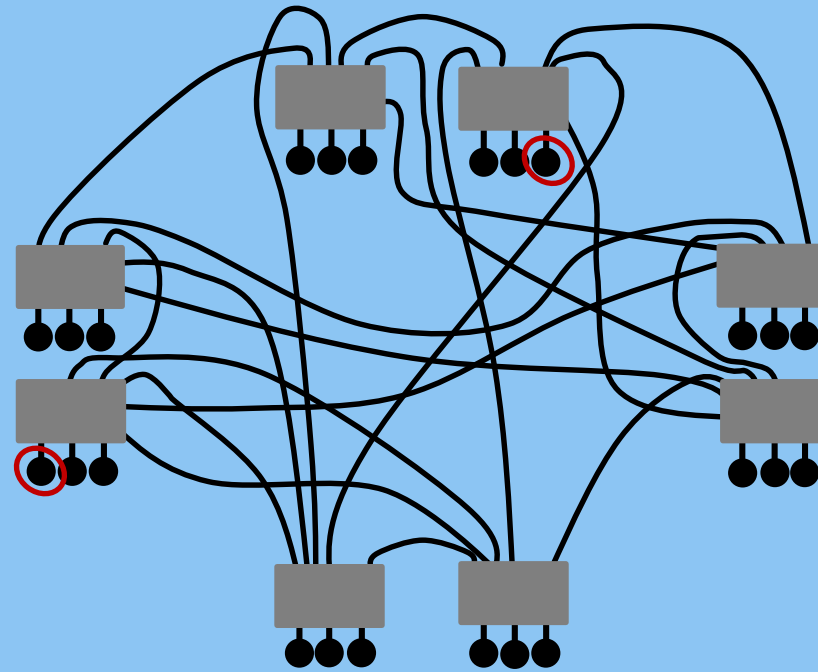


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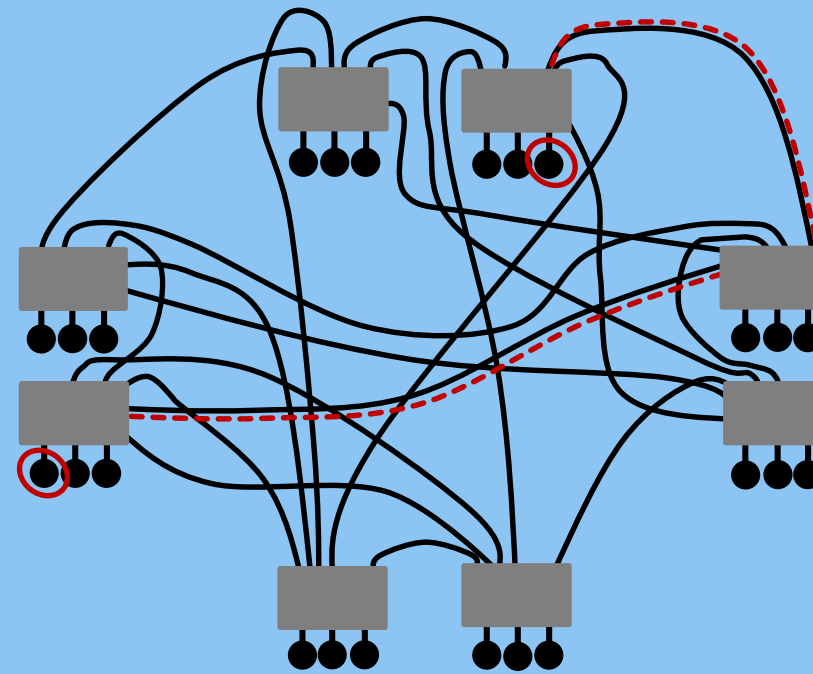


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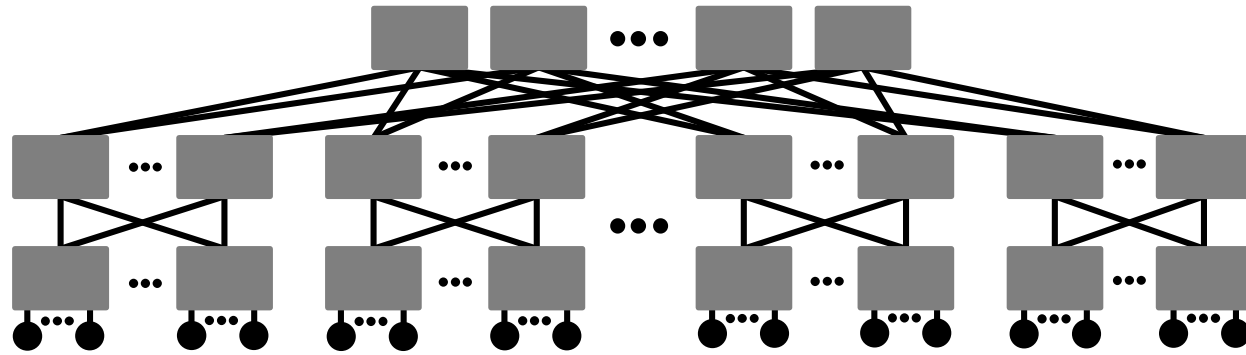
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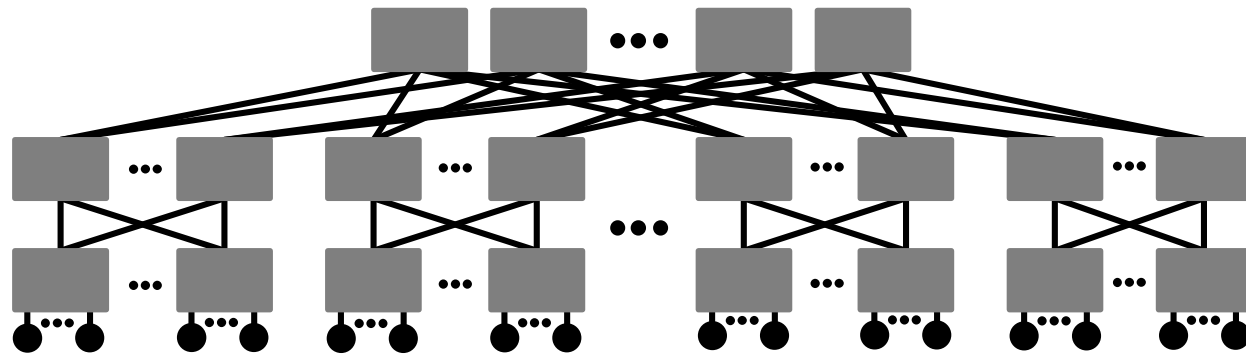
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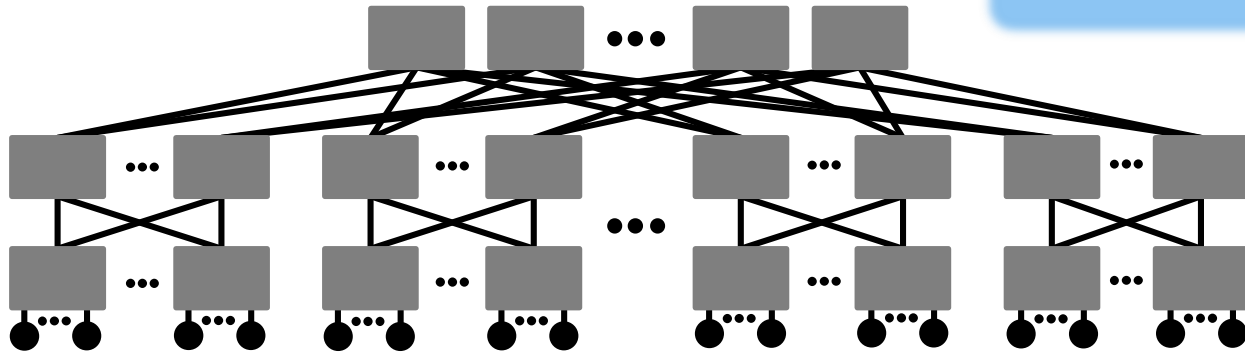


TSUBAME2.0

# DESIGNING AN EFFICIENT NETWORK TOPOLOGY

3-level fat tree [1]:

diameter = 4



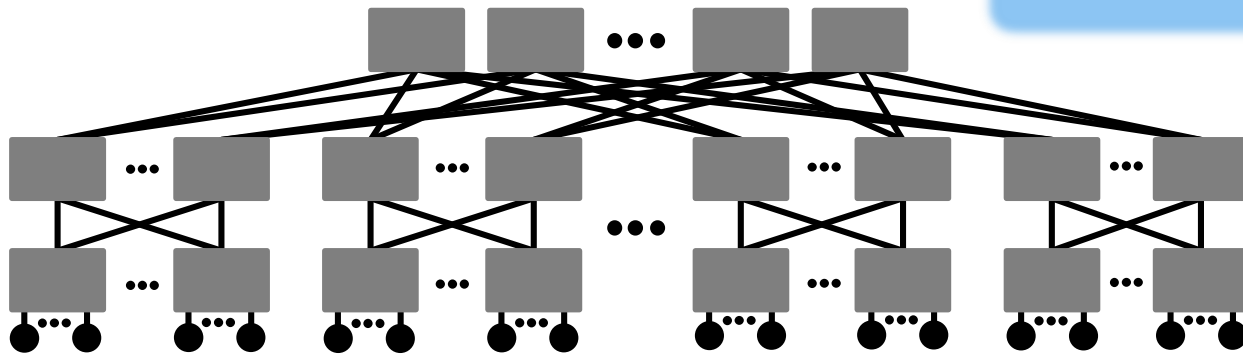
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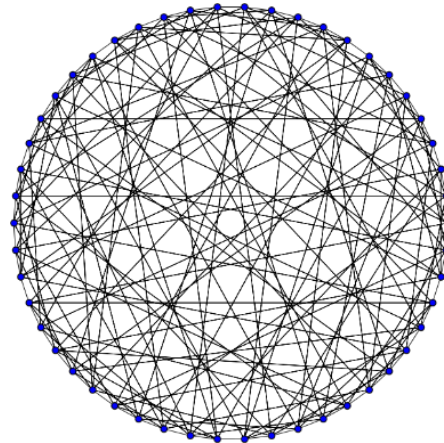
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Slim Fly [2] based on  
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[1] C. E. Leiserson. Fat-trees: Universal Networks for Hardware-Efficient Supercomputing. IEEE Transactions on Computers. 1985.

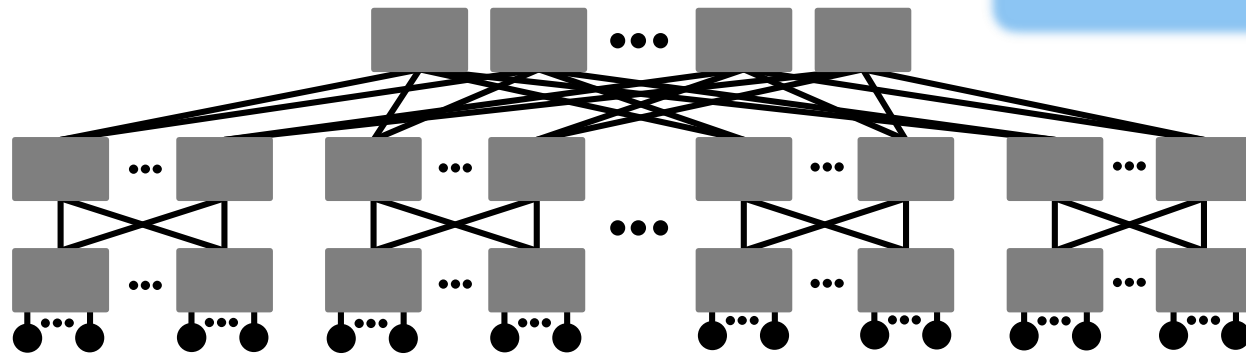
[2] M. Besta and T. Hoefler. Slim Fly: A Cost Effective Low-Diameter Network Topology. SC14.

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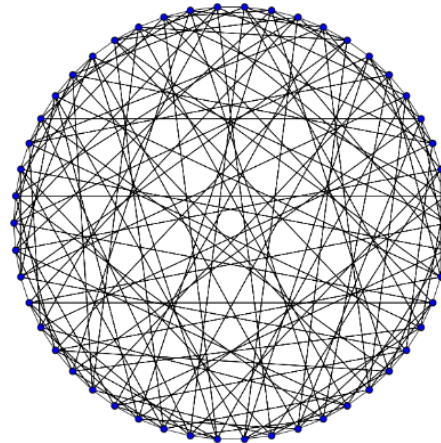
3-level fat tree [1]:

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**Slim Fly [2] based on the Hoffman-Singleton Graph [3]:**



diameter = 2  
> ~50% fewer routers  
> ~30% fewer cables

[1] C. E. Leiserson. Fat-trees: Universal Networks for Hardware-Efficient Supercomputing. IEEE Transactions on Computers. 1985.

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**Optimize towards the Moore Bound [1]:**  
the upper bound on the *number of vertices* in  
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$$MB(D, k) = 1$$



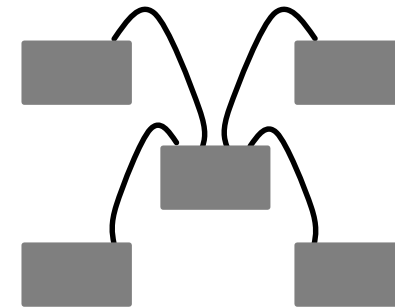


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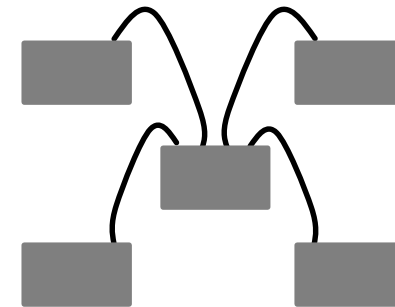


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$$MB(D, k) = 1 + k$$

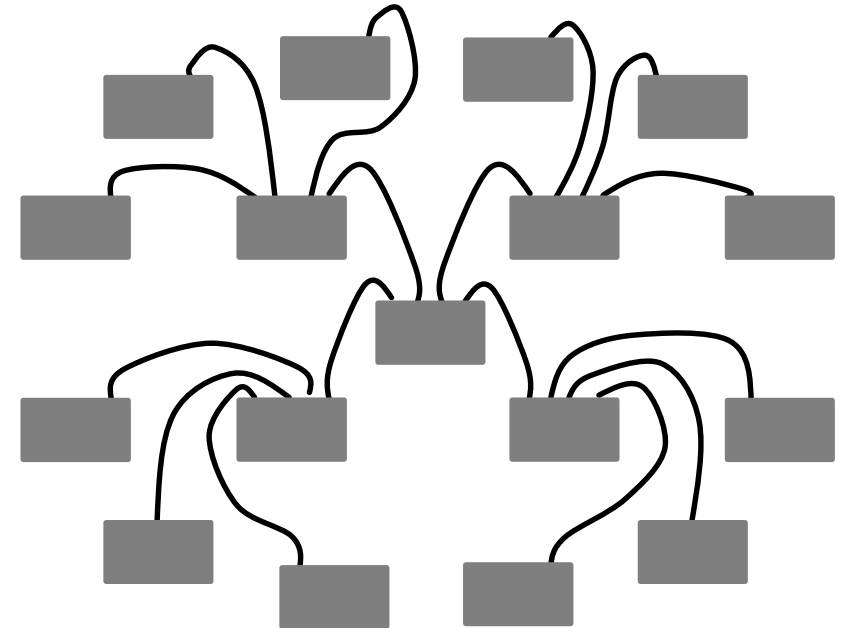


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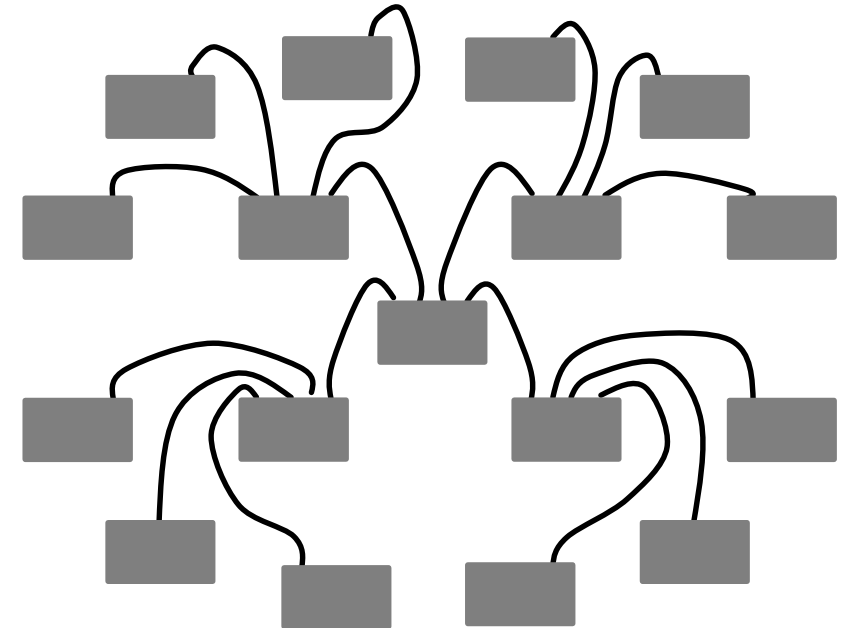


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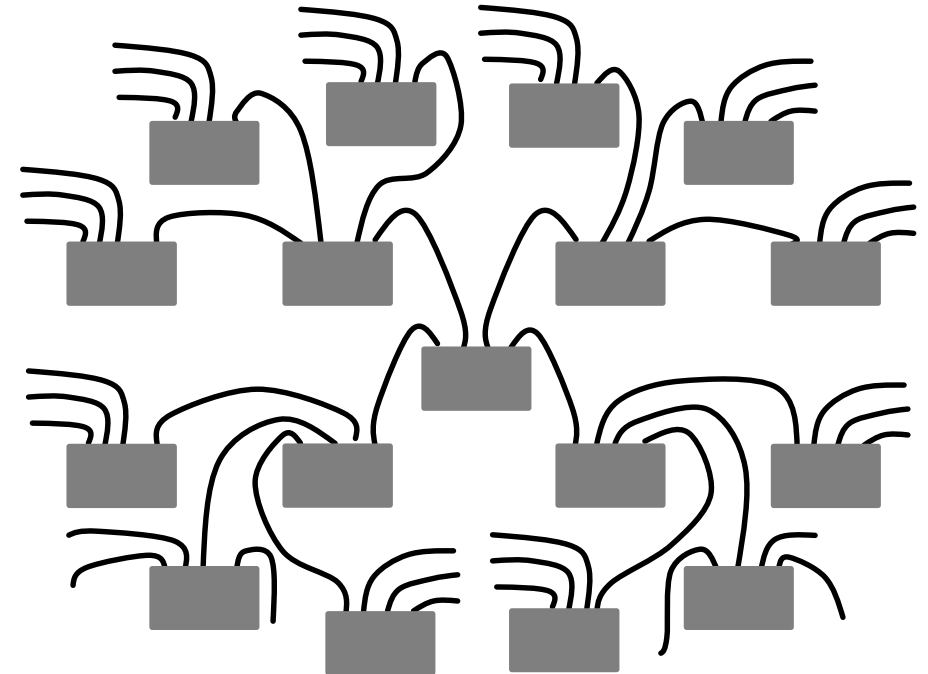


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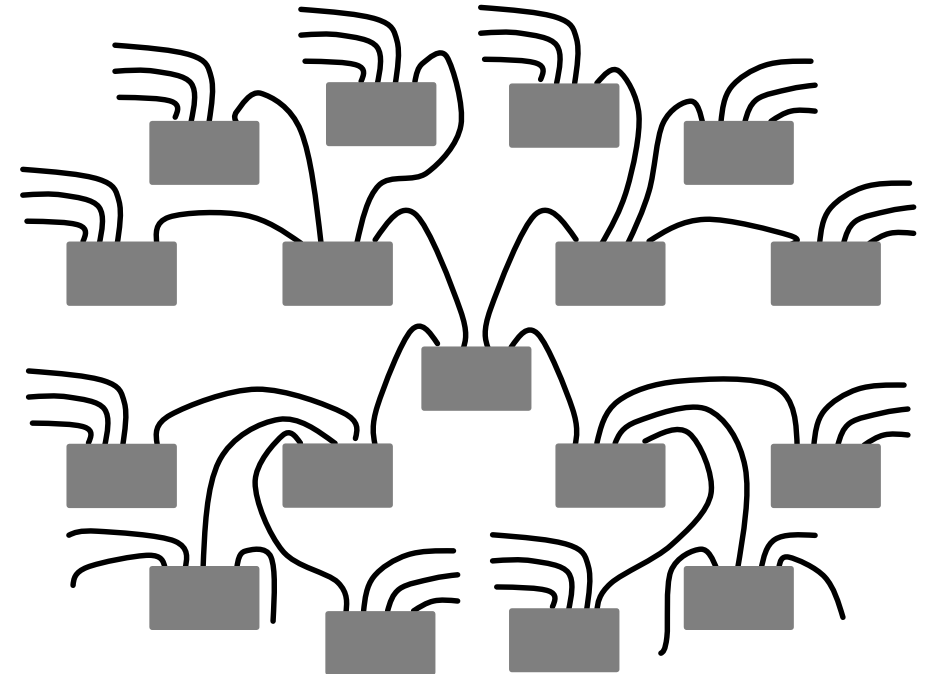
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$$\begin{aligned} MB(D, k) &= 1 + k + k(k - 1) \\ &\quad + k(k - 1)^2 + \dots \\ &= 1 + k \sum_{i=0}^{D-1} (k - 1)^i \end{aligned}$$



# DIAMETER-2 SLIM FLY

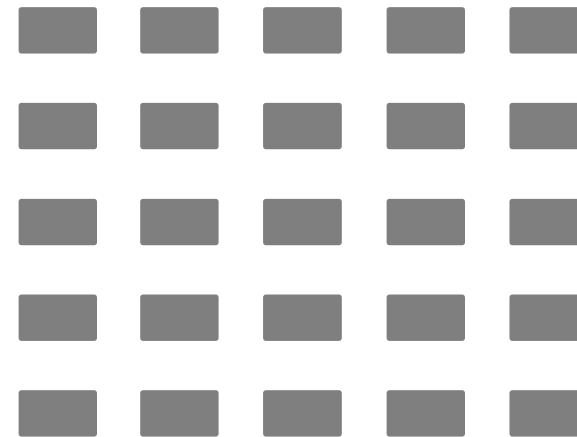
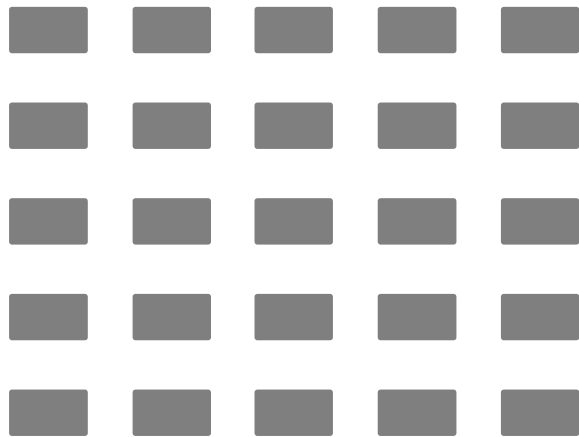
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- Example design for *diameter* = 2

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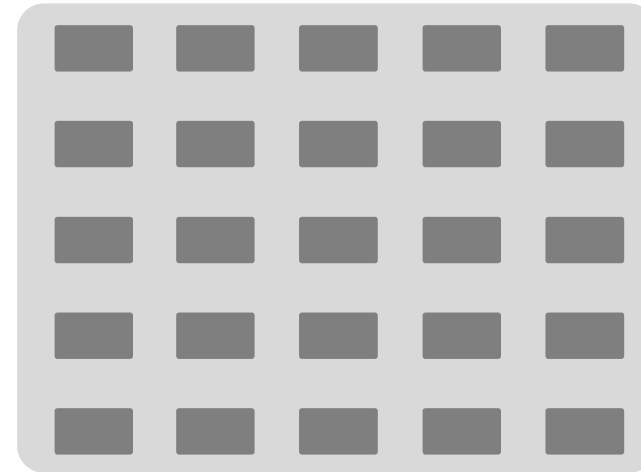
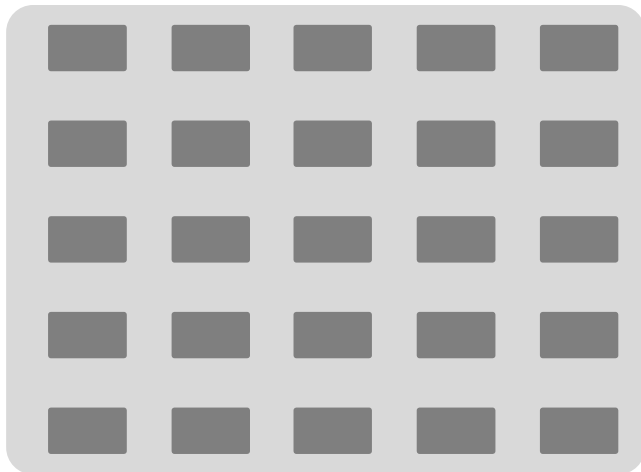


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A subgraph with  
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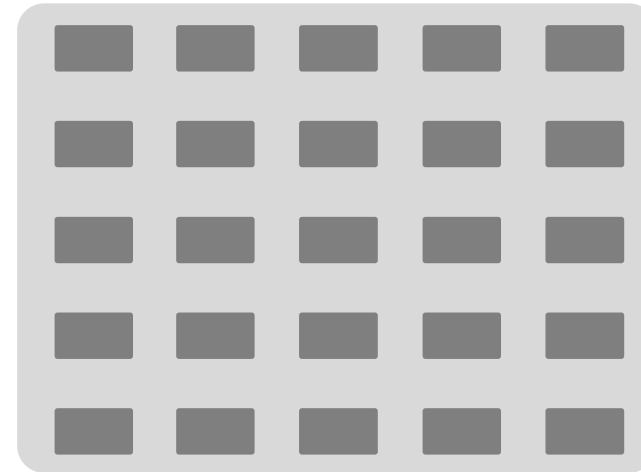
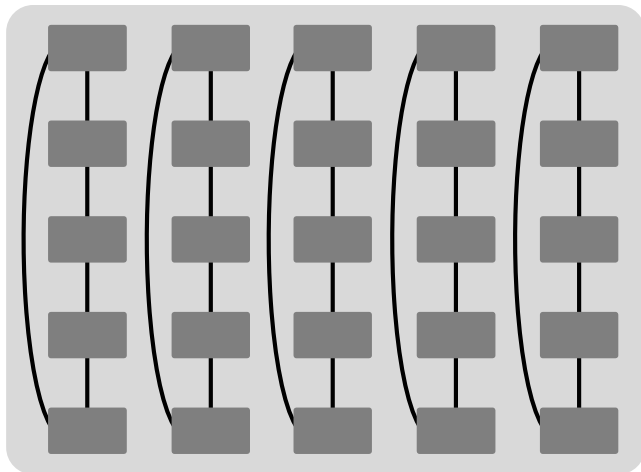


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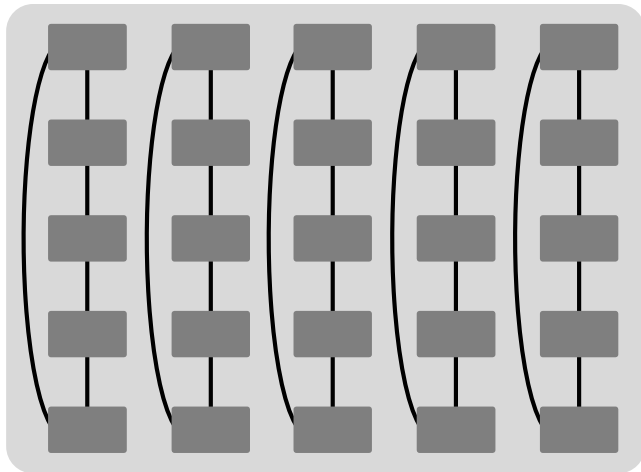


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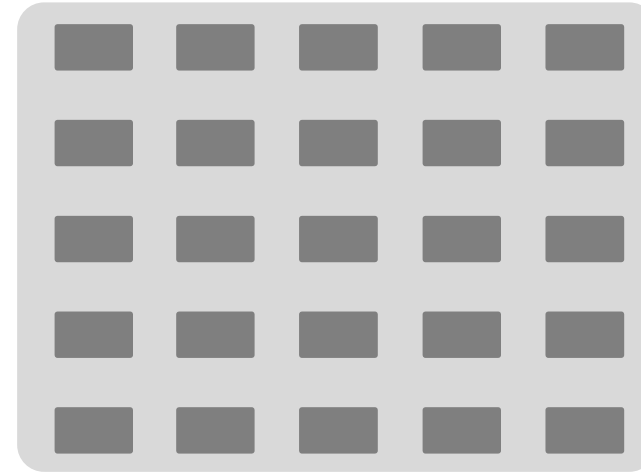
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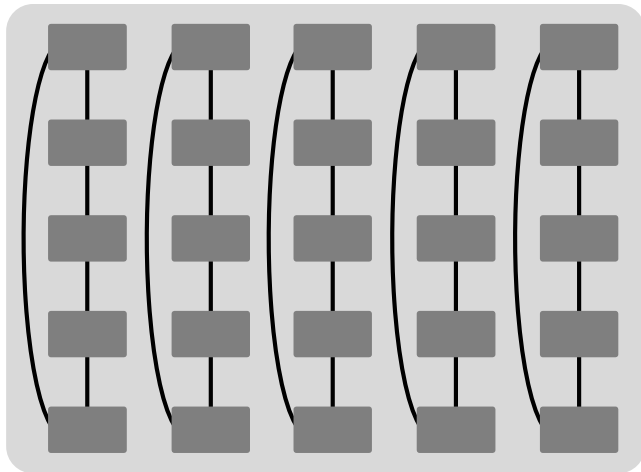


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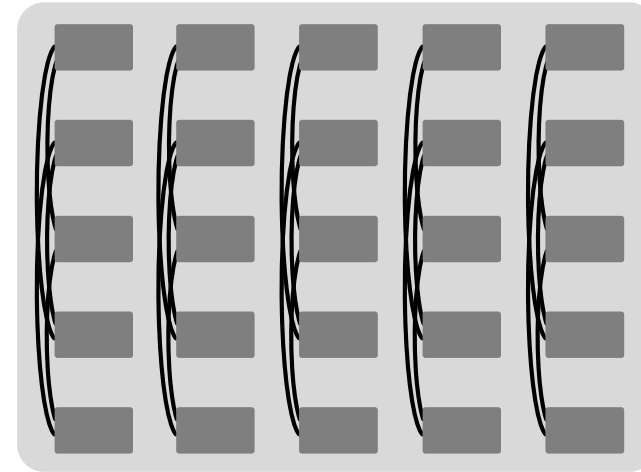
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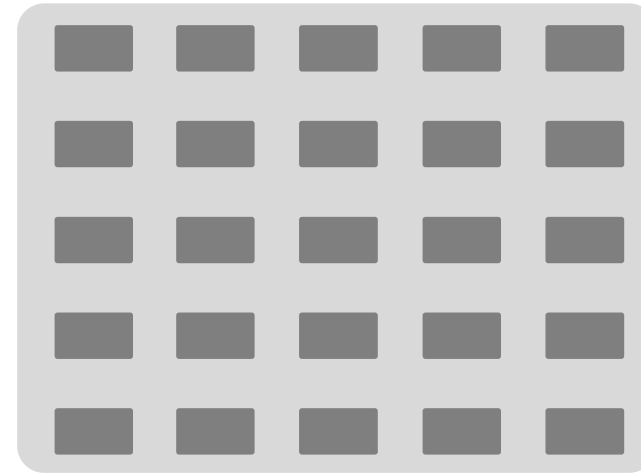
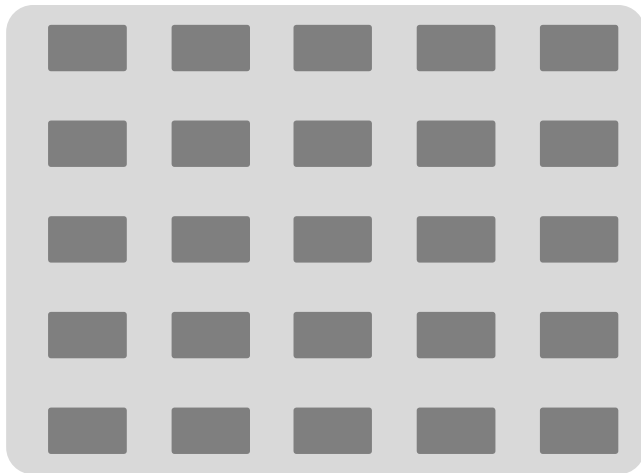
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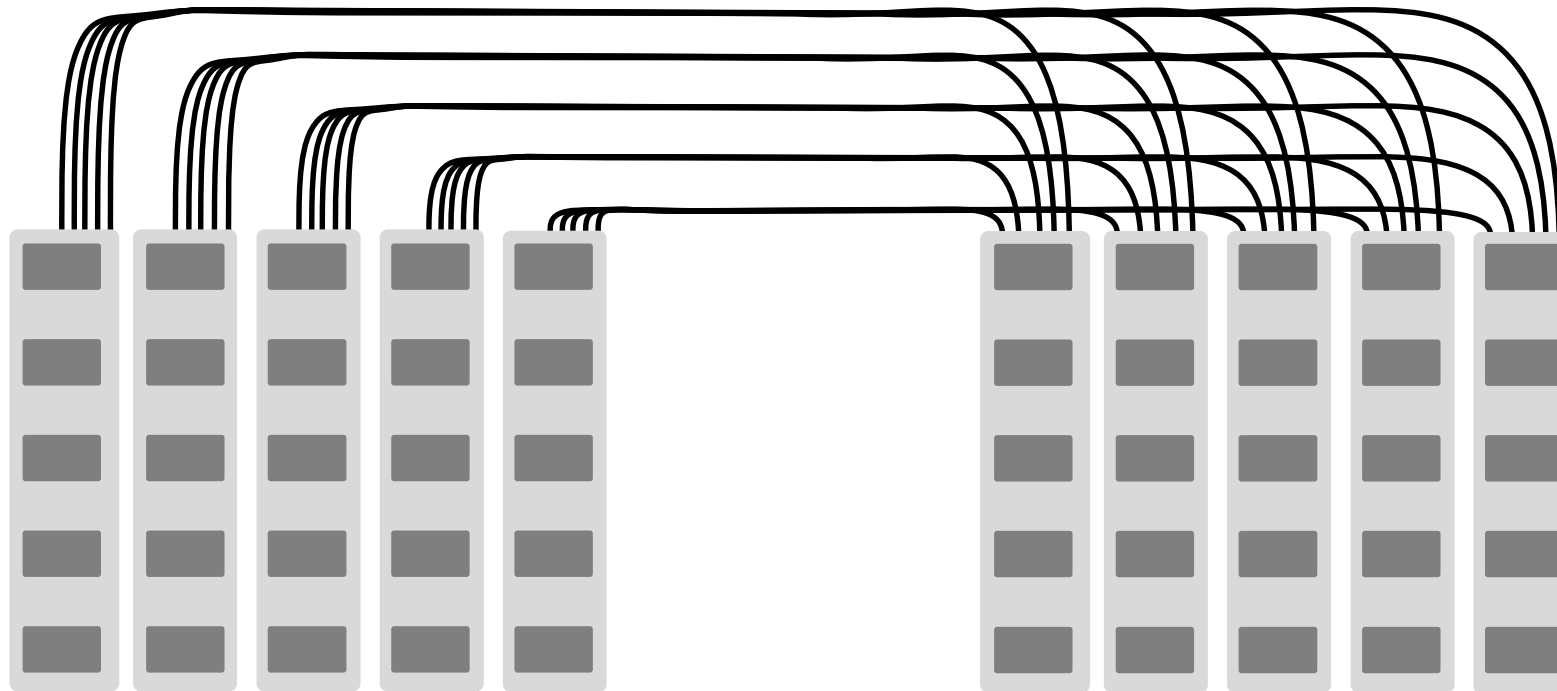
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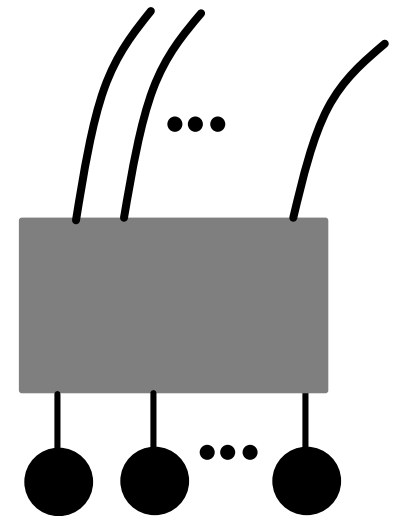
## STRUCTURE INTUITION



Groups form a fully-connected bipartite graph

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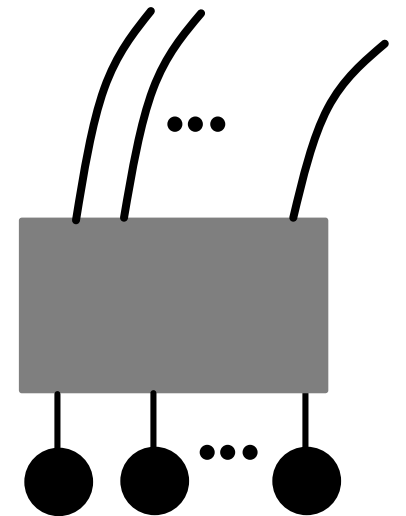
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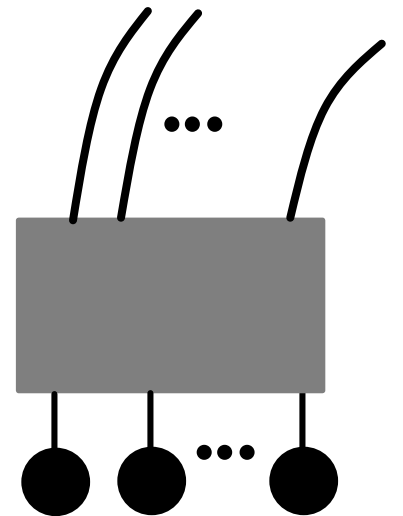


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- 1 Get load  $l$  per router-router channel (average number of routes per channel)

$$l = \frac{\text{total number of routes}}{\text{total number of channels}}$$



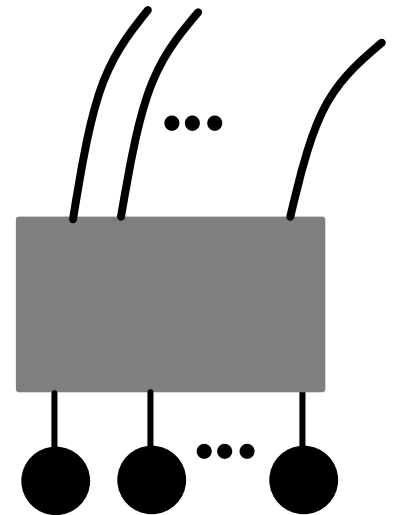
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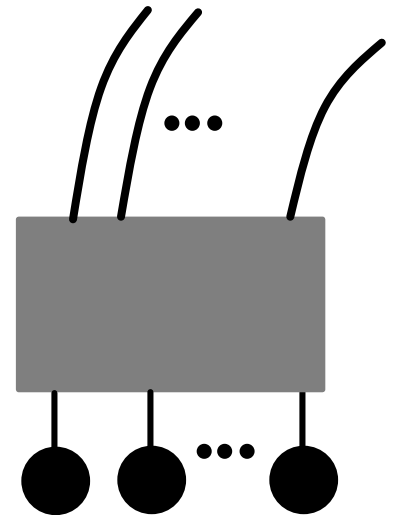
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- 2 Make the network balanced, i.e.,:



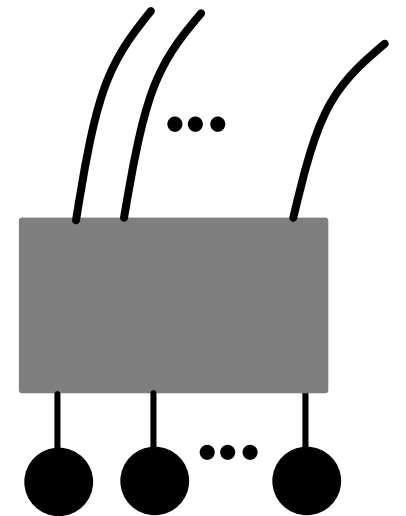
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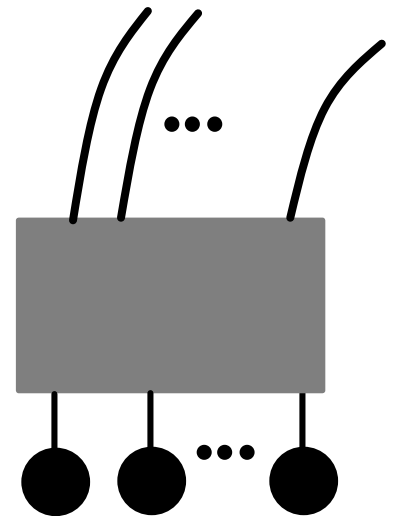
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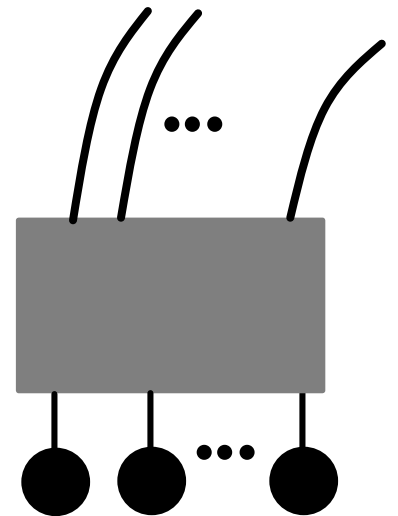
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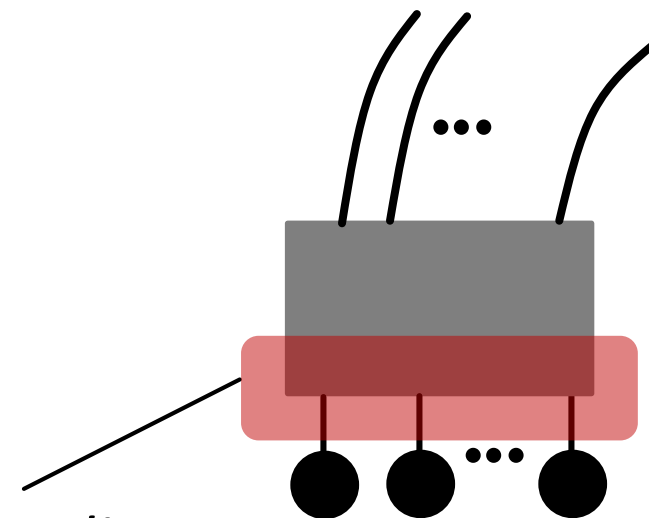
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*concentration = 33% of router radix*



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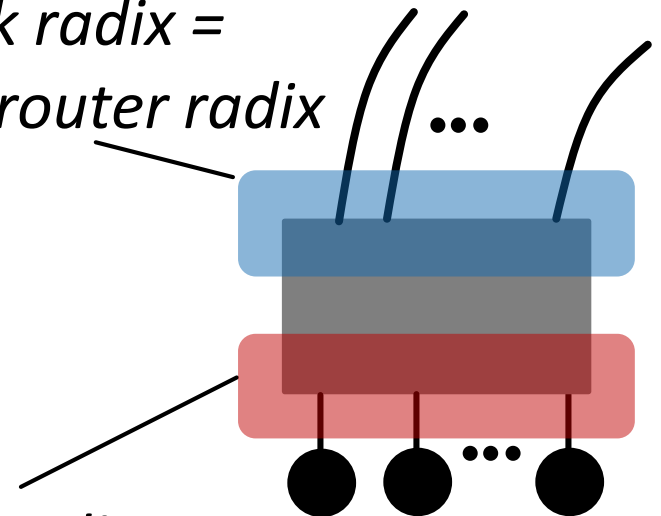
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*network radix =  
67% of router radix*



*concentration = 33% of router radix*



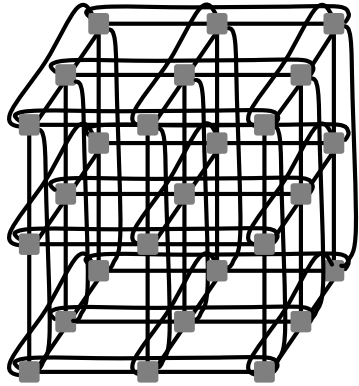
# COMPARISON TARGETS

## LOW-RADIX TOPOLOGIES

# COMPARISON TARGETS

## LOW-RADIX TOPOLOGIES

Torus 3D

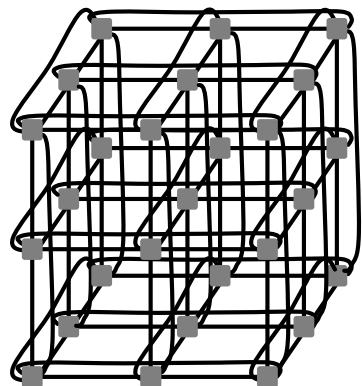


Cray XE6

# COMPARISON TARGETS

## LOW-RADIX TOPOLOGIES

Torus 3D

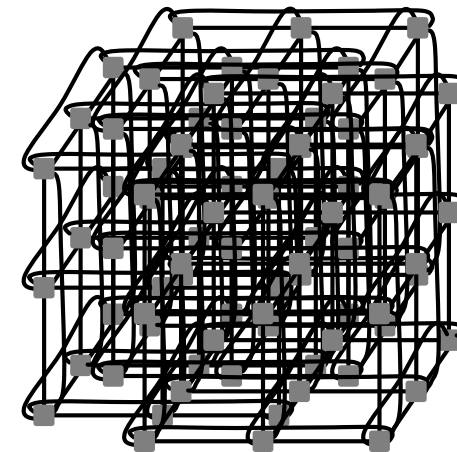


Cray XE6



IBM BG/Q

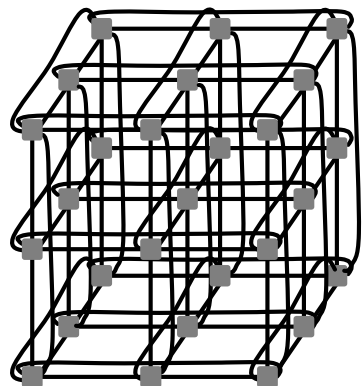
Torus 5D



# COMPARISON TARGETS

## LOW-RADIX TOPOLOGIES

Torus 3D

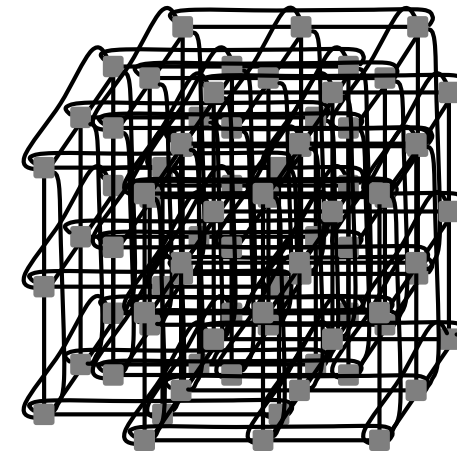


Cray XE6

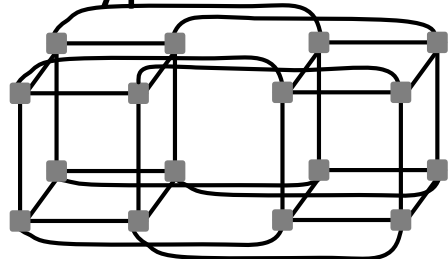


IBM BG/Q

Torus 5D



Hypercube

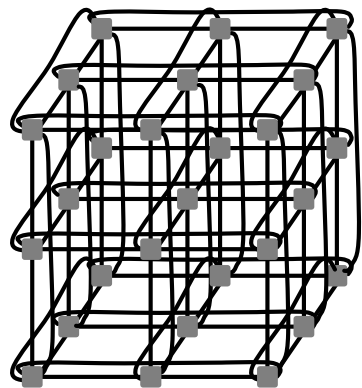


NASA Pleiades

# COMPARISON TARGETS

## LOW-RADIX TOPOLOGIES

Torus 3D

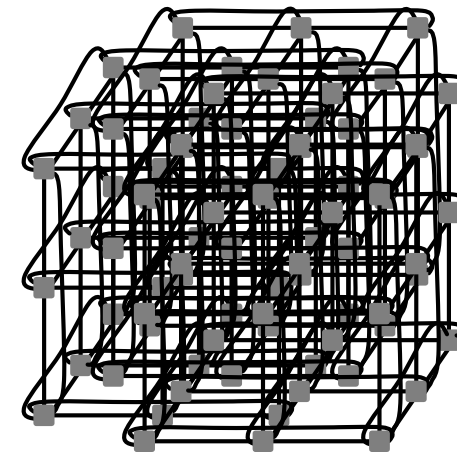


Cray XE6

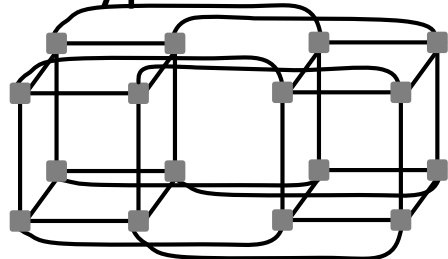


IBM BG/Q

Torus 5D



Hypercube

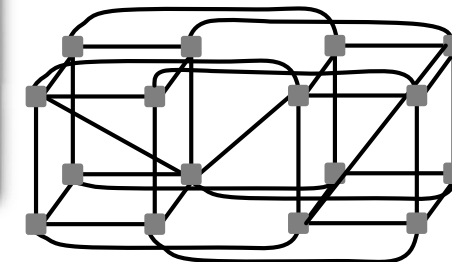


NASA Pleiades



Infinetics

Long Hop [1]



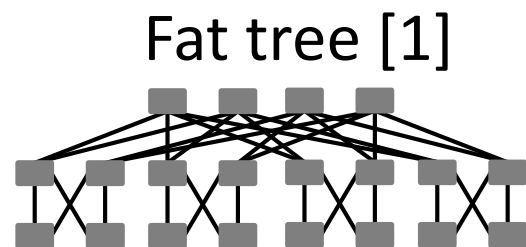
[1] Tomic, Ratko V. Optimal networks from error correcting codes. 2013 ACM/IEEE Symposium on Architectures for Networking and Communications Systems (ANCS)

# COMPARISON TARGETS

## HIGH-RADIX TOPOLOGIES

# COMPARISON TARGETS

## HIGH-RADIX TOPOLOGIES

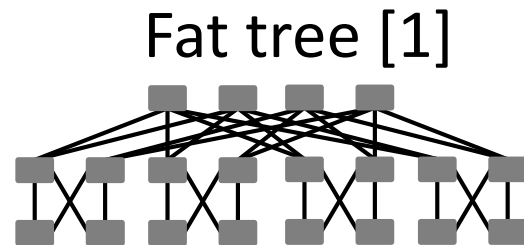


TSUBAME2.0

[1] C. E. Leiserson. Fat-trees: universal networks for hardware-efficient supercomputing. IEEE Transactions on Computers. 1985

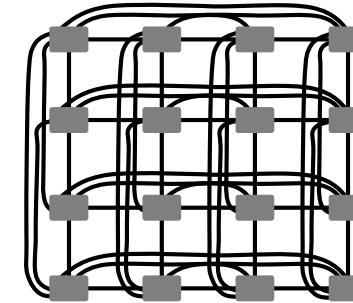
# COMPARISON TARGETS

## HIGH-RADIX TOPOLOGIES



TSUBAME2.0

## Flattened Butterfly [2]



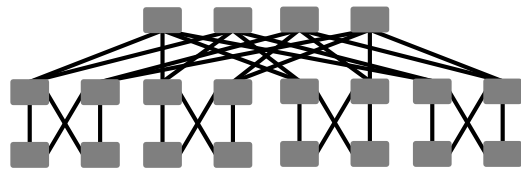
- [1] C. E. Leiserson. Fat-trees: universal networks for hardware-efficient supercomputing. IEEE Transactions on Computers. 1985
- [2] J. Kim, W. J. Dally, D. Abts. Flattened butterfly: a cost-efficient topology for high-radix networks. ISCA'07



# COMPARISON TARGETS

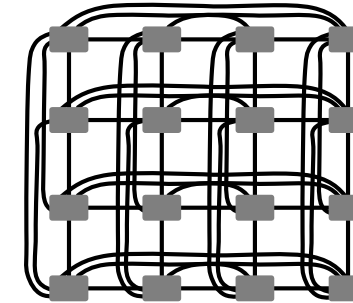
## HIGH-RADIX TOPOLOGIES

Fat tree [1]

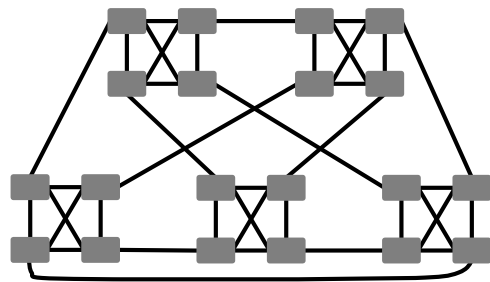


TSUBAME2.0

Flattened Butterfly [2]



Dragonfly [3]

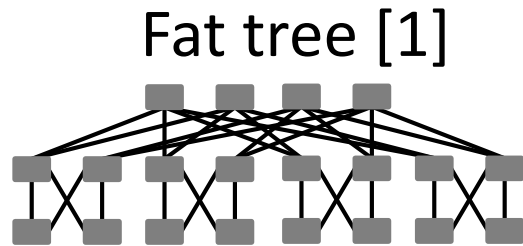


Cray Cascade

- [1] C. E. Leiserson. Fat-trees: universal networks for hardware-efficient supercomputing. IEEE Transactions on Computers. 1985
- [2] J. Kim, W. J. Dally, D. Abts. Flattened butterfly: a cost-efficient topology for high-radix networks. ISCA'07
- [3] J. Kim, W. J. Dally, S. Scott, D. Abts. Technology-Driven, Highly-Scalable Dragonfly Topology. ISCA'08

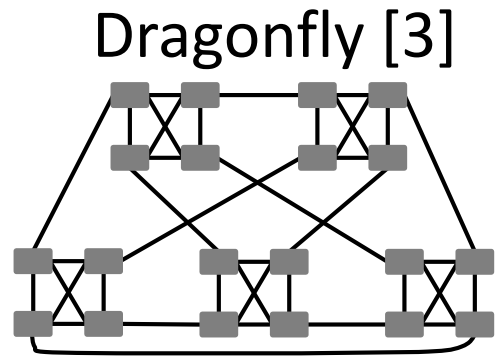
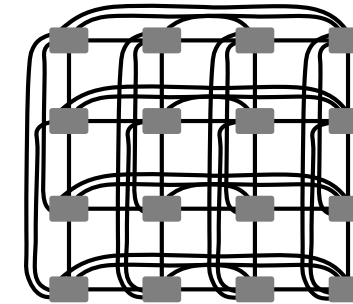
# COMPARISON TARGETS

## HIGH-RADIX TOPOLOGIES



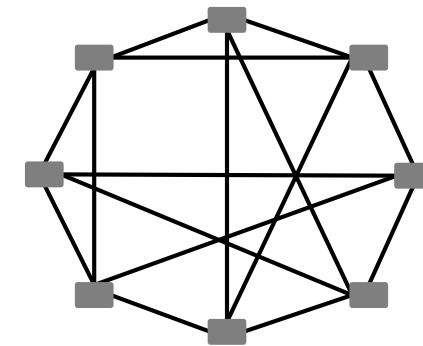
TSUBAME2.0

### Flattened Butterfly [2]



Cray Cascade

### Random Topologies [4]



[1] C. E. Leiserson. Fat-trees: universal networks for hardware-efficient supercomputing. IEEE Transactions on Computers. 1985

[2] J. Kim, W. J. Dally, D. Abts. Flattened butterfly: a cost-efficient topology for high-radix networks. ISCA'07

[3] J. Kim, W. J. Dally, S. Scott, D. Abts. Technology-Driven, Highly-Scalable Dragonfly Topology. ISCA'08

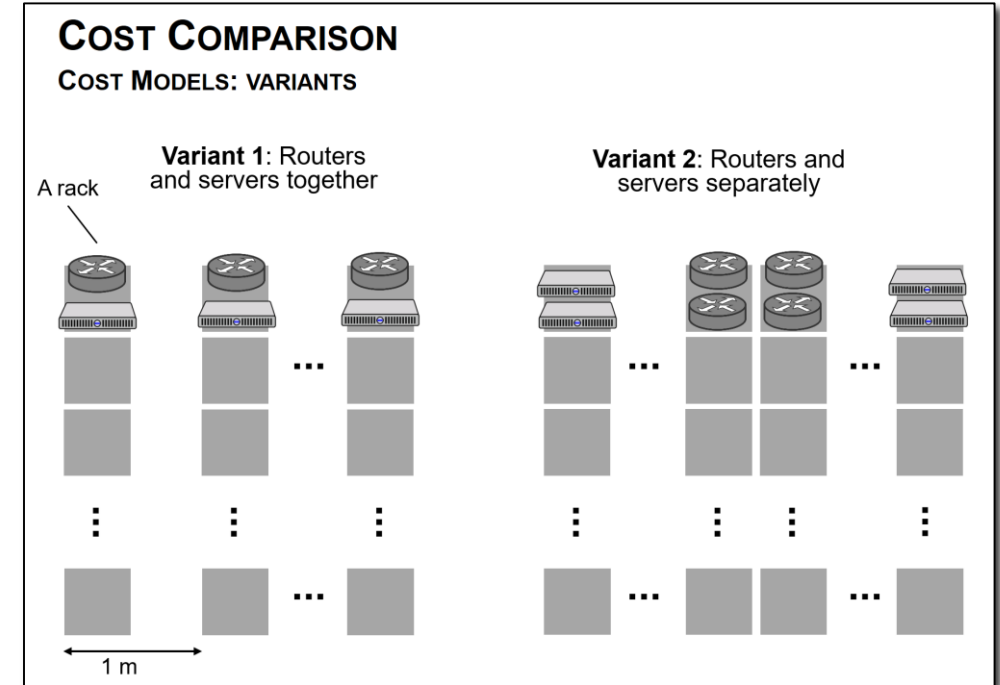
[4] M. Koibuchi, H. Matsutani, H. Amano, D. F. Hsu, H. Casanova. A case for random shortcut topologies for HPC interconnects. ISCA'12

# **COST OF NETWORK CONSTRUCTION**

## **MODELS, VARIANTS**

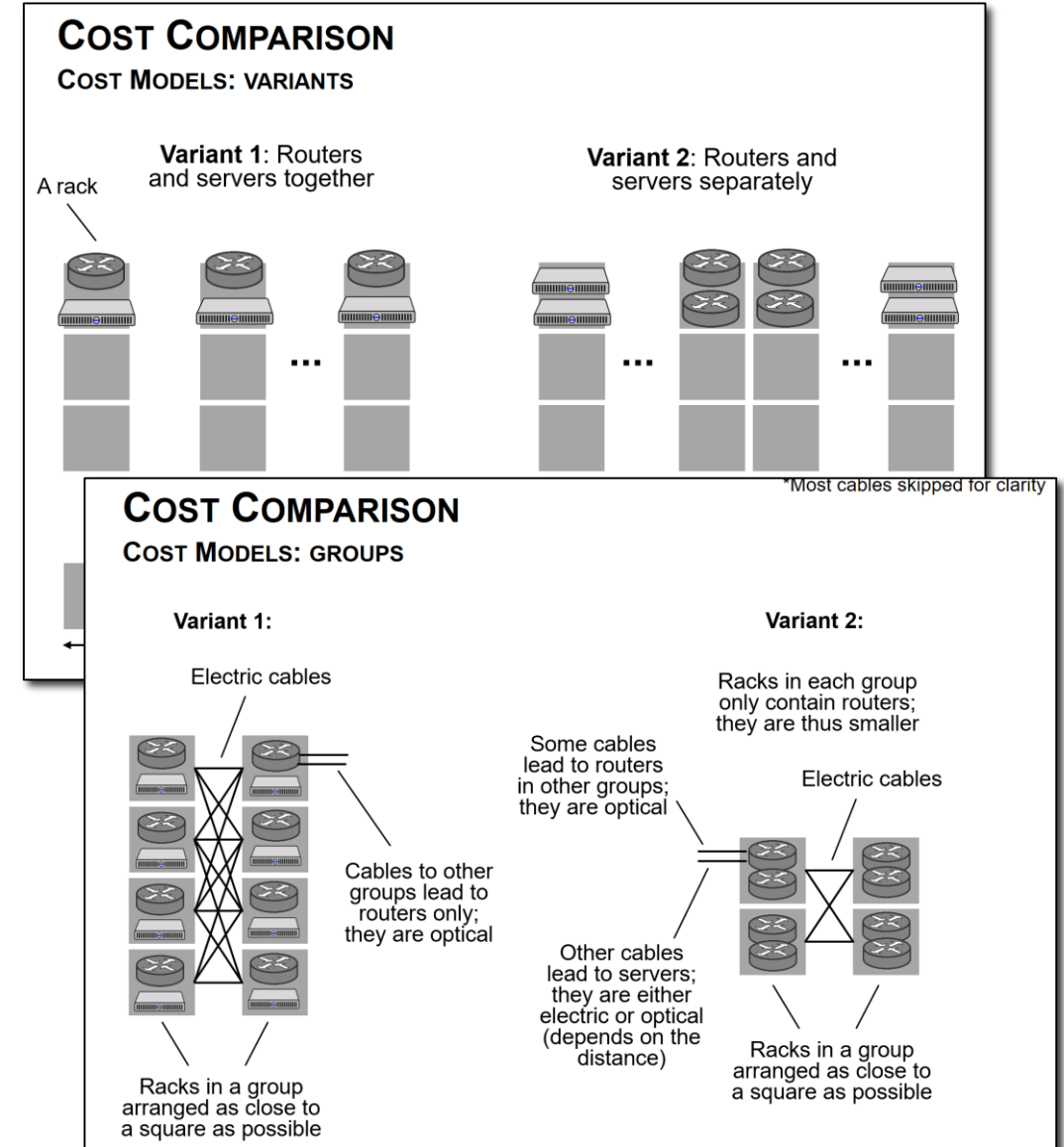
# COST OF NETWORK CONSTRUCTION MODELS, VARIANTS

## Cluster structure



# COST OF NETWORK CONSTRUCTION MODELS, VARIANTS

## Cluster structure

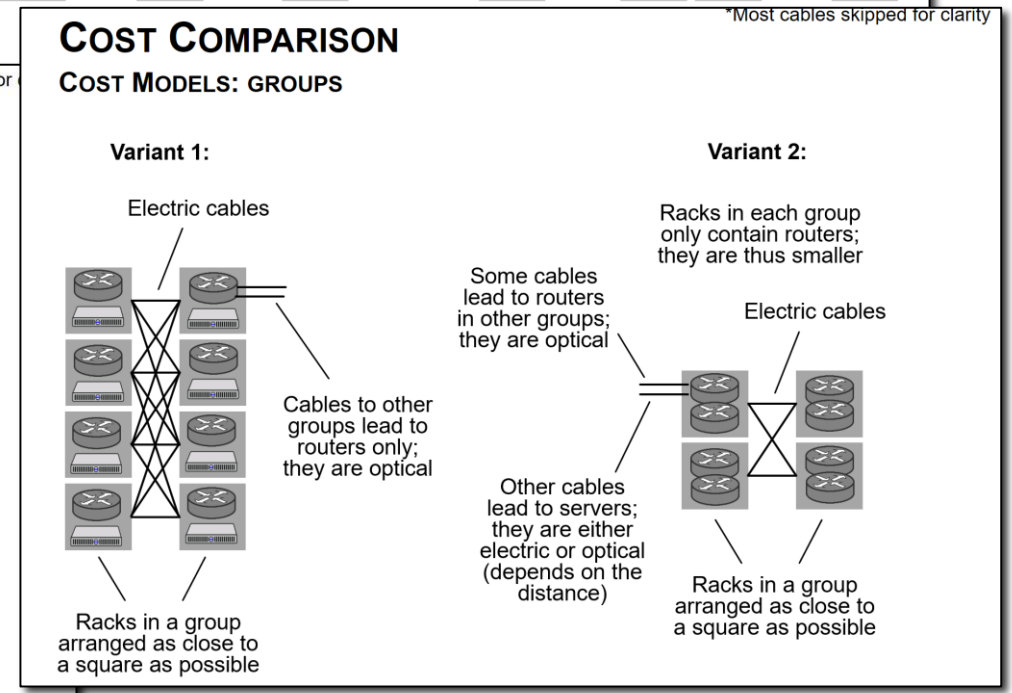
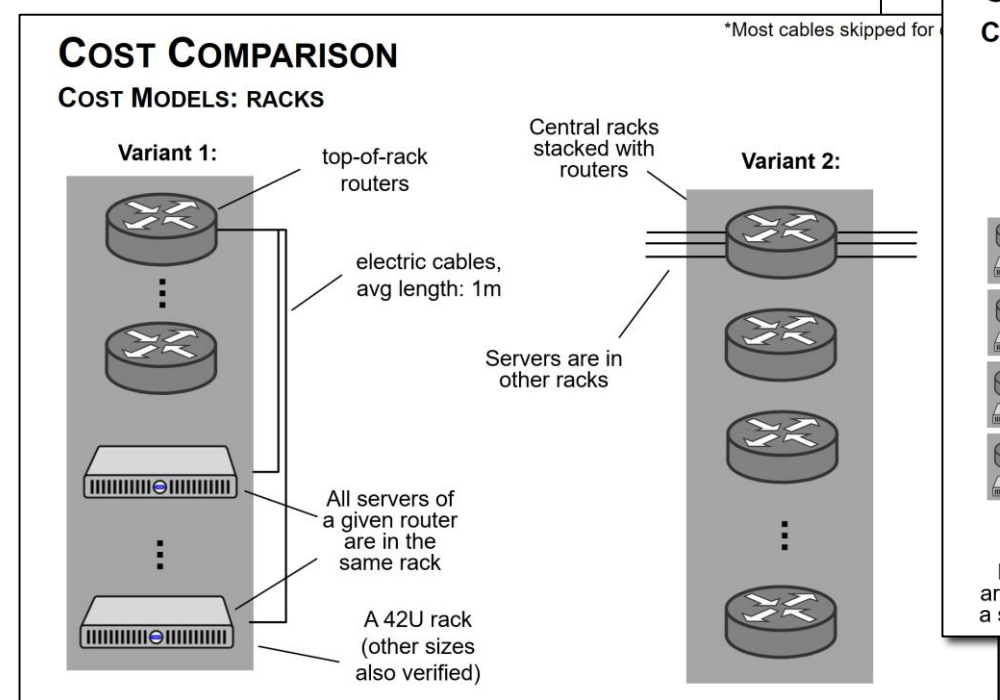
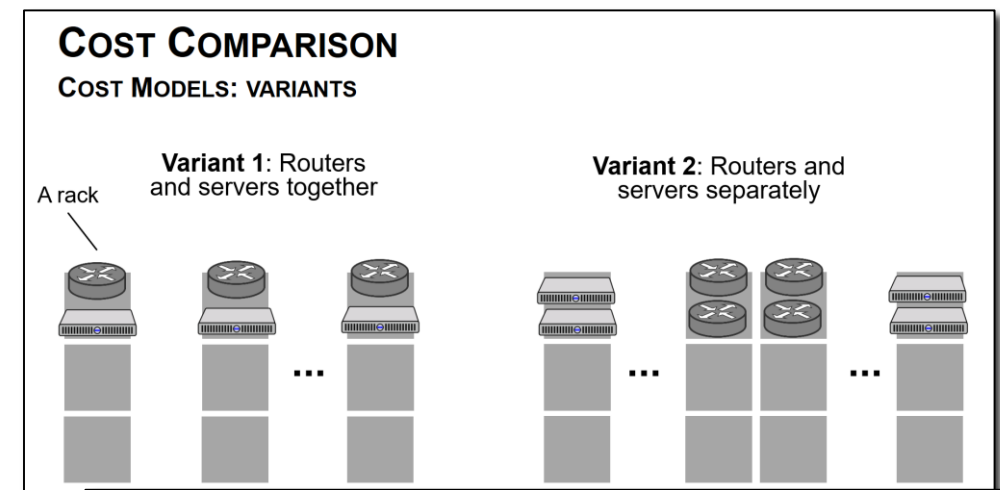


## Structure of router groups

# COST OF NETWORK CONSTRUCTION MODELS, VARIANTS

## Cluster structure

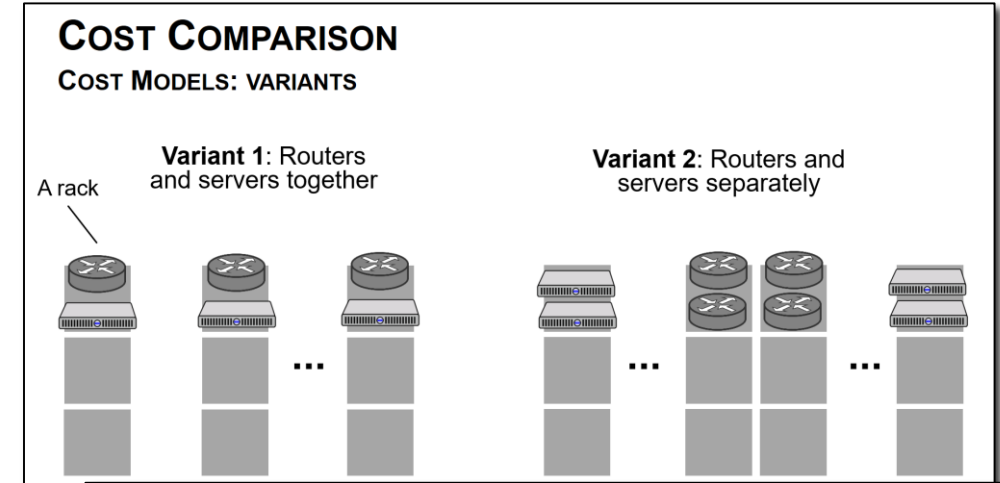
## Rack structure



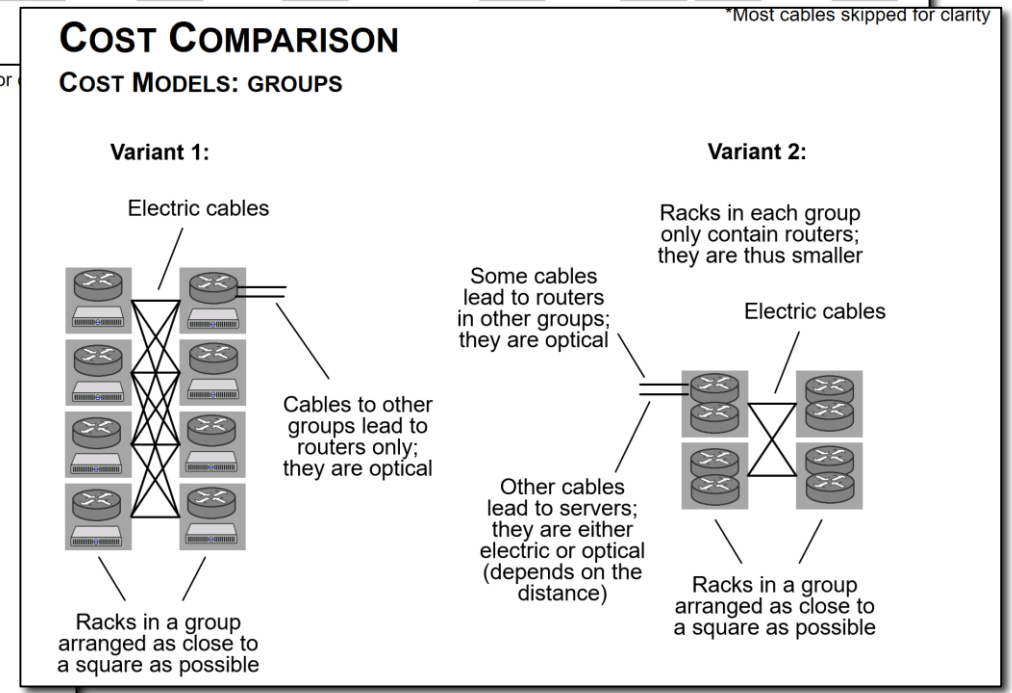
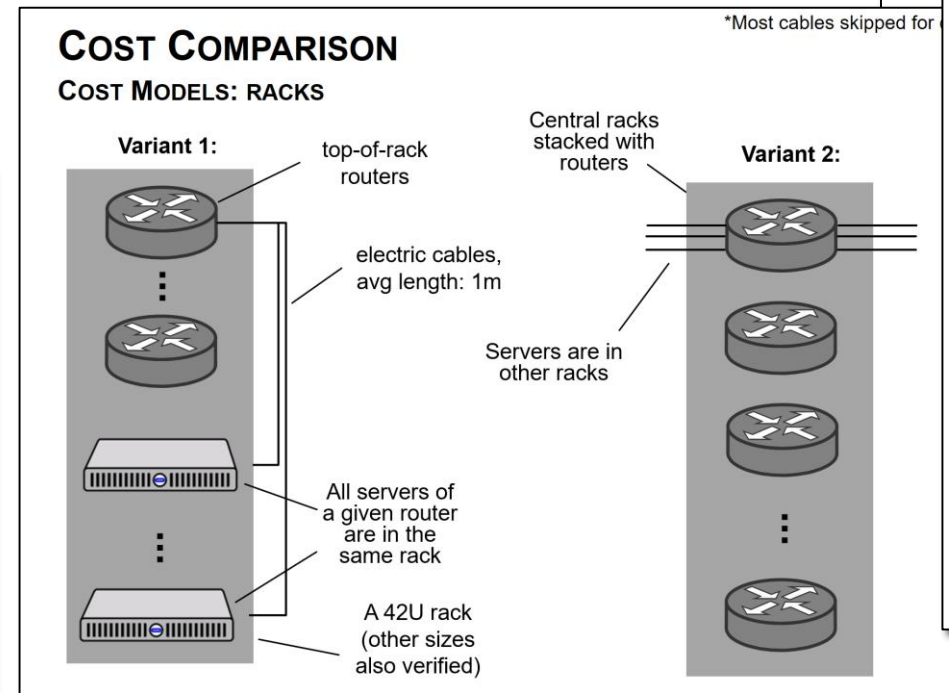
## Structure of router groups

# COST OF NETWORK CONSTRUCTION MODELS, VARIANTS

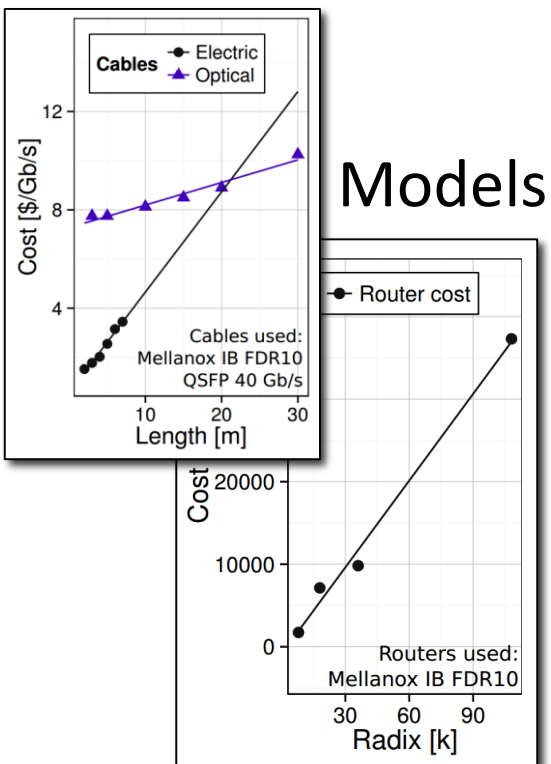
## Cluster structure



## Rack structure



## Structure of router groups

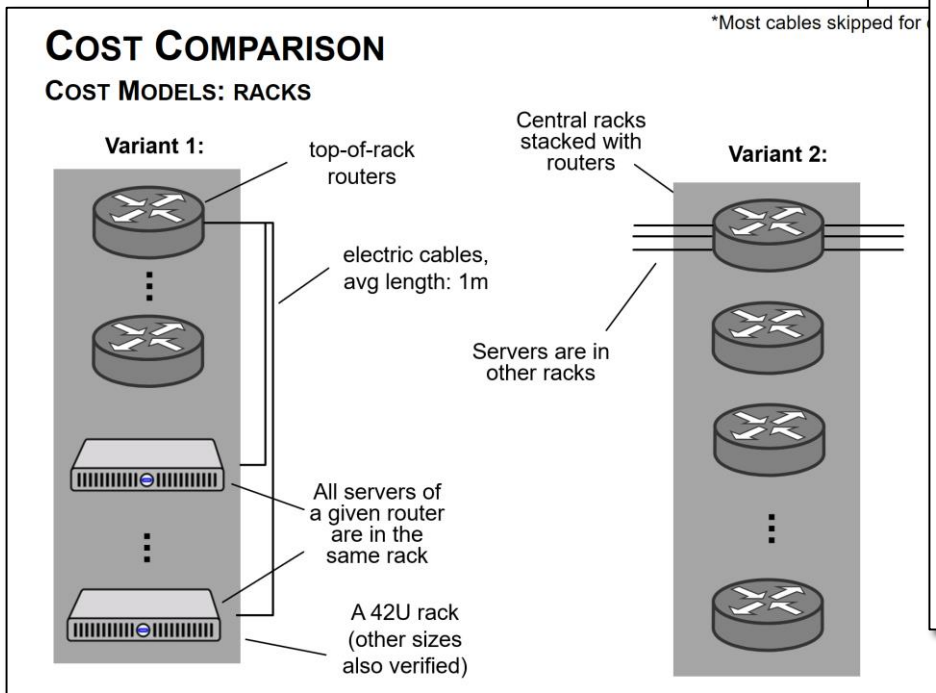


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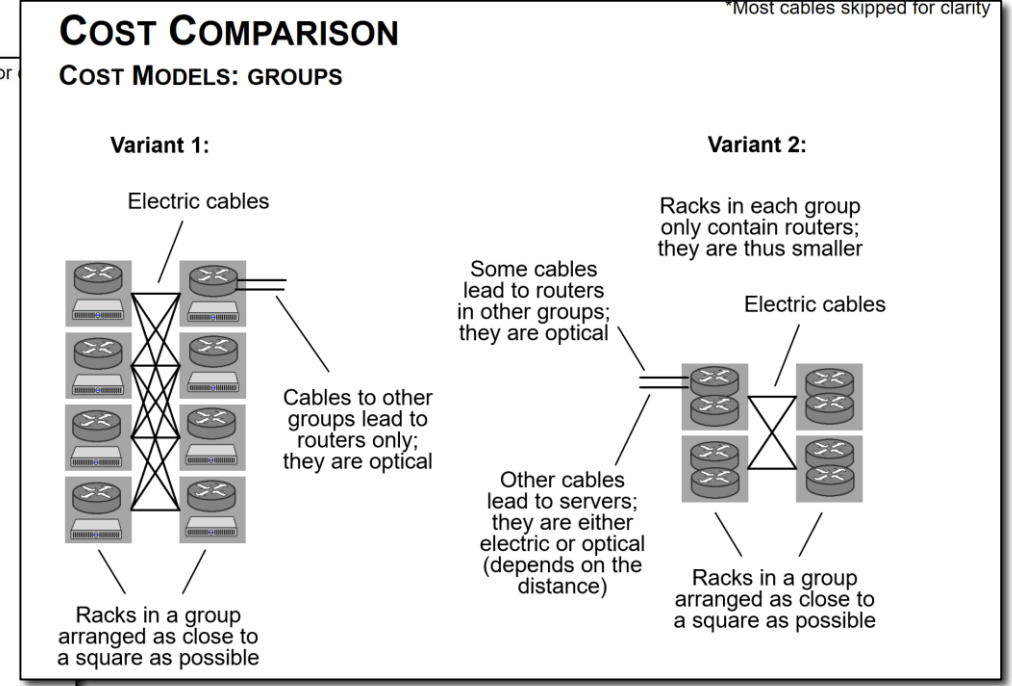
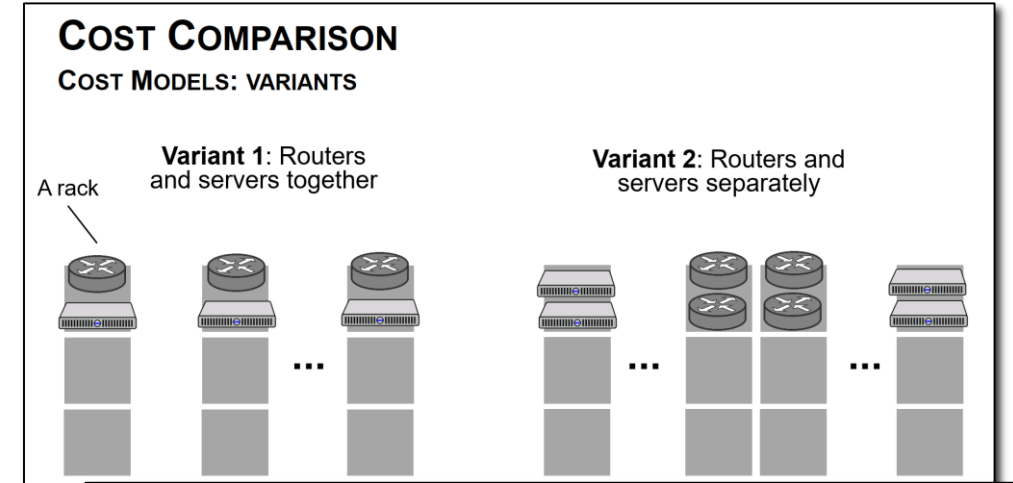
## Routers



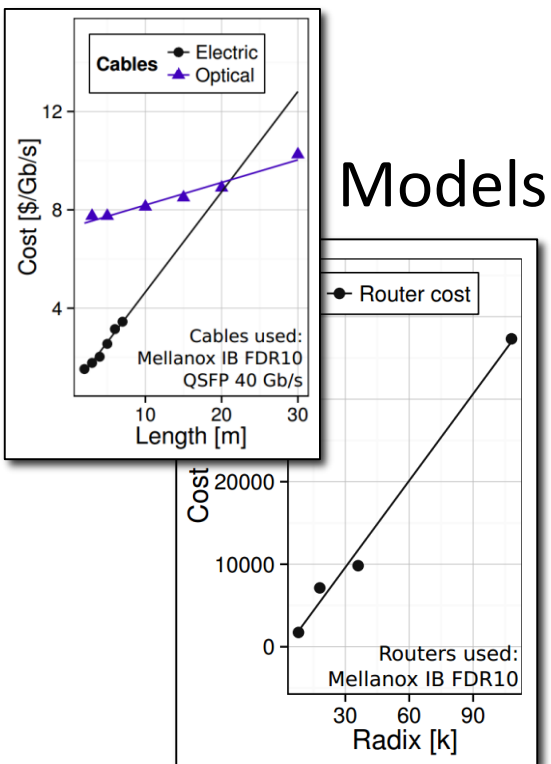
## Rack structure



## Cluster structure



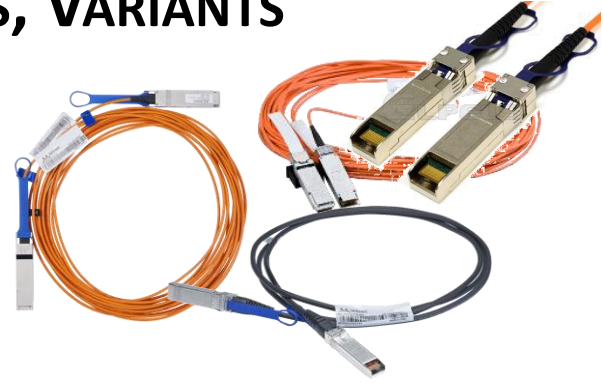
## Structure of router groups





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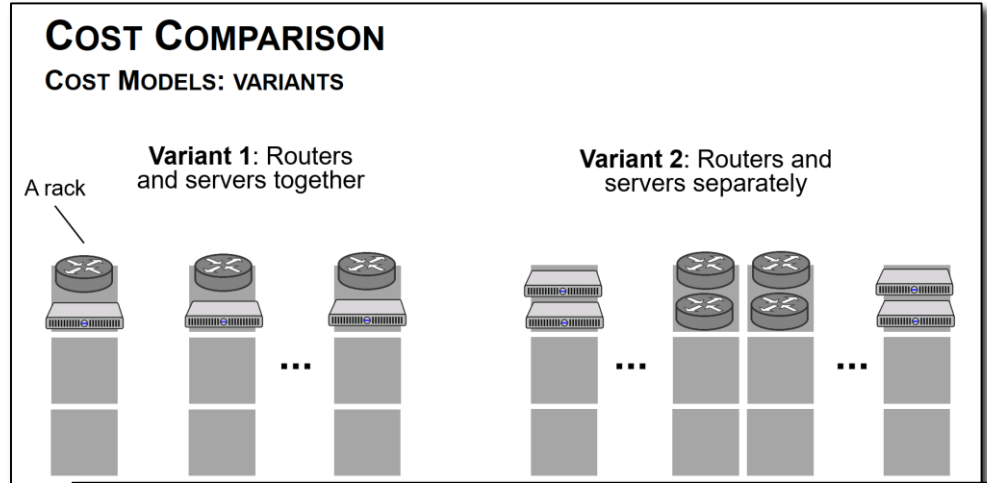
## Cables



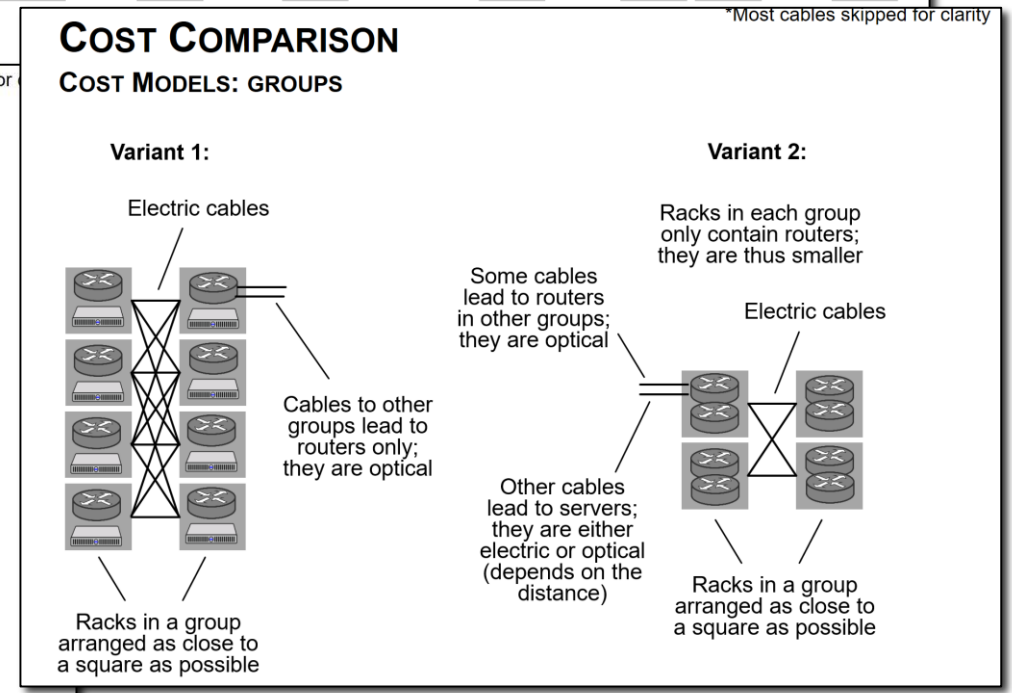
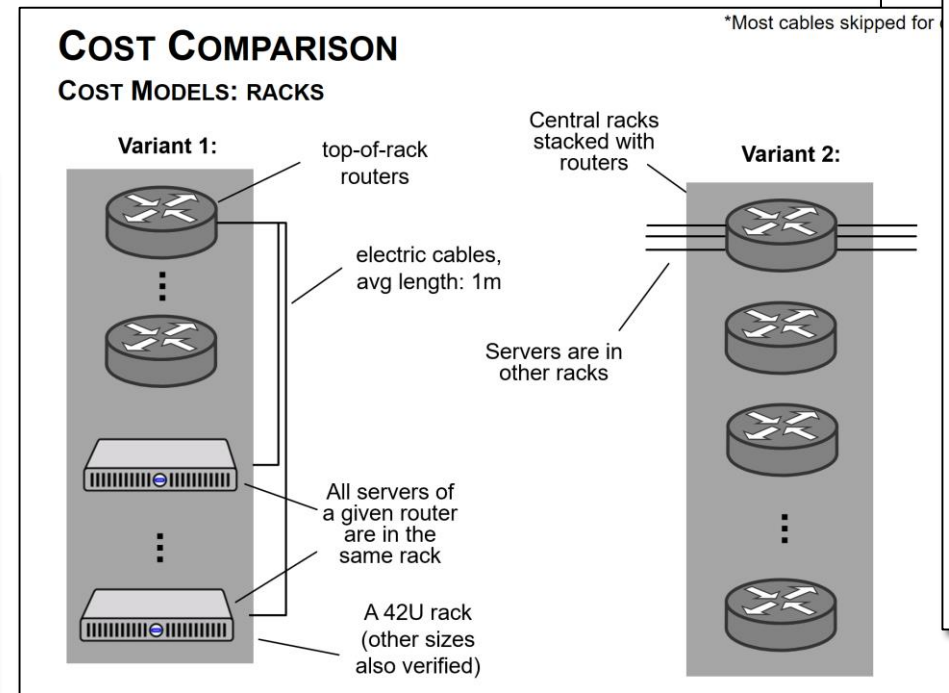
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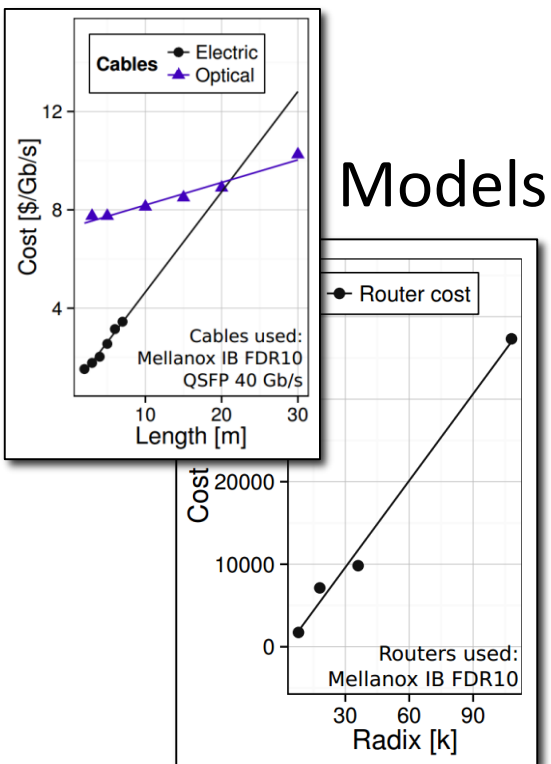
## Cluster structure



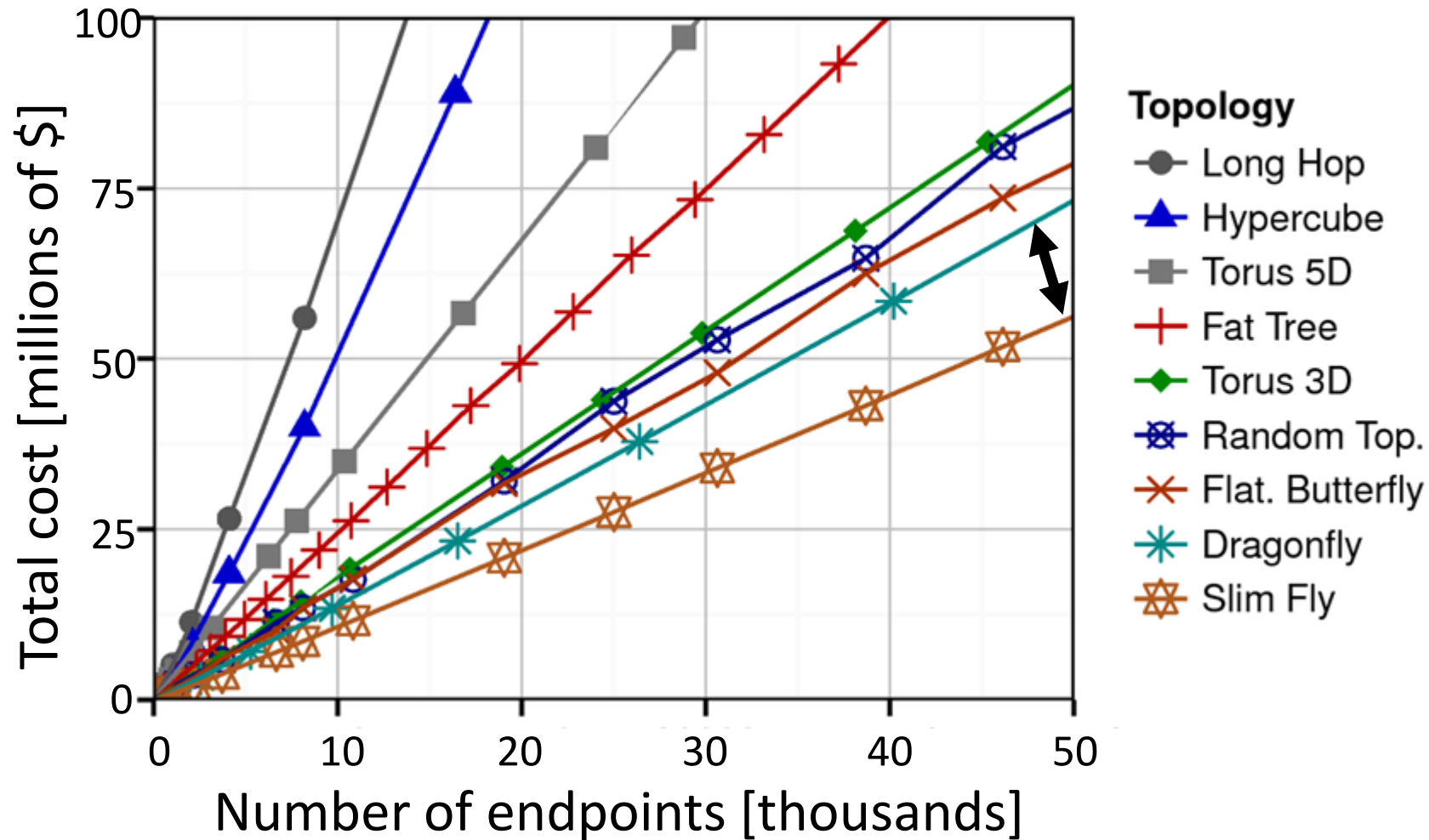
## Rack structure



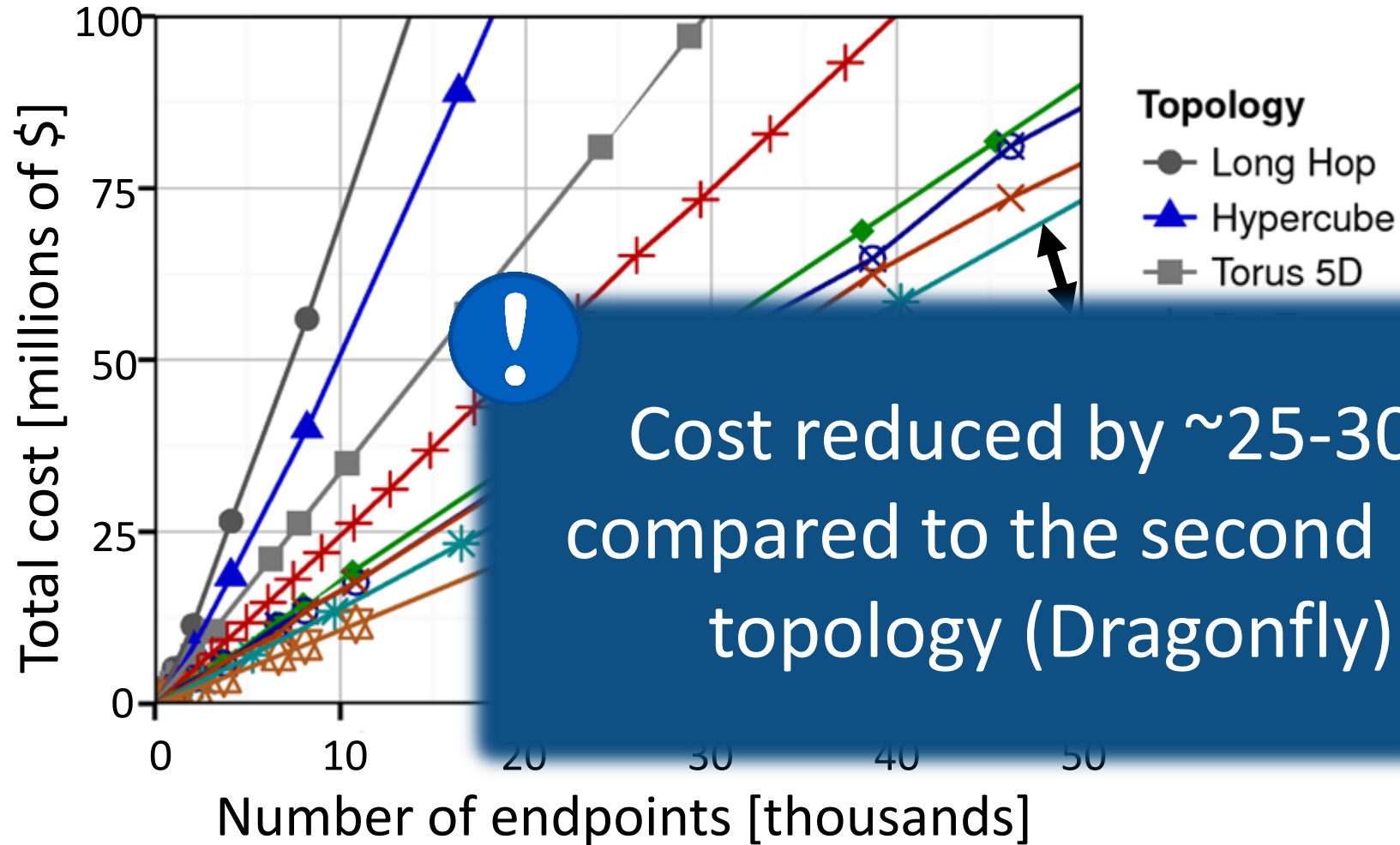
## Structure of router groups



# RESULTS: COST OF NETWORK CONSTRUCTION

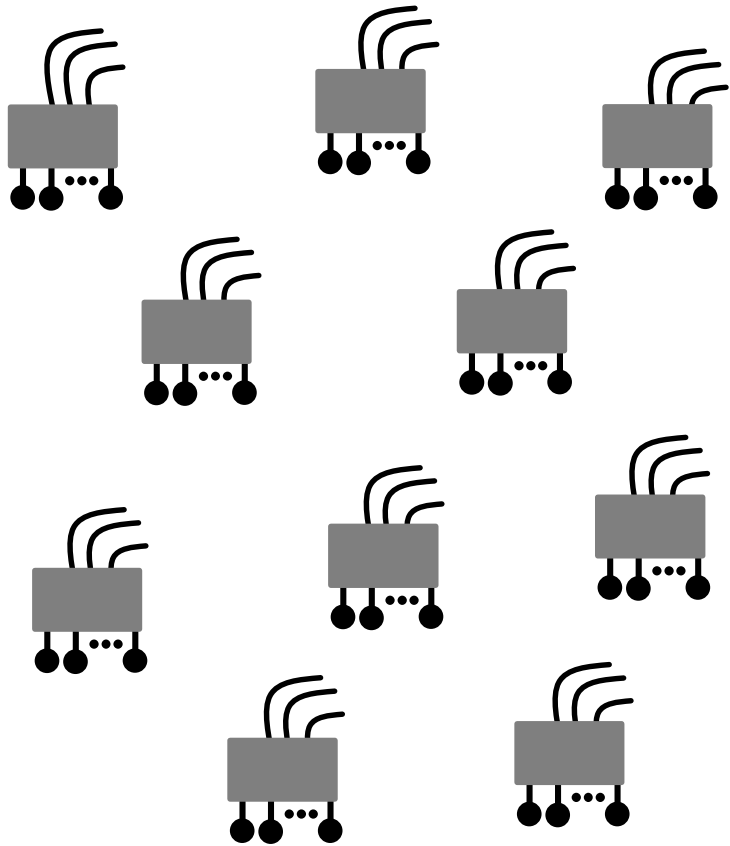


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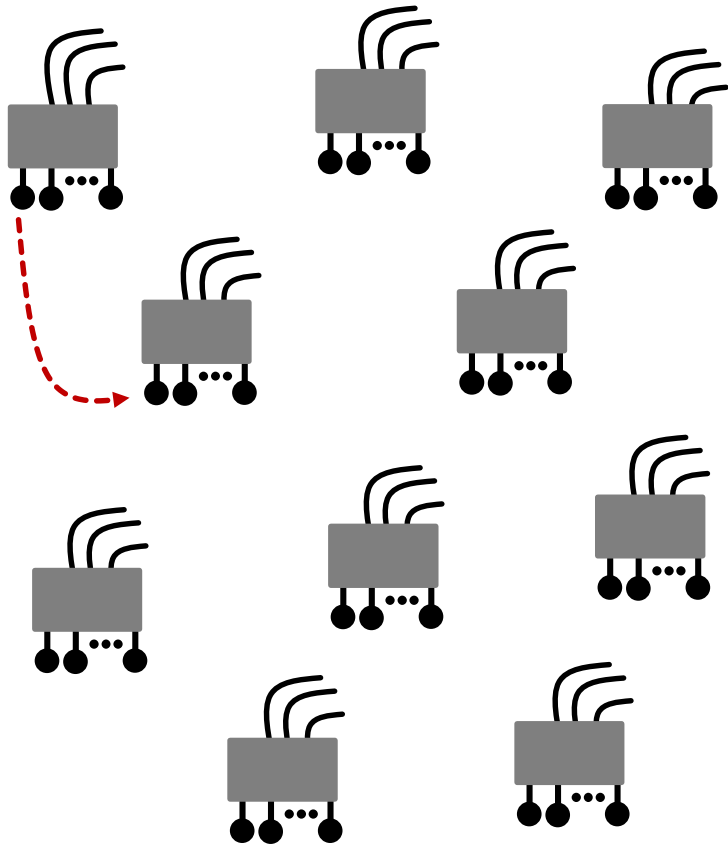


# RESULTS: PERFORMANCE

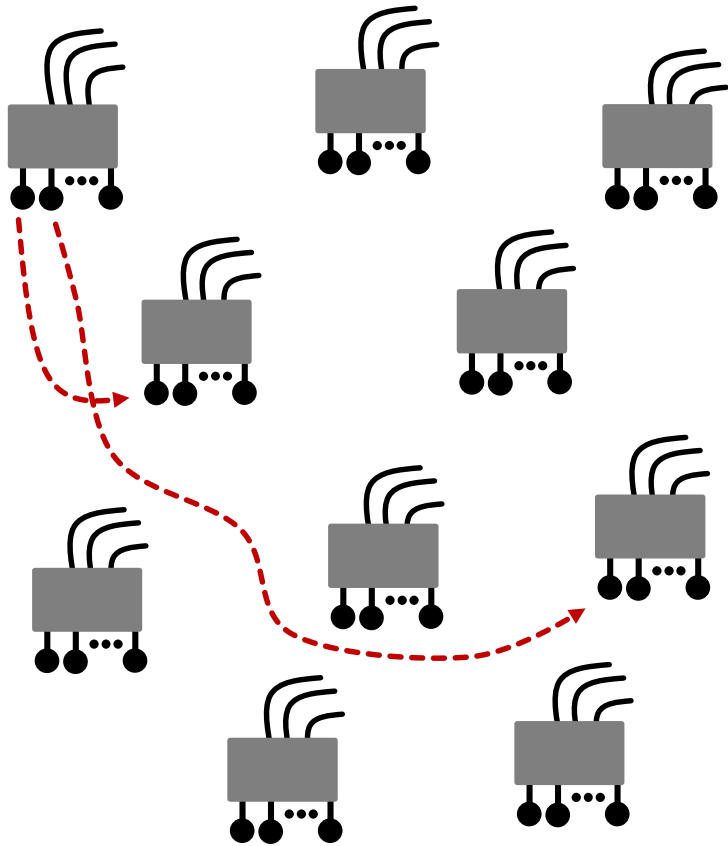
# RESULTS: PERFORMANCE



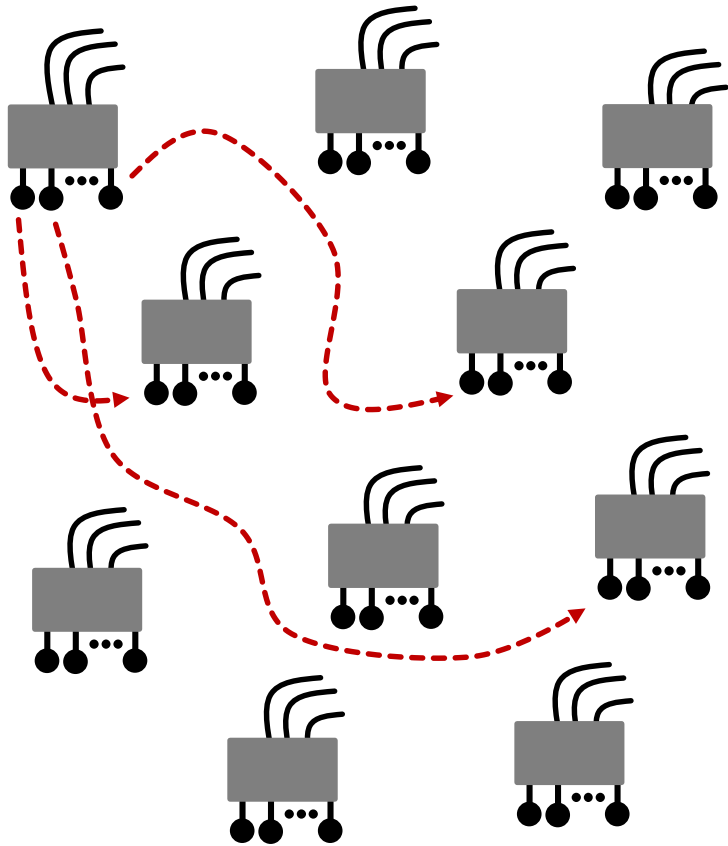
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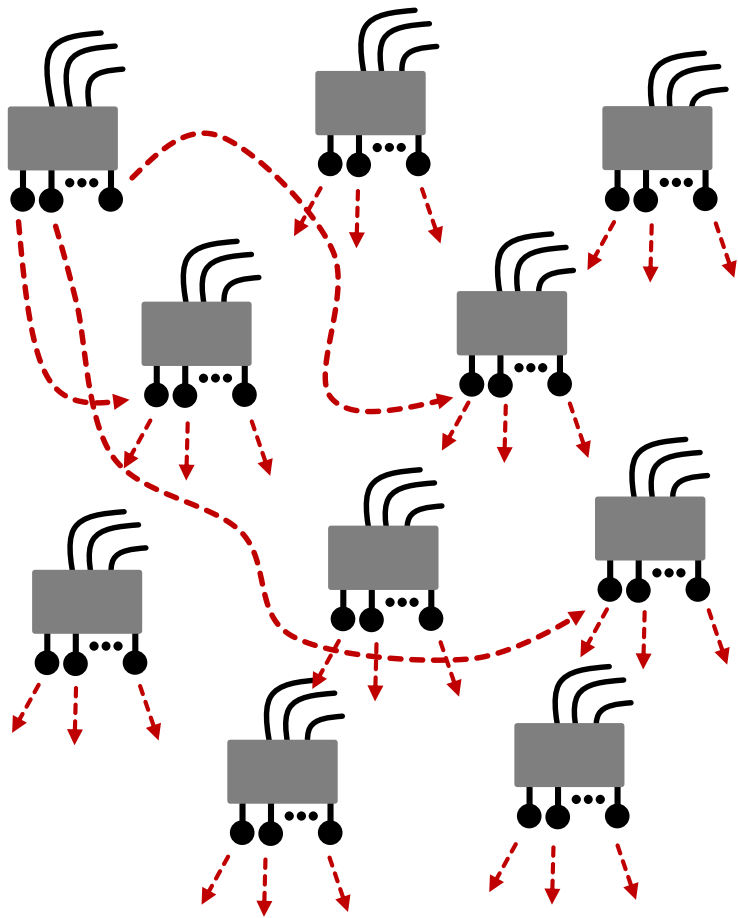


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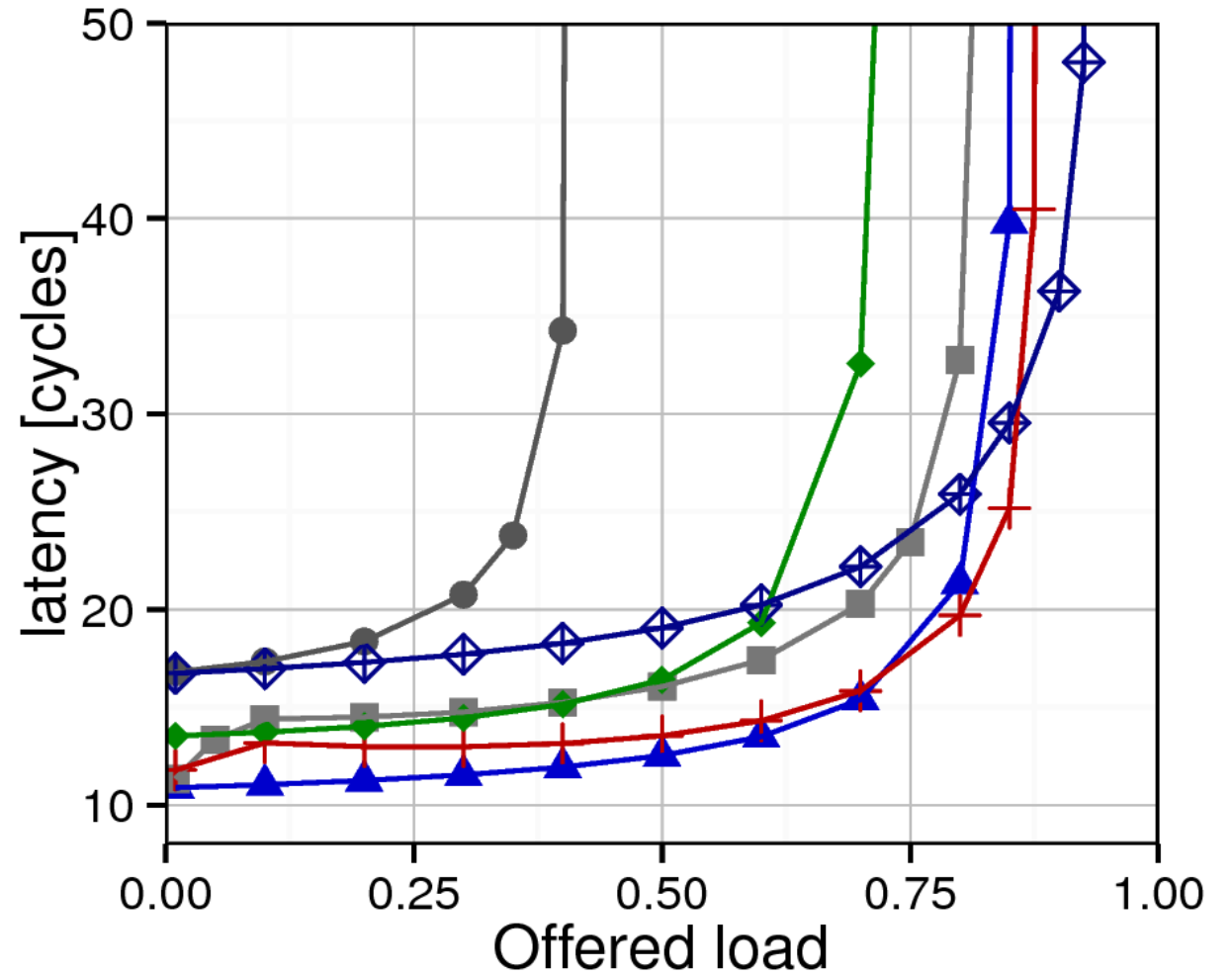
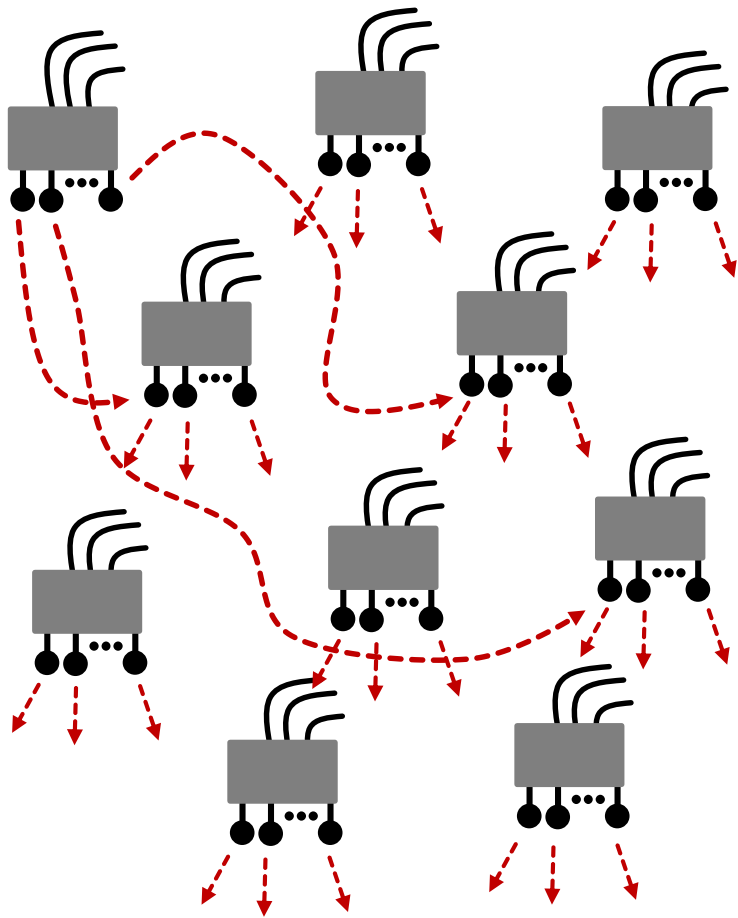




# RESULTS: PERFORMANCE



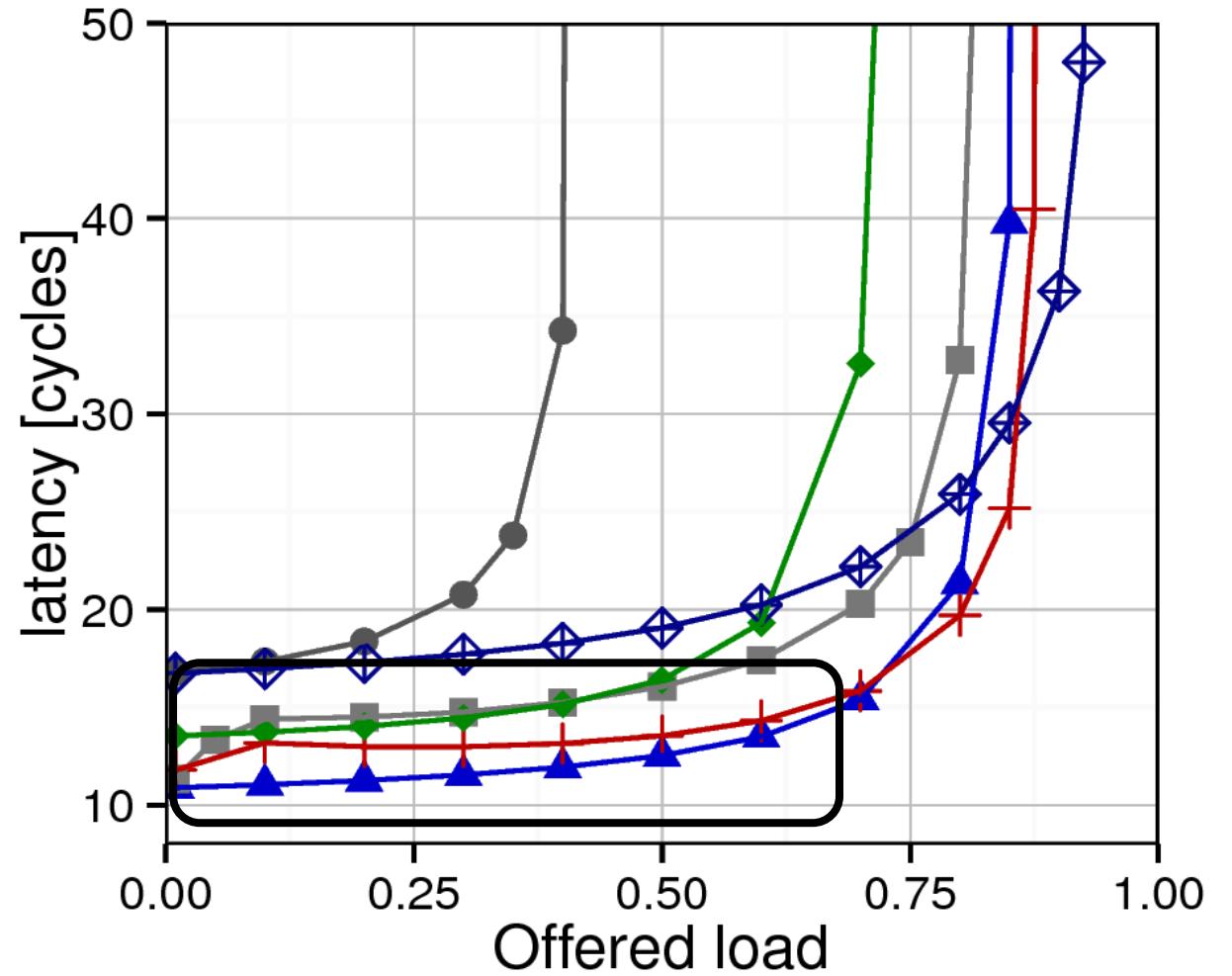
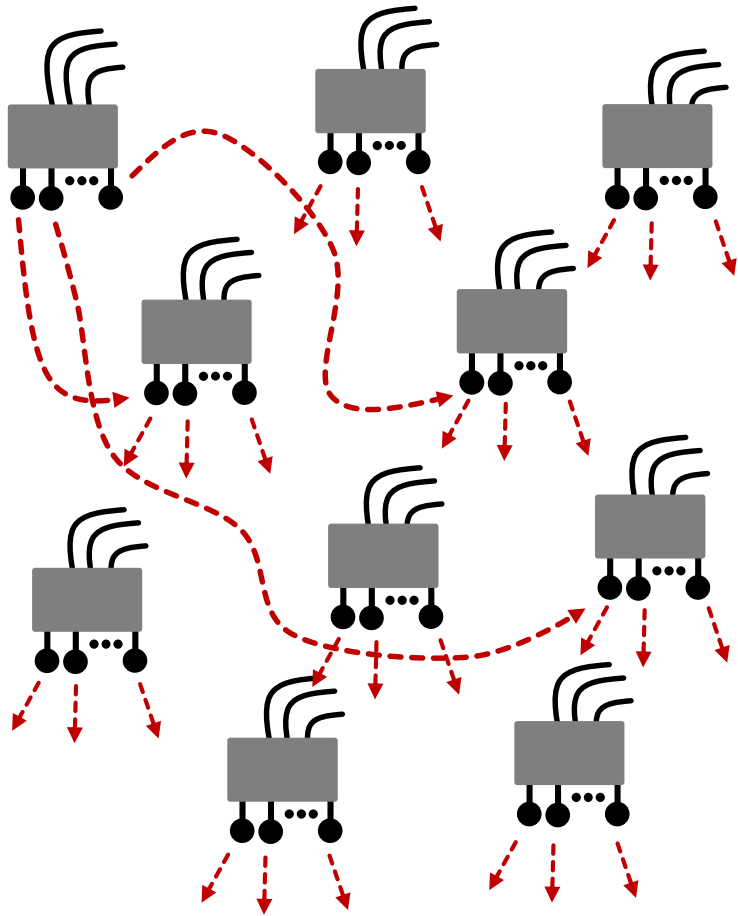
# RESULTS: PERFORMANCE



## Routing protocol

- Slim Fly (Valiant)
- ▲ Slim Fly (Minimum)
- Slim Fly (UGAL-L)
- ✦ Slim Fly (UGAL-G)
- ◆ Dragonfly (UGAL-L)
- ◇ Fat Tree (ANCA)

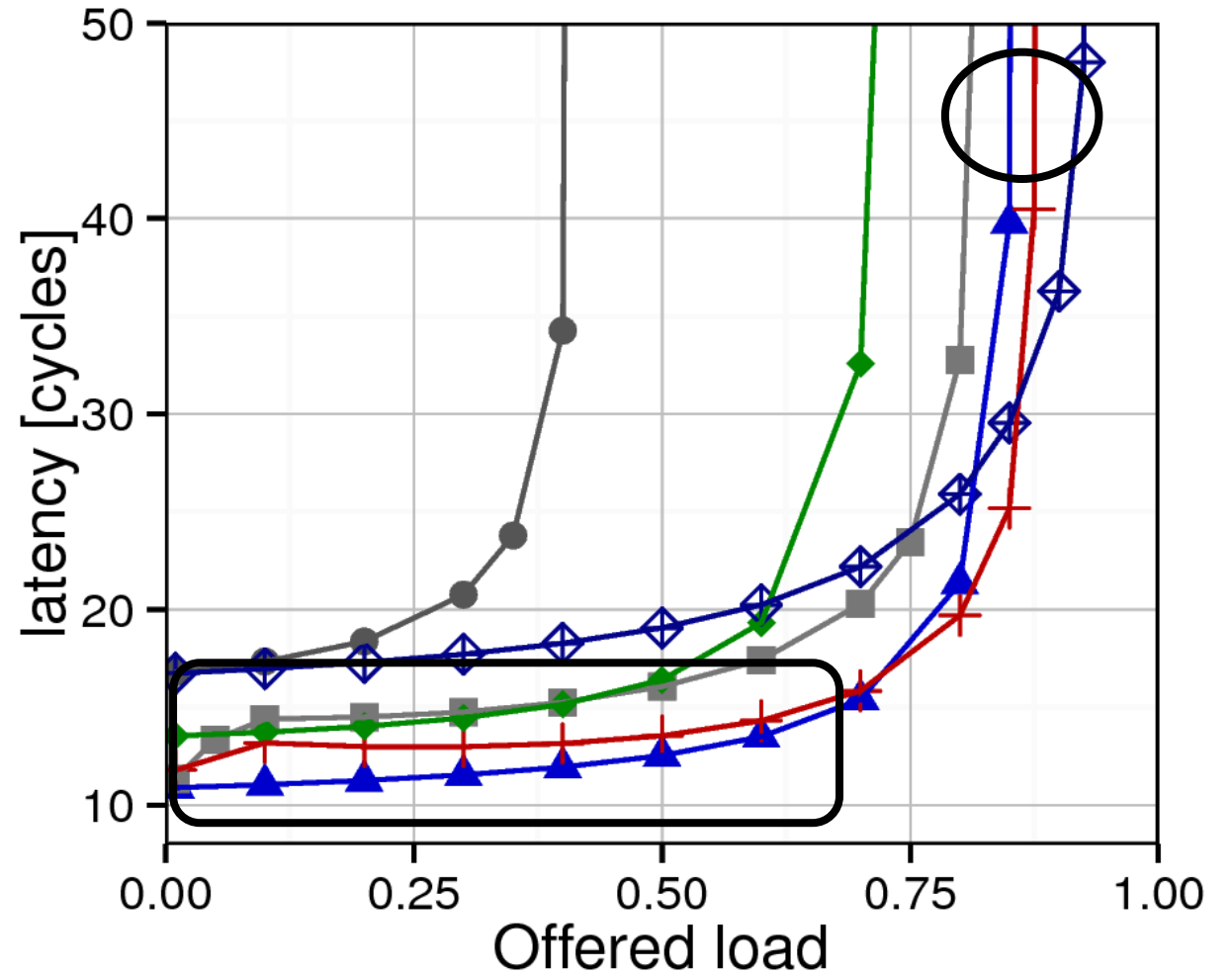
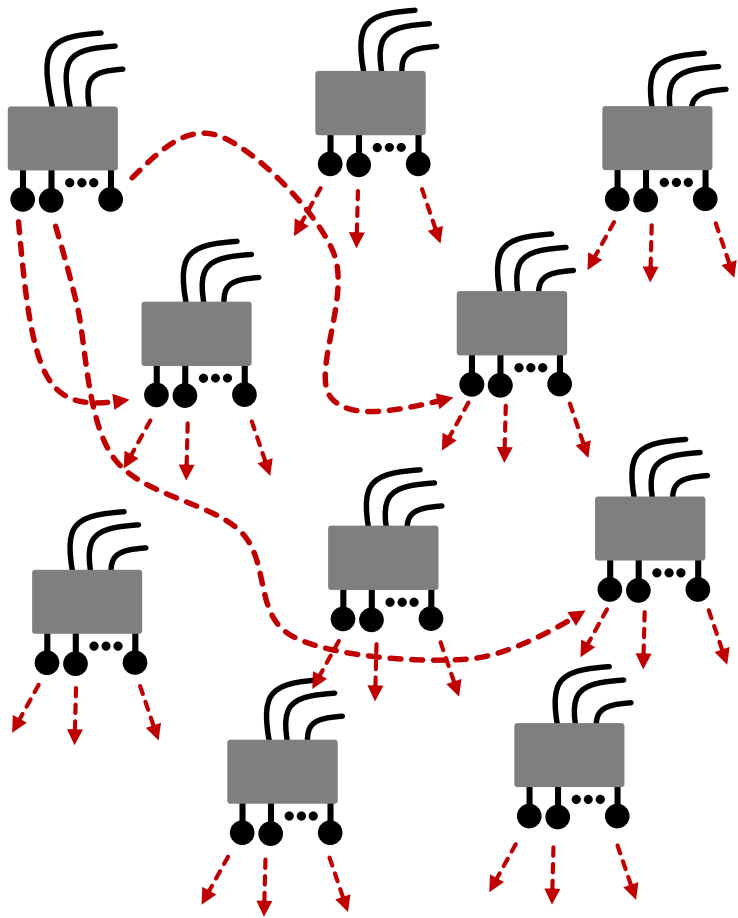
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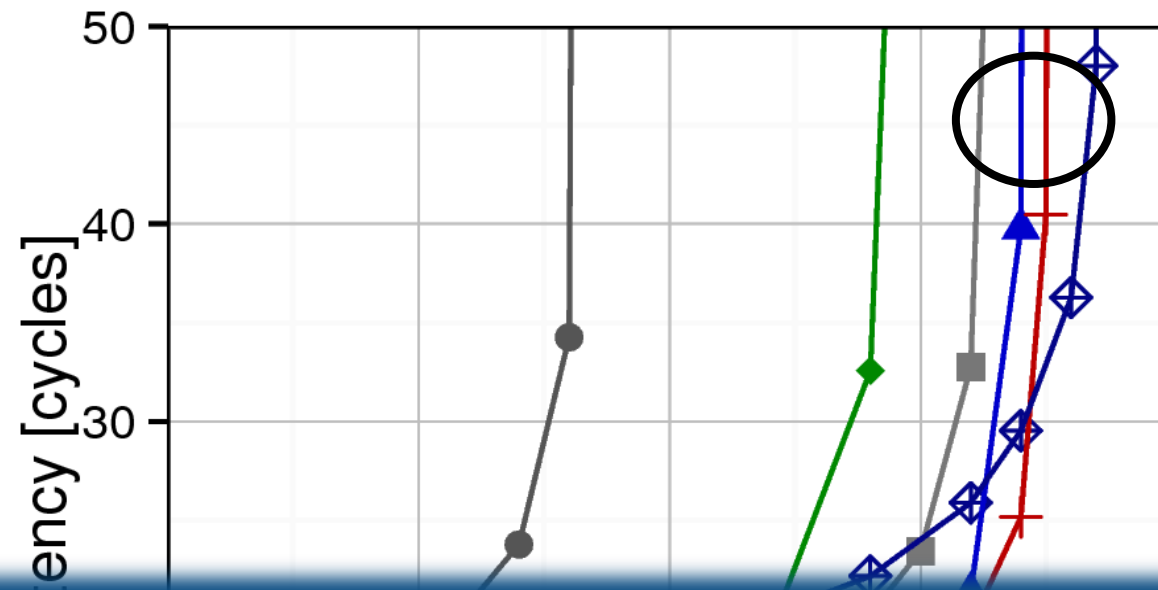
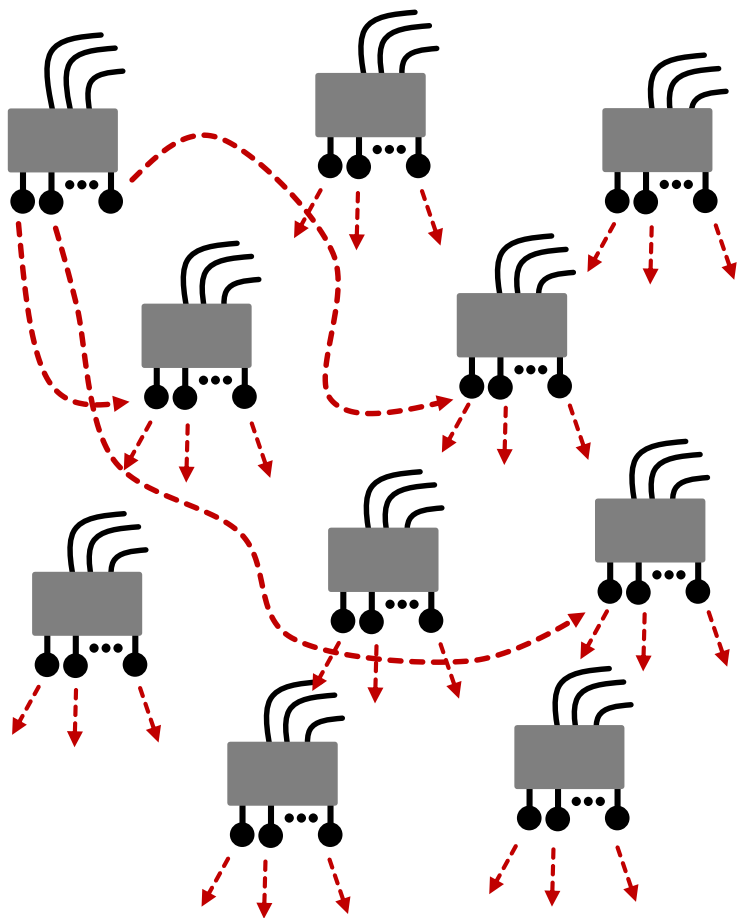
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# RESULTS: PERFORMANCE



Lowest latency, throughput better than that of Dragonfly and negligibly lower than that of Fat tree.

- ▲ Slim Fly (Minimum)
- Slim Fly (UGAL-L)
- ◆ Dragonfly (UGAL-L)
- ◆ Fat Tree (ANCA)

# RESULTS: A CASE STUDY



## RESULTS: A CASE STUDY

- A Slim Fly with;
  - $N = 10,830$
  - $k = 43$
  - $N_r = 722$

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Topology					
Endpoints ( $N$ )	19,876	40,200	20,736	58,806	<b>10,830</b>
Routers ( $N_r$ )	2,311	4,020	1,728	5,346	<b>722</b>
Radix ( $k$ )	<b>43</b>	<b>43</b>	<b>43</b>	<b>43</b>	<b>43</b>
Electric cables	19,414	32,488	9,504	56,133	<b>6,669</b>
Fiber cables	40,215	33,842	20,736	29,524	<b>6,869</b>
Cost per node [\$]	2,346	1,743	1,570	1,438	<b>1,033</b>
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Topology					
Endpoints ( $N$ )	<b>10,718</b>	<b>9,702</b>	<b>10,000</b>	<b>9,702</b>	<b>10,830</b>
Routers ( $N_r$ )	1,531	1,386	1,000	1,386	<b>722</b>
Radix ( $k$ )	35	28	33	27	<b>43</b>
Electric cables	7,350	6,837	4,500	9,009	<b>6,669</b>
Fiber cables	24,806	7,716	10,000	4,900	<b>6,869</b>
Cost per node [\$]	2,315	1,566	1,535	1,342	<b>1,033</b>
Power per node [W]	14.0	11.2	10.8	10.8	<b>8.02</b>

# RESULTS: A CASE STUDY

- A Slim Fly with;
  - $N = 10,830$
  - $k = 43$
  - $N_r = 722$

Topology	Fat tree	Random	Flat. Butterfly	Dragonfly	Slim Fly
Endpoints ( $N$ )	19,876	40,200	20,736	58,806	<b>10,830</b>
Routers ( $N_r$ )	2,311	4,020	1,728	5,346	<b>722</b>
Radix ( $k$ )	<b>43</b>	<b>43</b>	<b>43</b>	<b>43</b>	<b>43</b>
Electric cables	19,414	32,488	9,504	56,133	<b>6,669</b>
Fiber cables	40,215	33,842	20,736	29,524	<b>6,869</b>
Cost per node [\$]	2,346	1,743	1,570	1,438	<b>1,033</b>
Power per node [W]	14.0	12.04	10.8	10.9	<b>8.02</b>

Topology					Slim Fly
Endpoints ( $N$ )	<b>10,718</b>	<b>9,702</b>	<b>10,000</b>	<b>9,702</b>	<b>10,830</b>
Routers ( $N_r$ )	1,531	1,386	1,000	1,386	<b>722</b>
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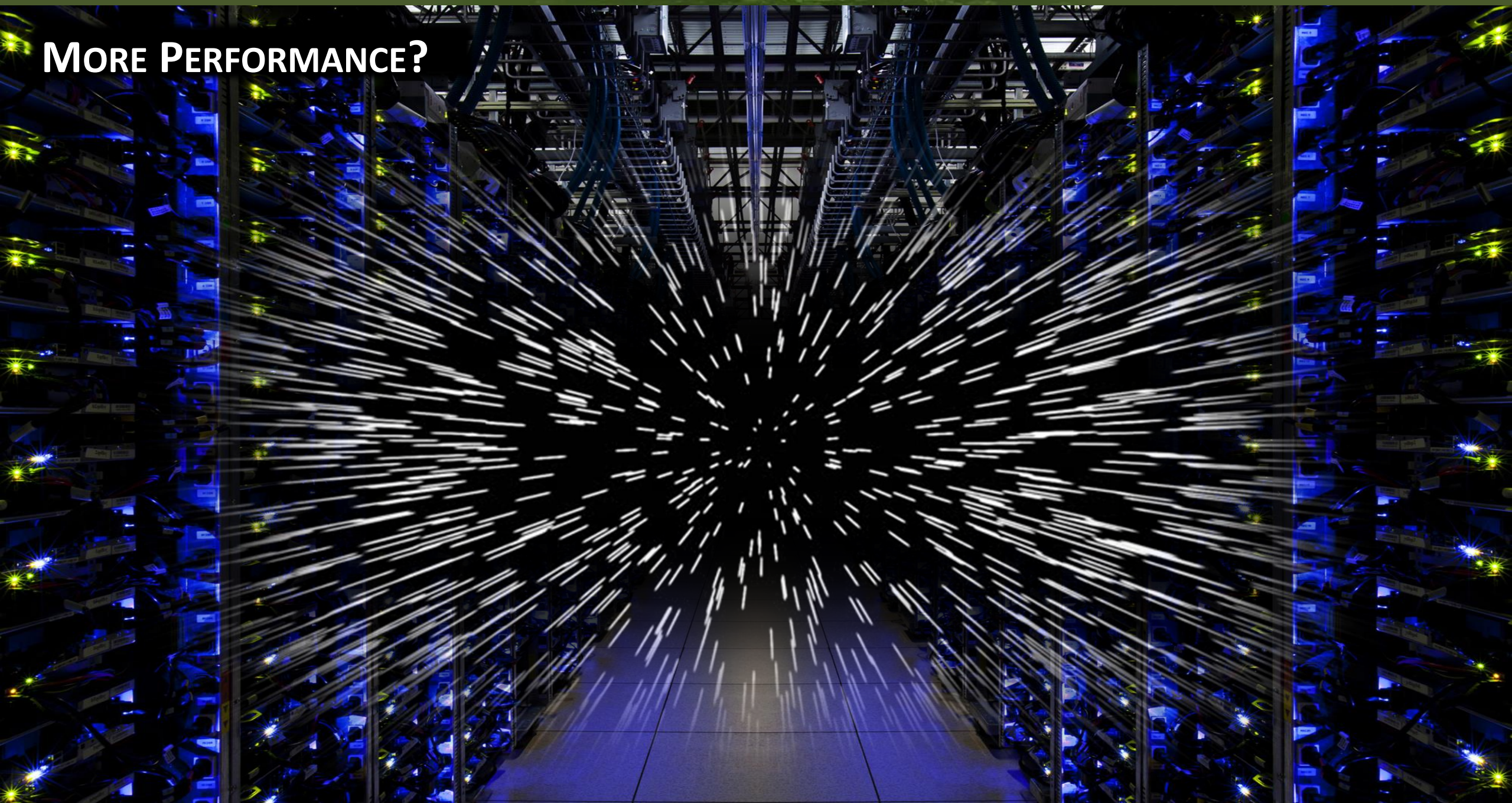
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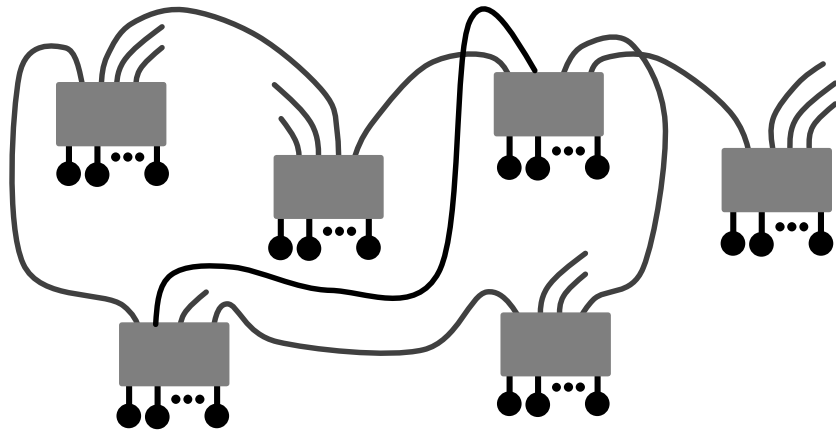
# MORE PERFORMANCE?



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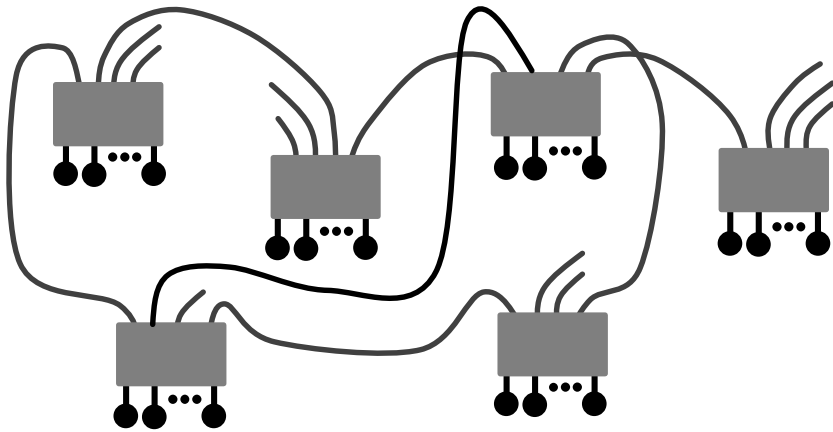


# MULTI PATHING?



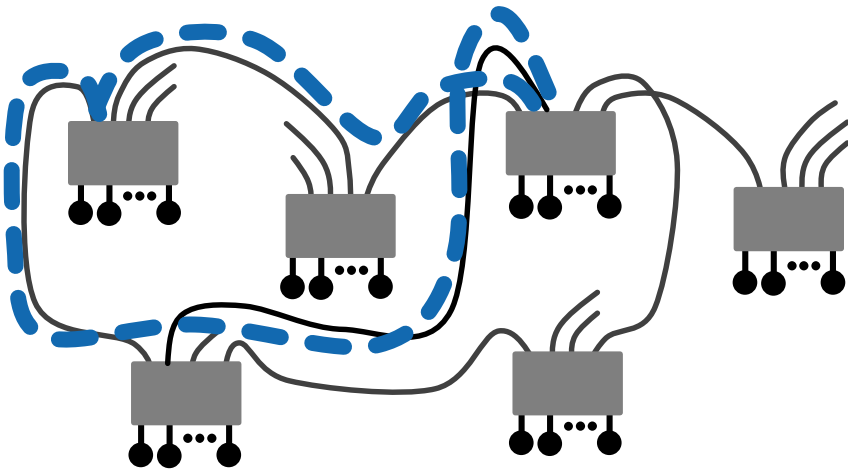
# MULTI PATHING?

## Minimal paths?



# MULTI PATHING?

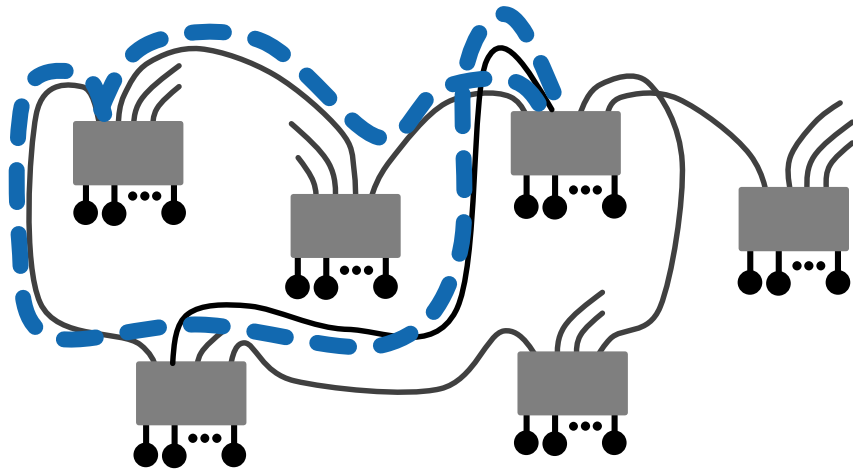
Minimal paths?



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Minimal paths?

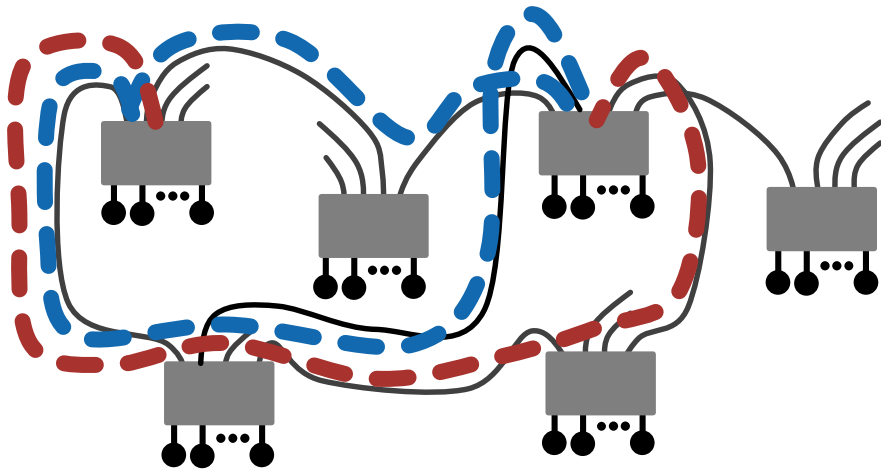
Non-minimal paths?



# MULTI PATHING?

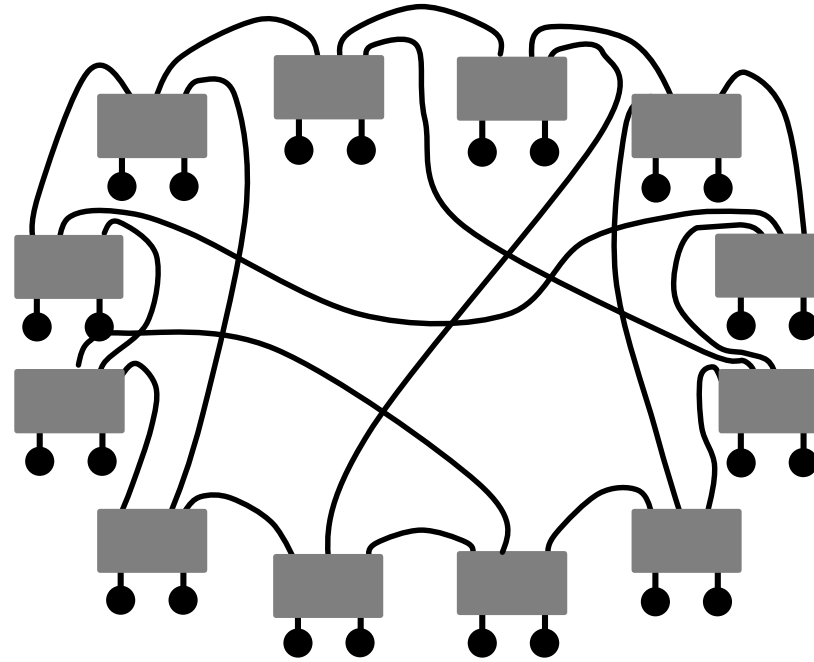
Minimal paths?

Non-minimal paths?



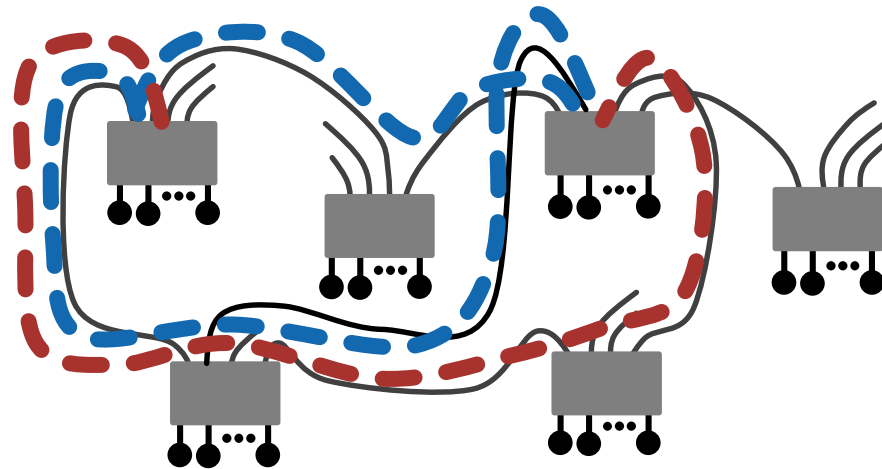


# MULTI PATHING?

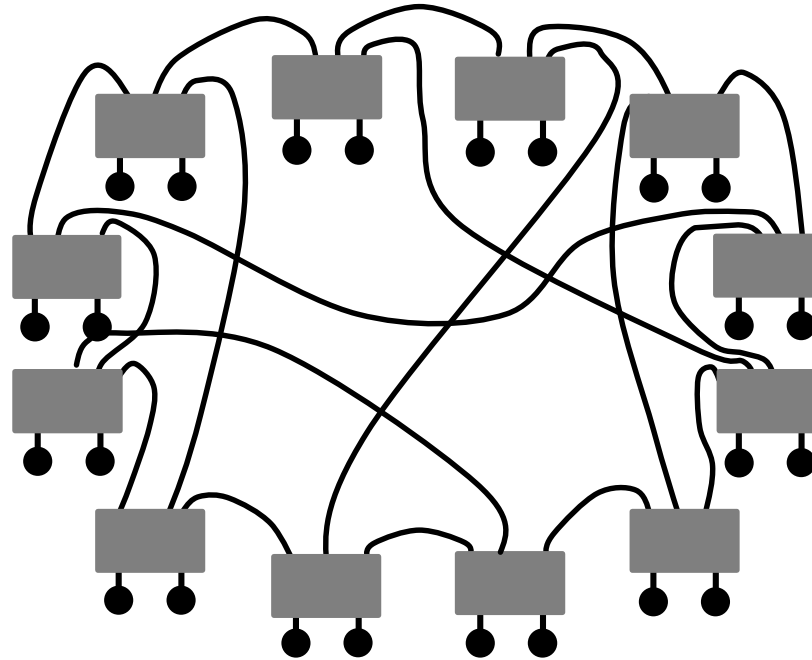


Minimal paths?

Non-minimal paths?

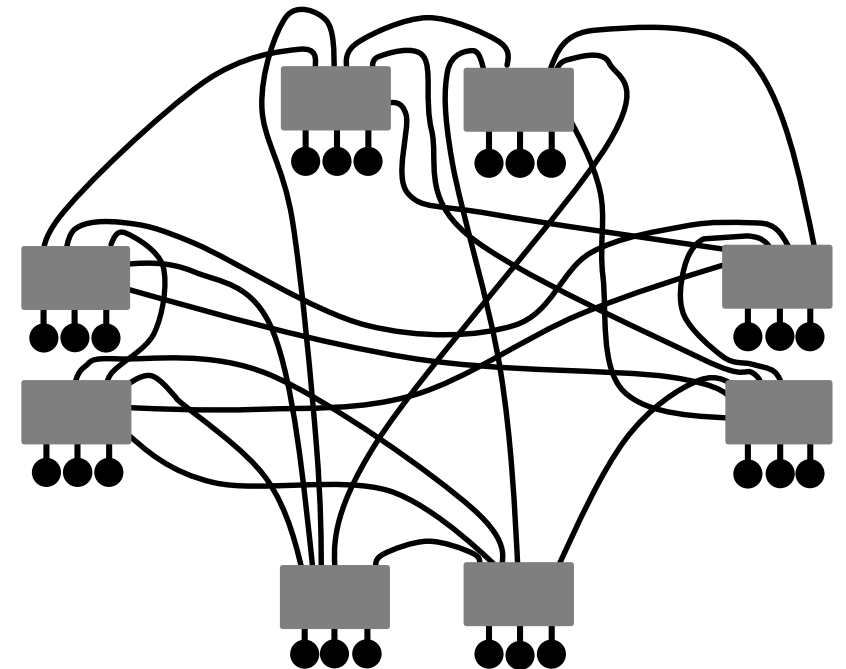
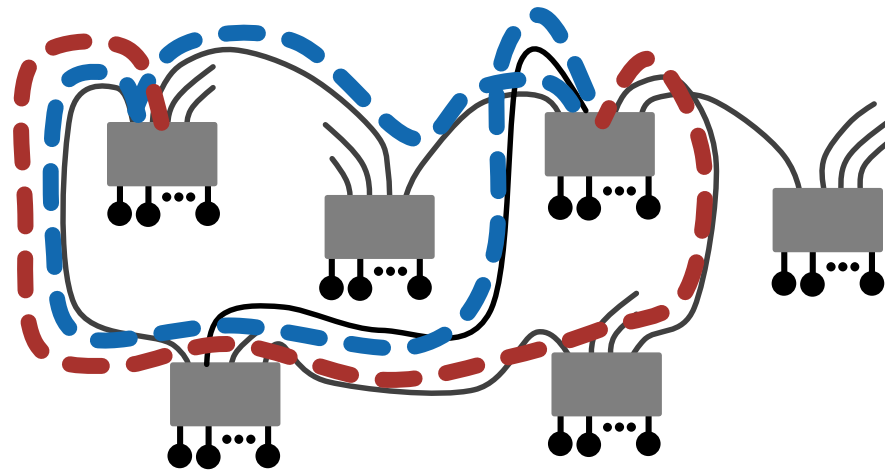


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Minimal paths?

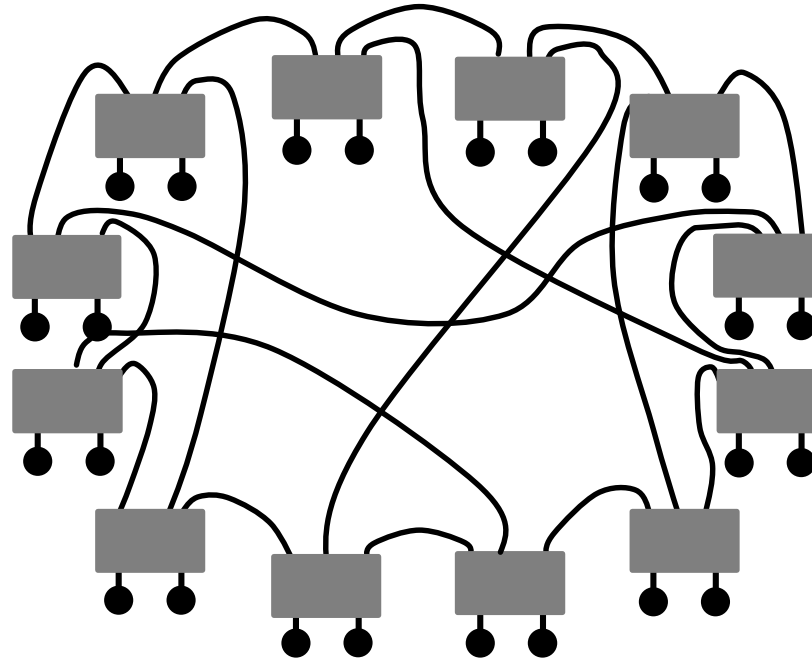
Non-minimal paths?



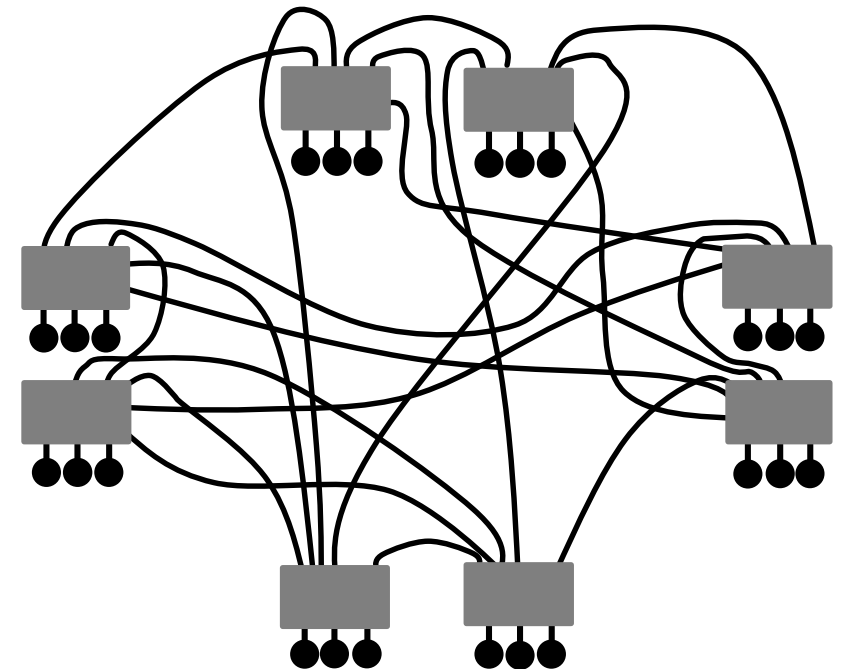
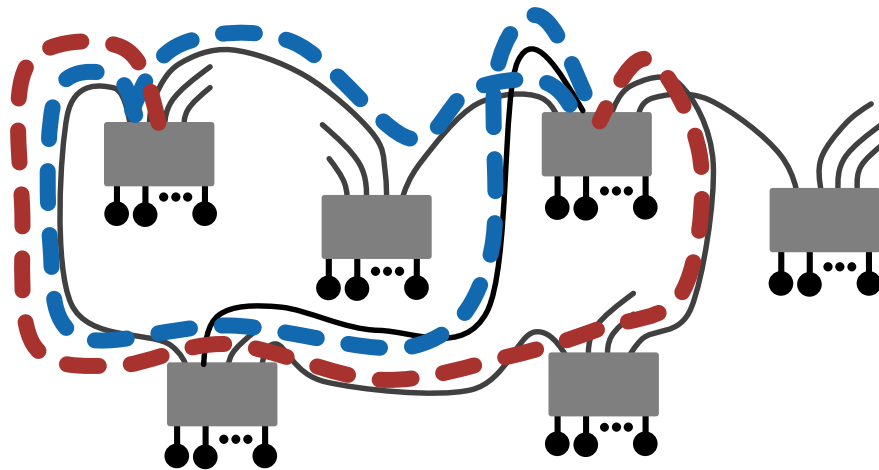
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Minimal paths?

Non-minimal paths?



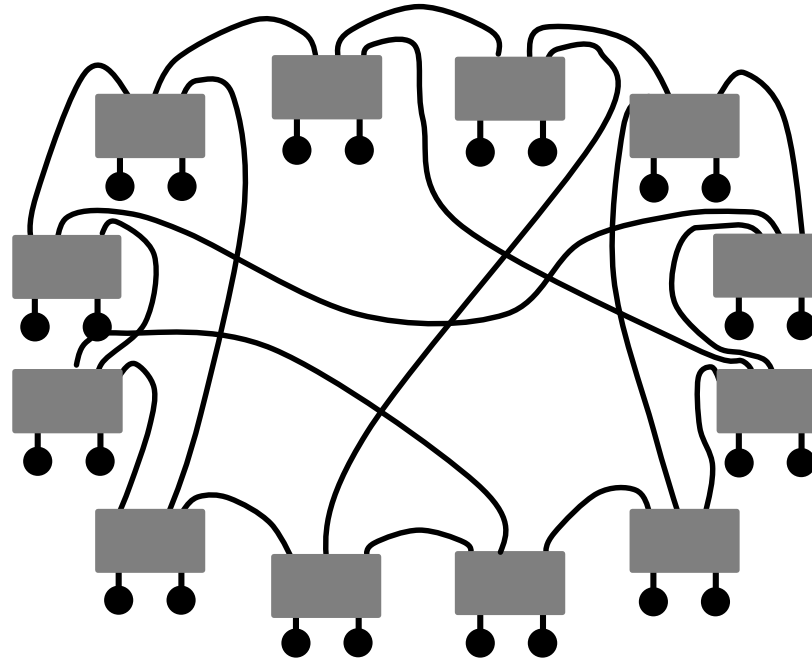
Expanders  
are sparse...



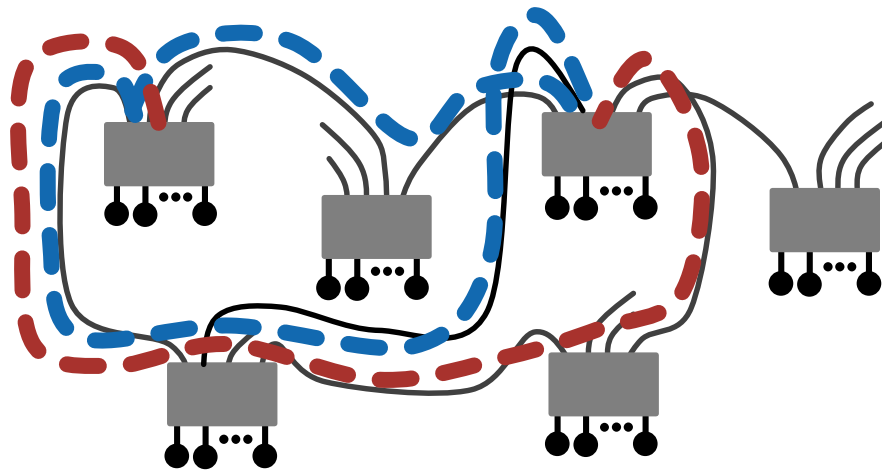
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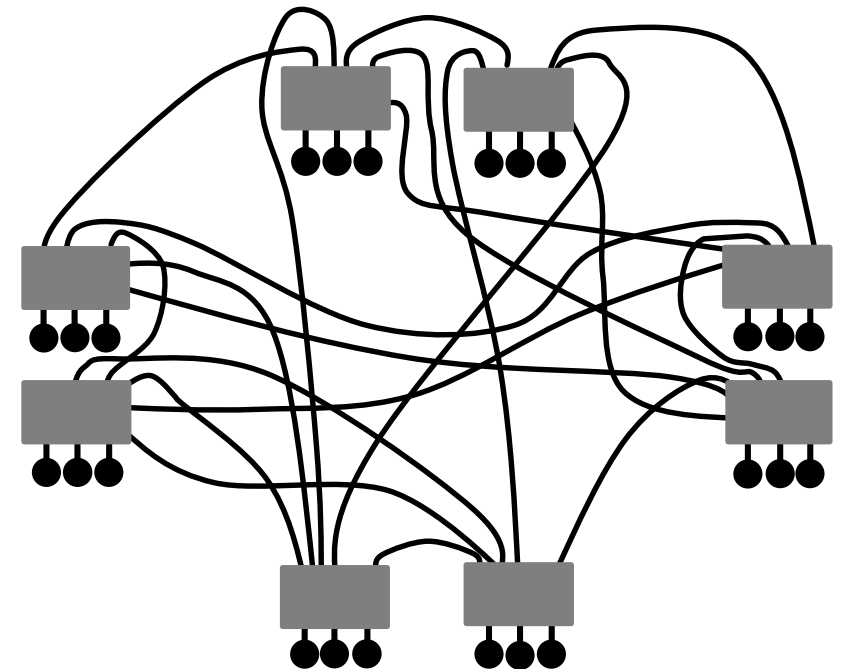
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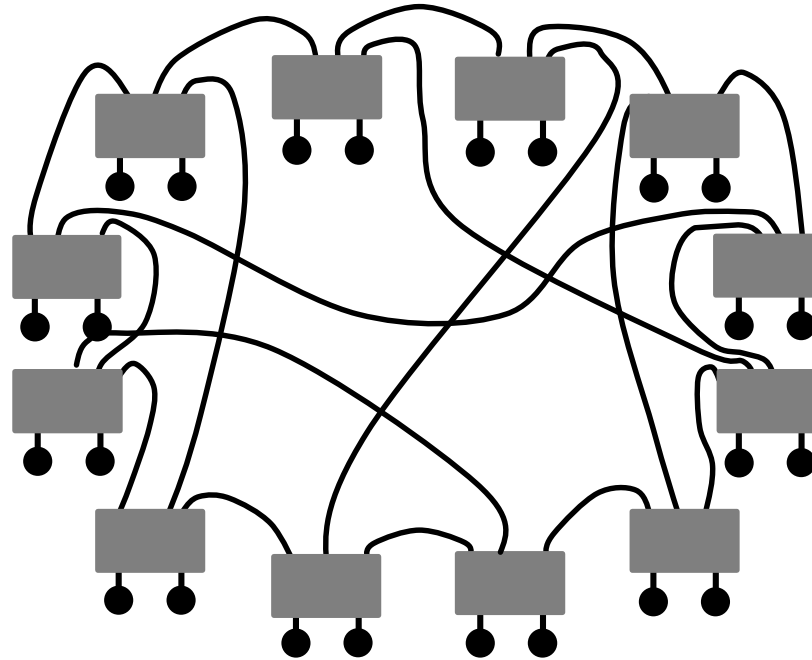
IS THERE  
ENOUGH  
PATH  
DIVERSITY?



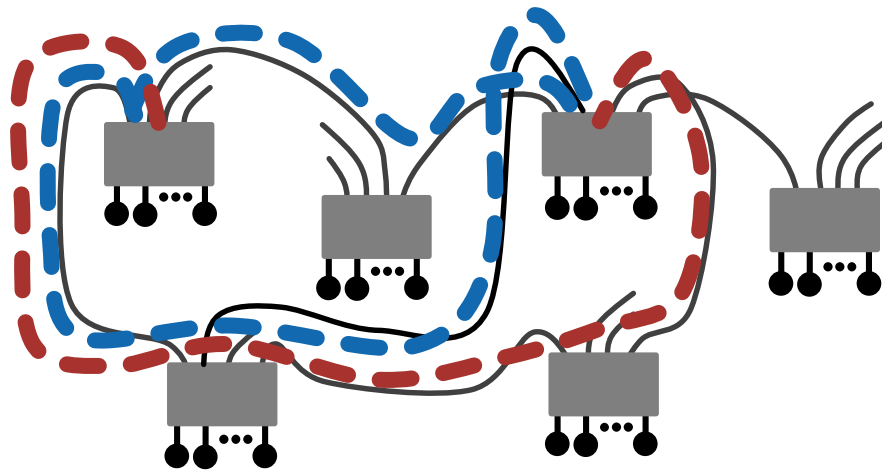
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Minimal paths?

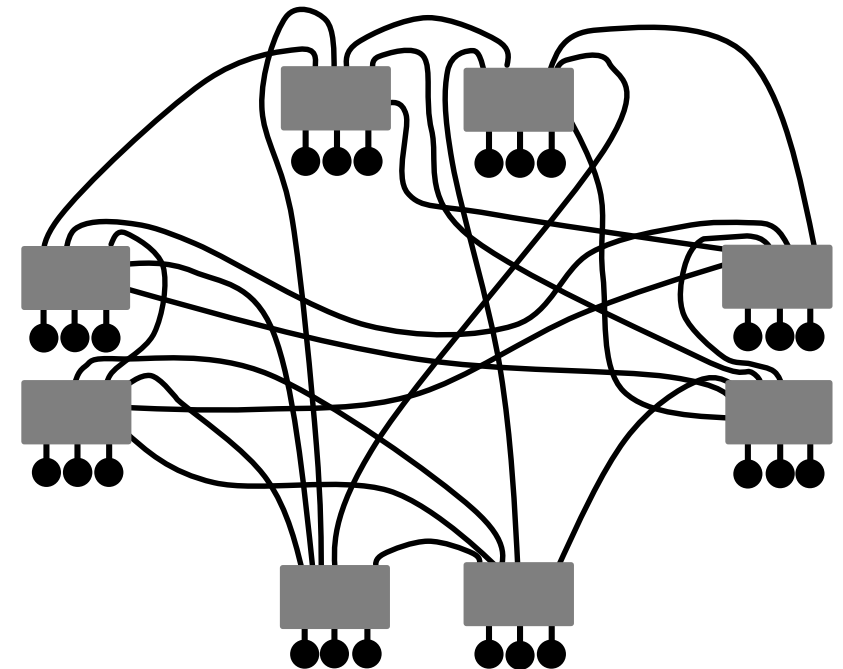
Non-minimal paths?



Expanders  
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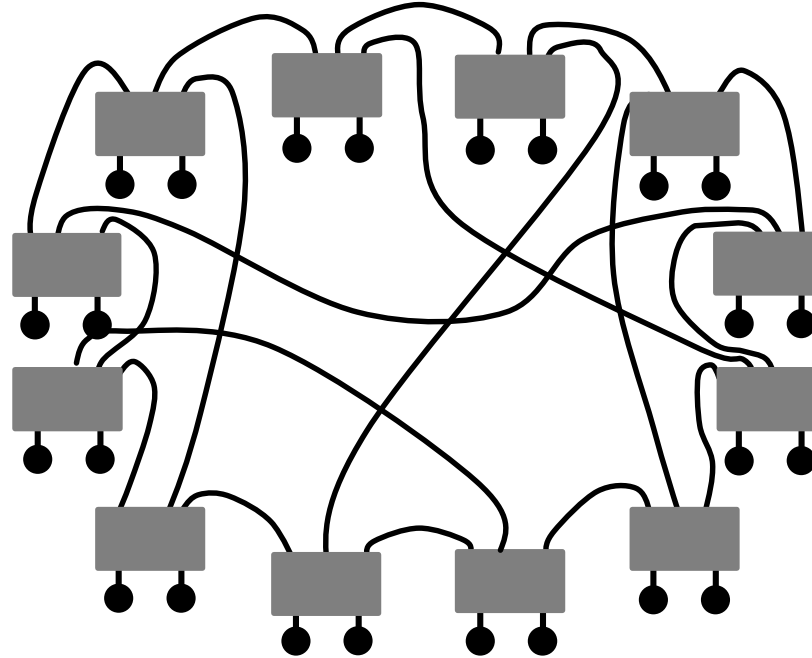


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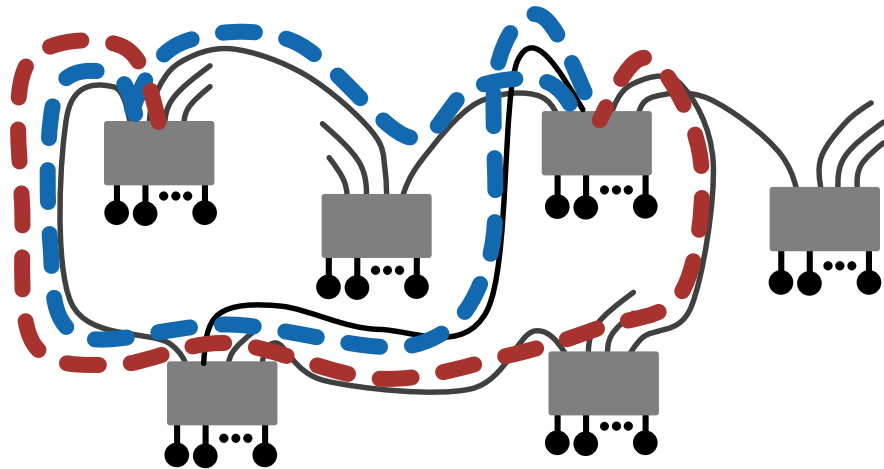
**HOW TO USE MULTIPATHING?**

Minimal paths?

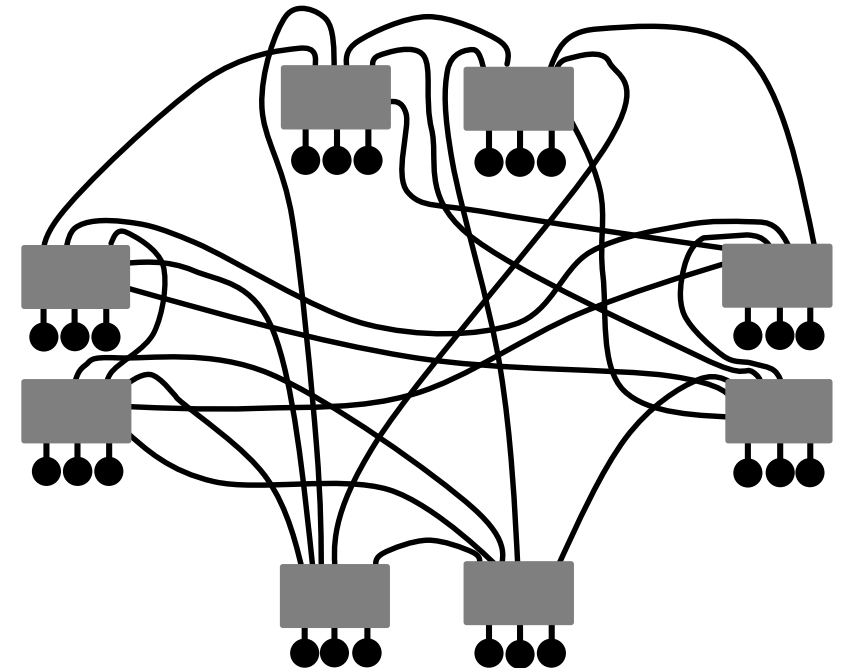
Non-minimal paths?



Expanders  
are sparse...

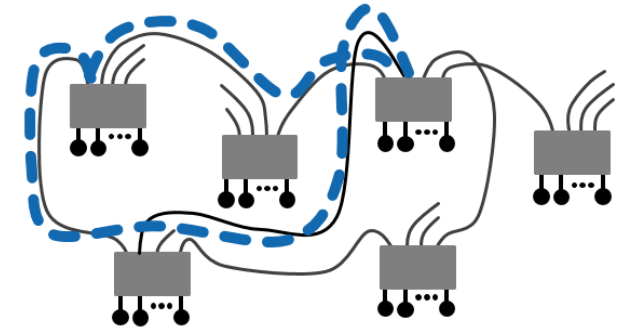


**IS THERE ENOUGH PATH DIVERSITY?**

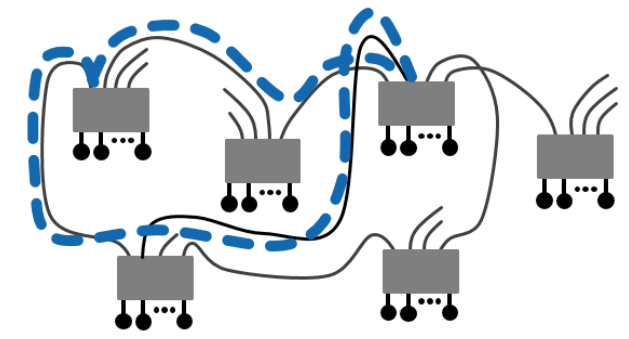


## RESULTS: MINIMAL PATHS

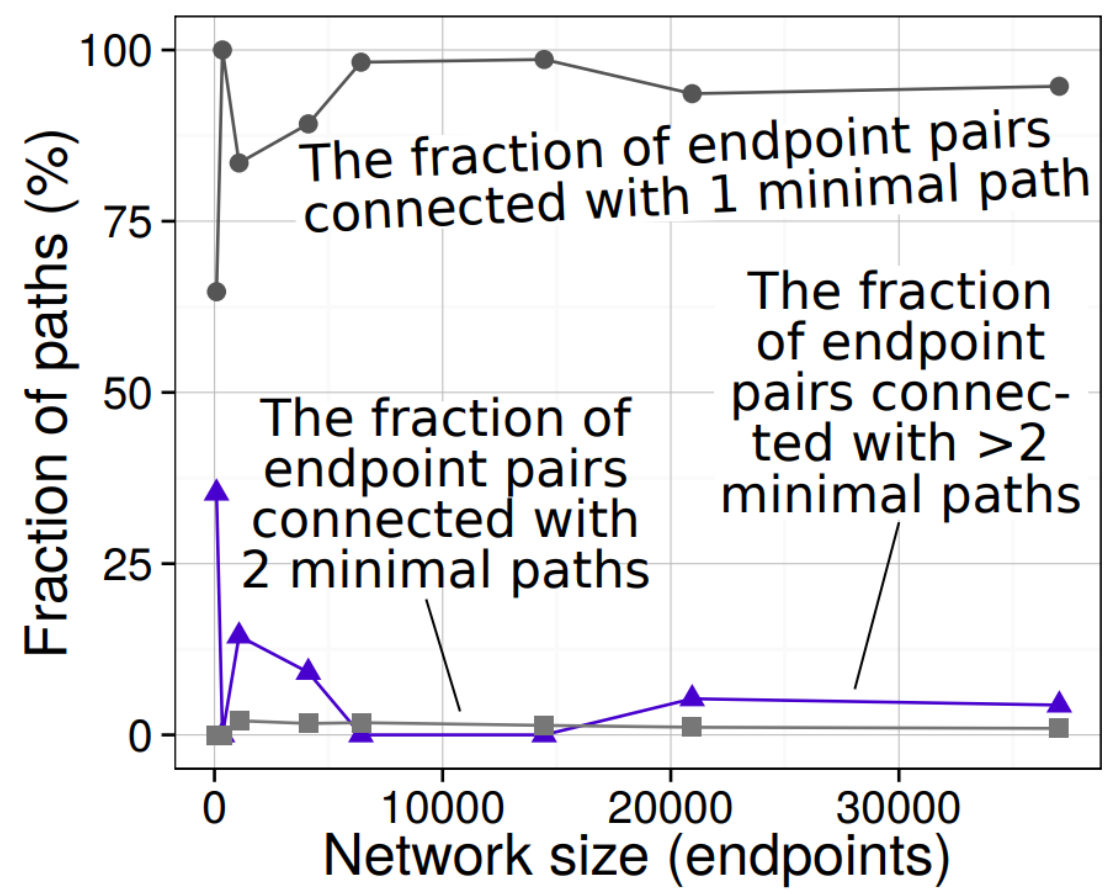
Slim Fly



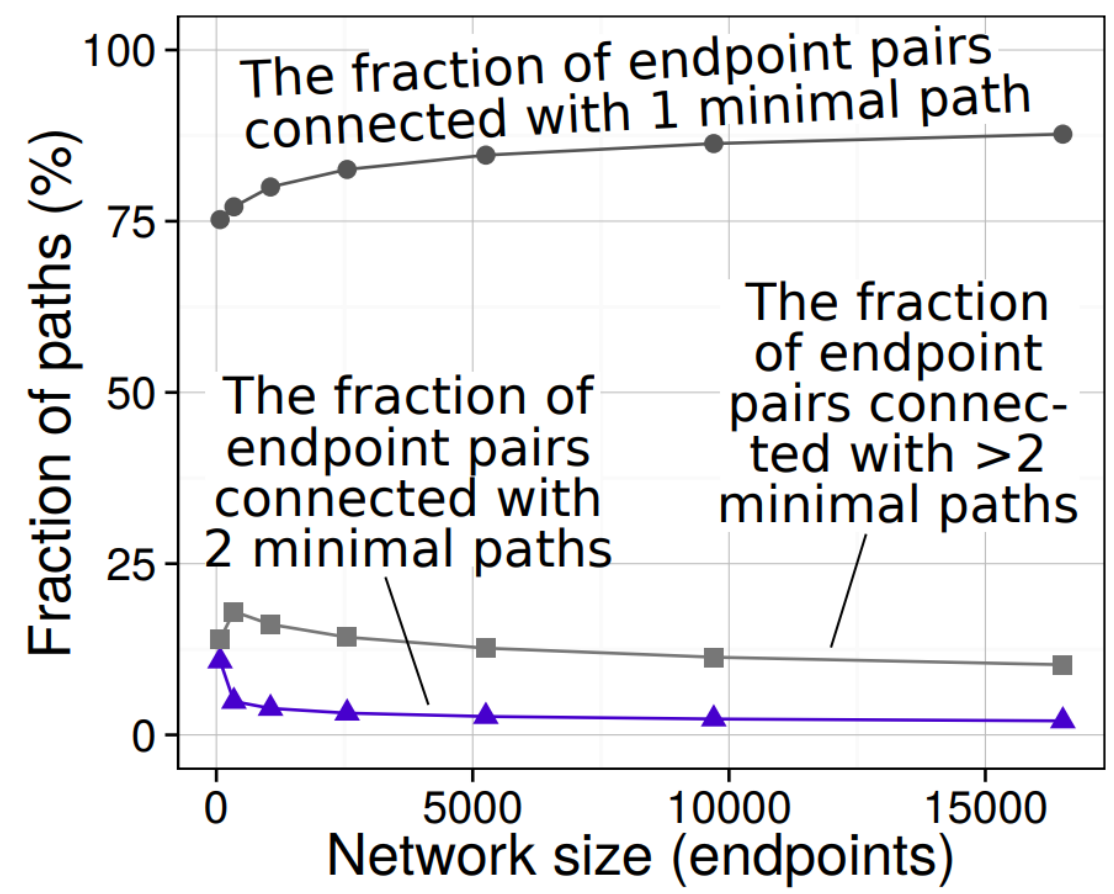
# RESULTS: MINIMAL PATHS



## Slim Fly

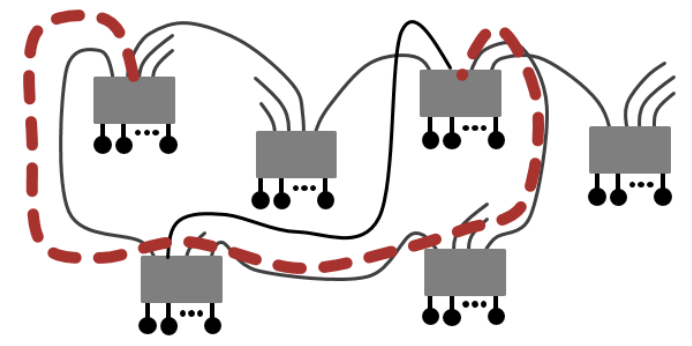
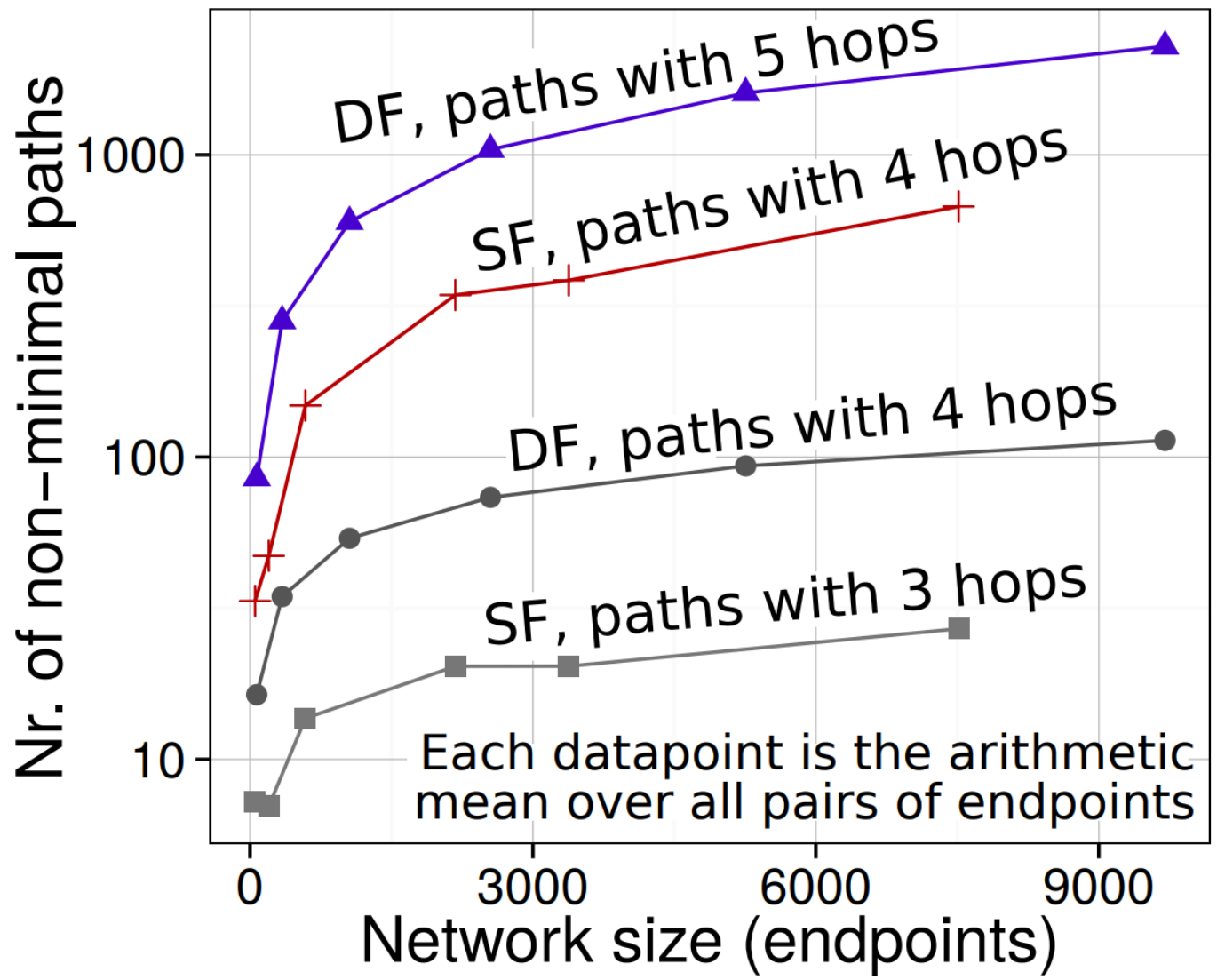


## Dragonfly

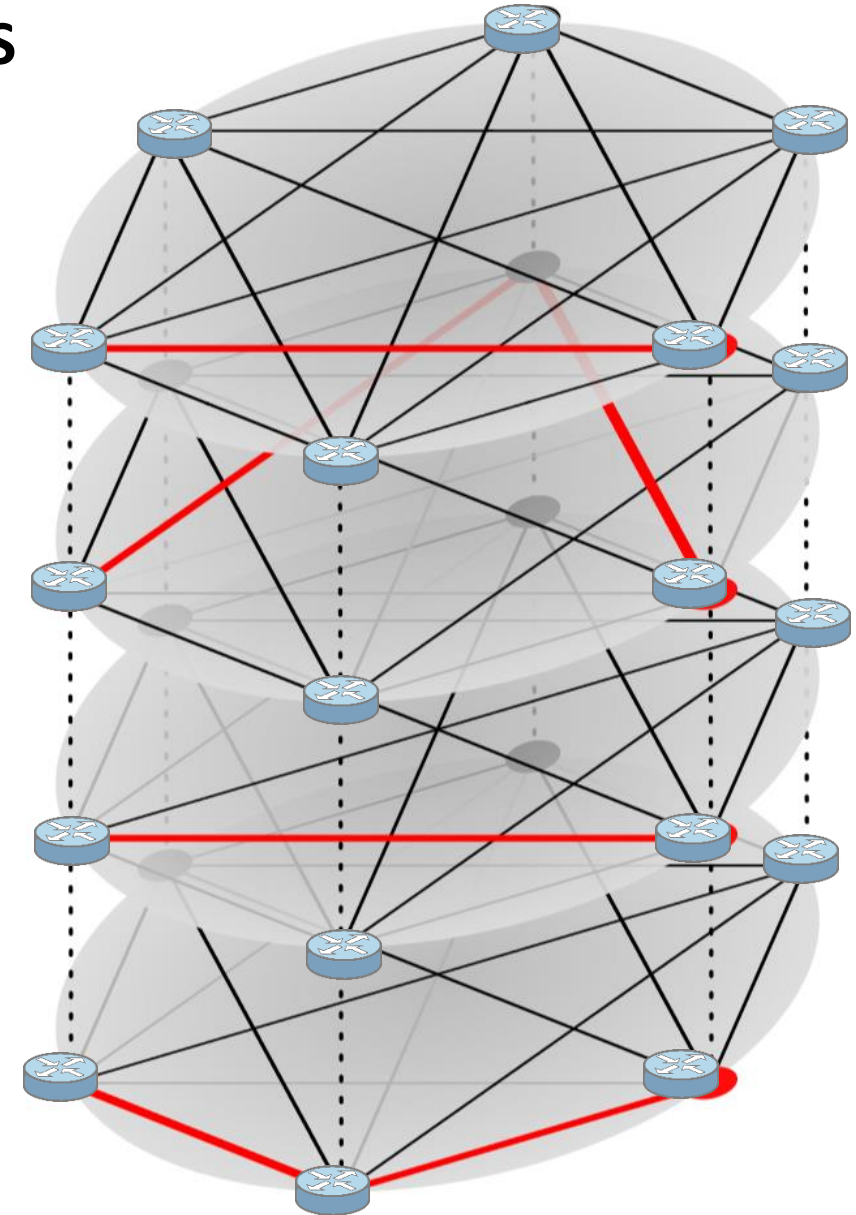




# RESULTS: NON-MINIMAL PATHS

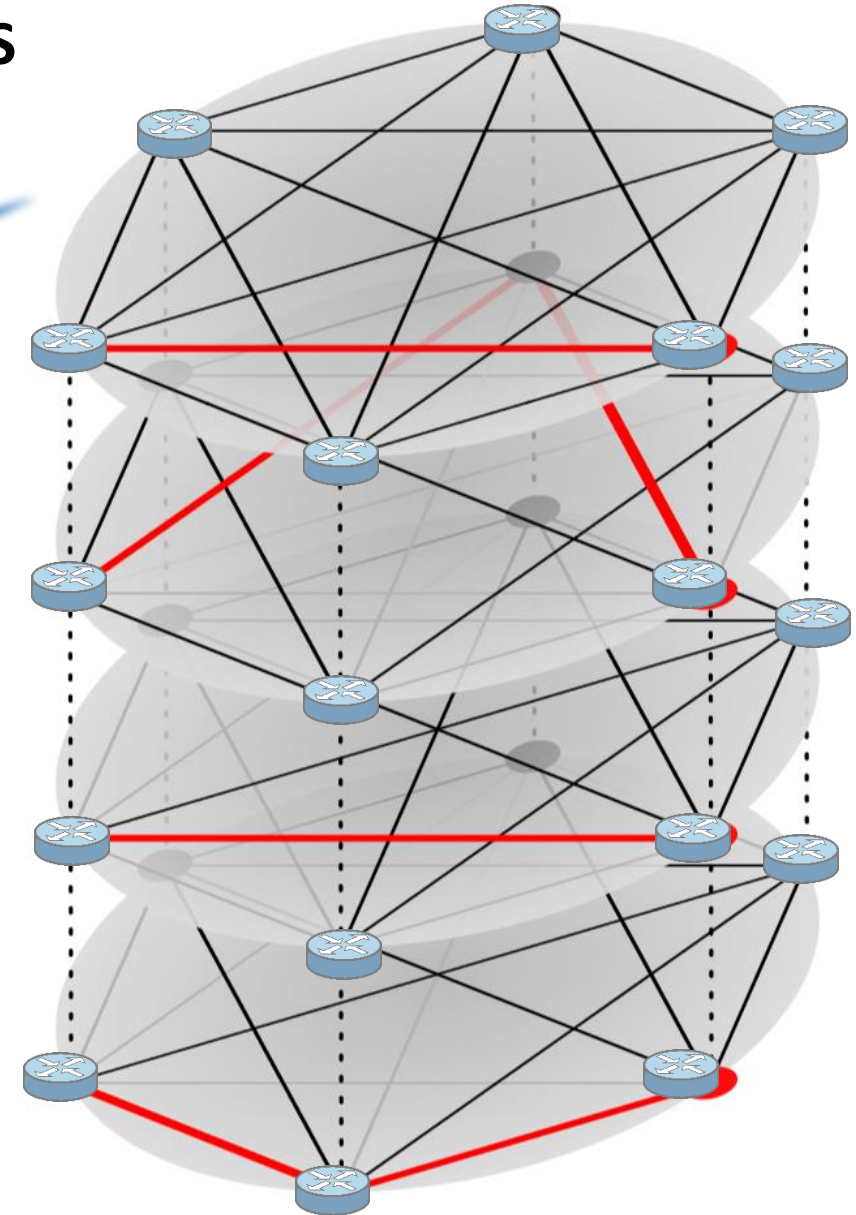


# DEPLOYMENT: LAYERS OF “ALMOST” MINIMAL PATHS



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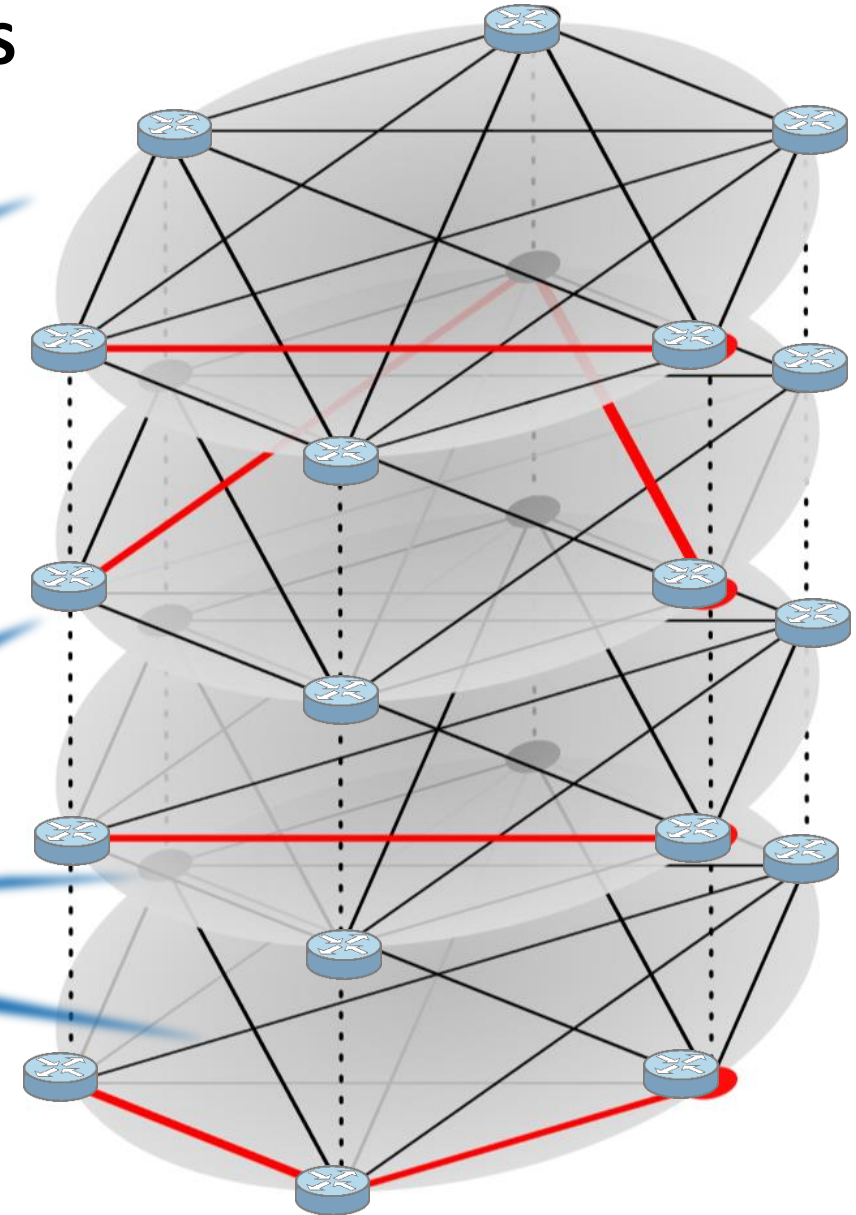
Layer 1: include all links and route minimally



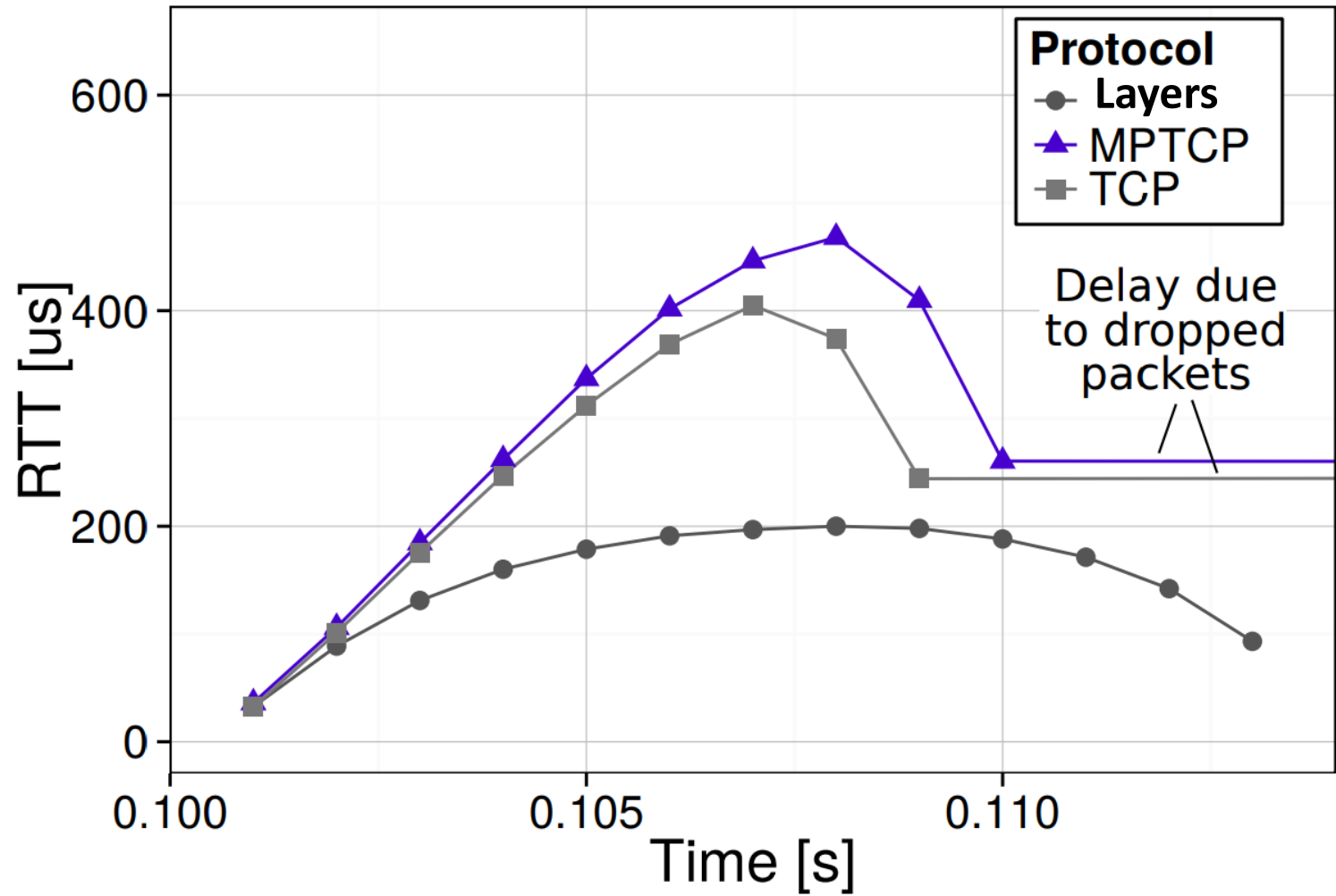
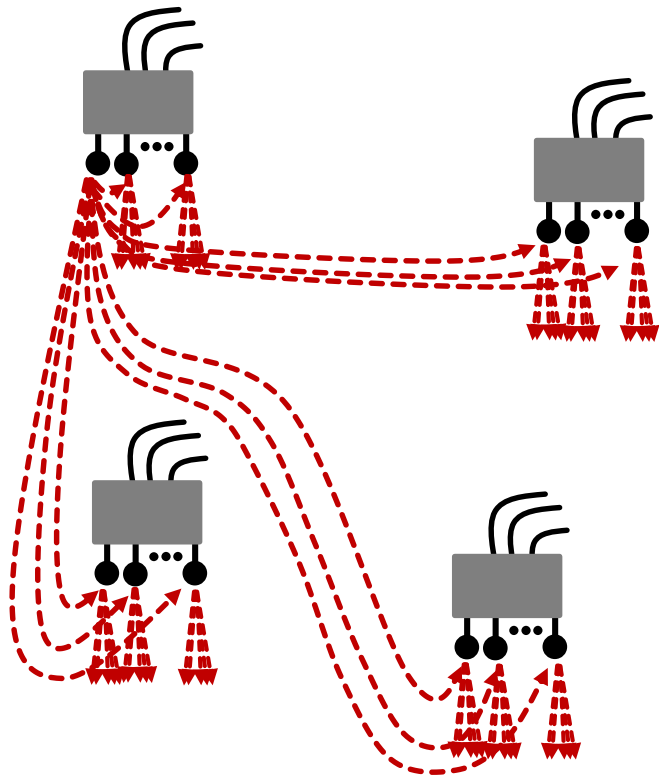
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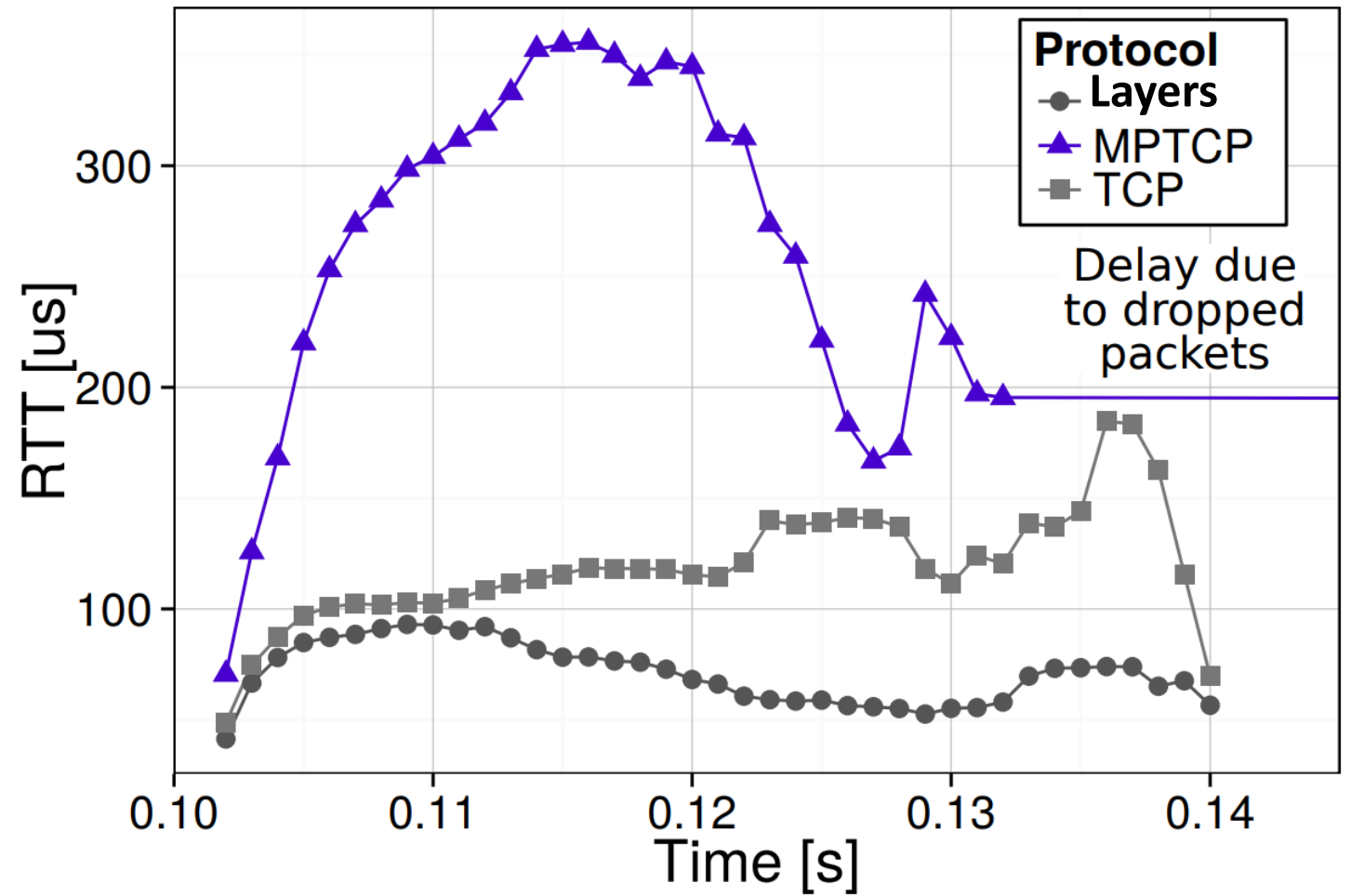
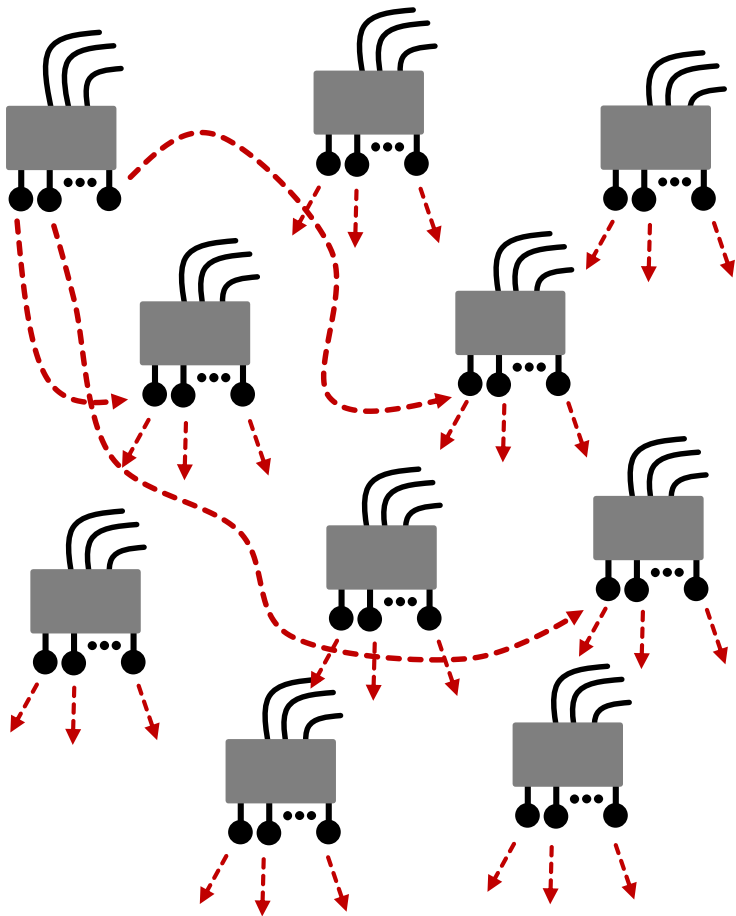
Layers 2-n: remove some links (e.g., select uniformly at random) and route minimally



# RESULTS: ALL-TO-ALL

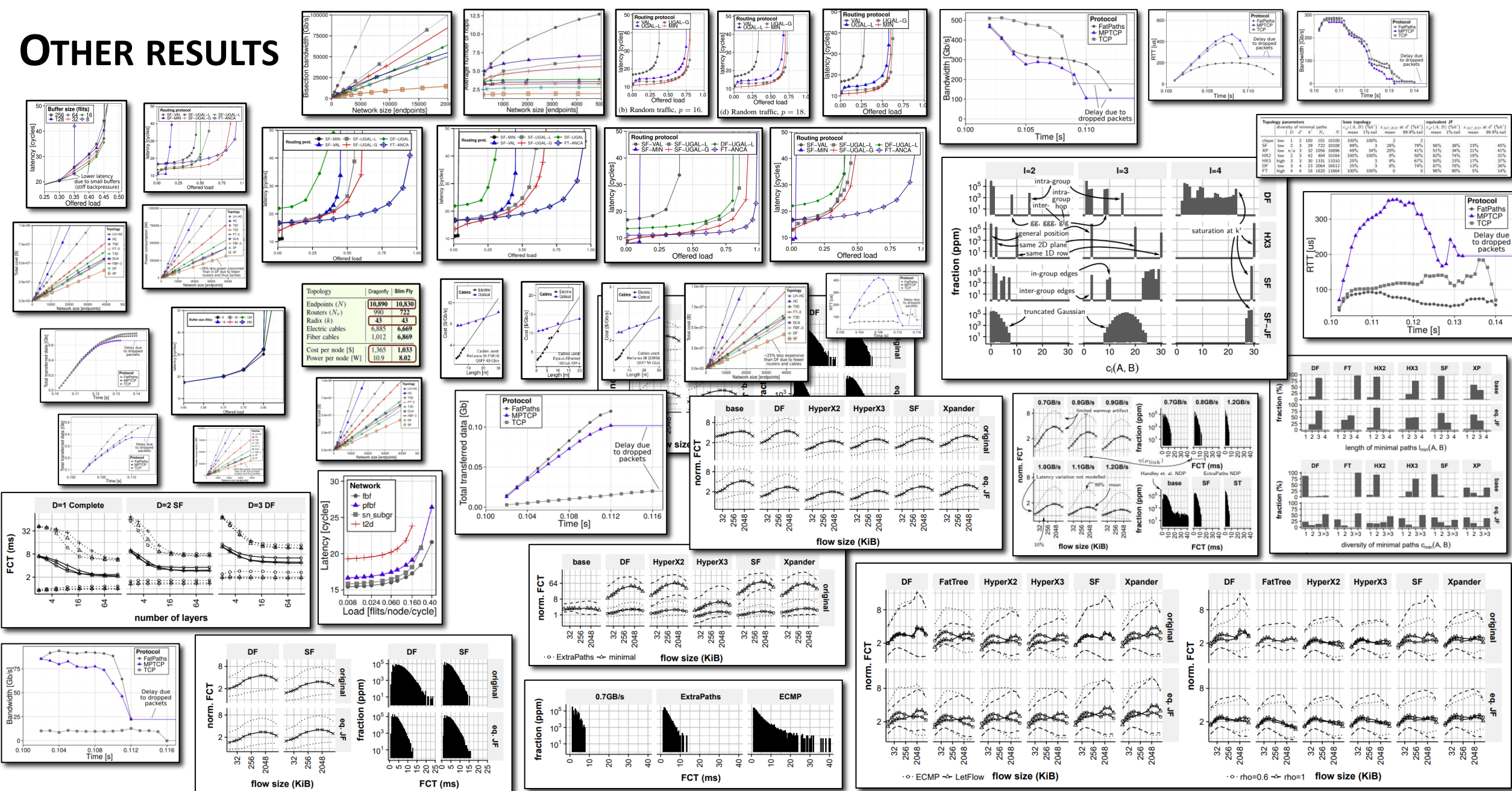


# RESULTS: RANDOM UNIFORM



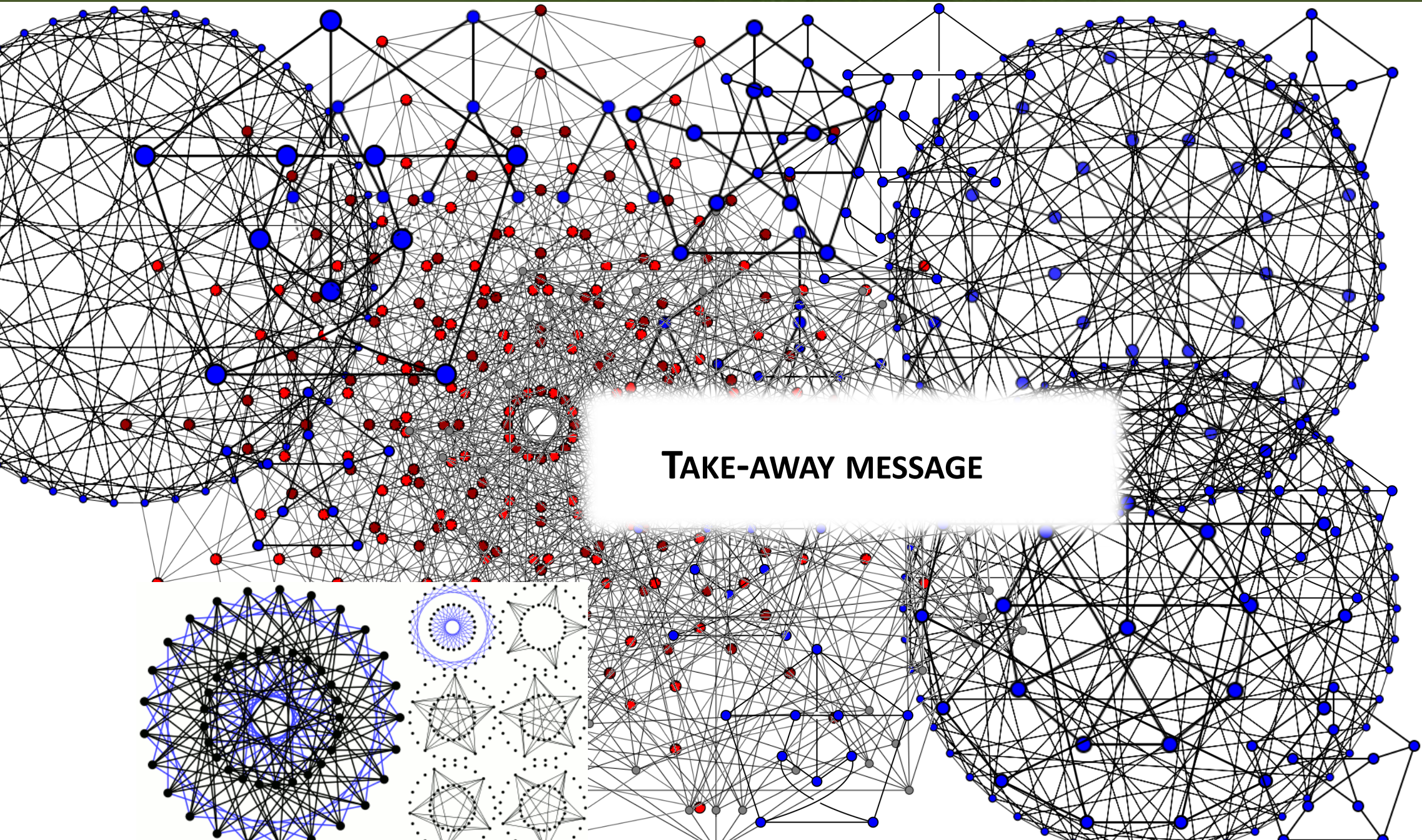
## OTHER RESULTS

# OTHER RESULTS





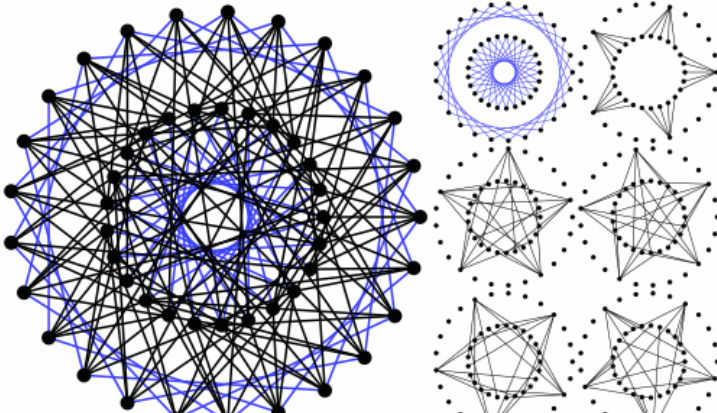






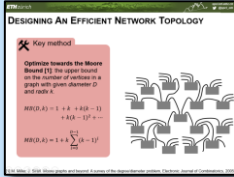
A LOWEST-DIAMETER TOPOLOGY

TAKE-AWAY MESSAGE



## A LOWEST-DIAMETER TOPOLOGY

- Approaching the Moore Bound
- Resilient



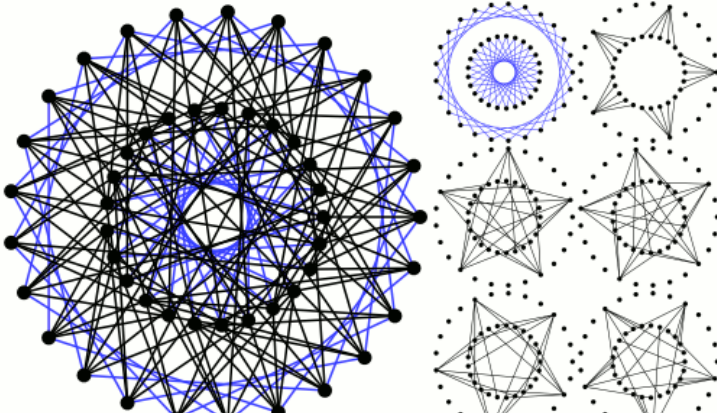
**STRUCTURE ANALYSIS**  
Resiliency\*

- Disconnection metrics\*
- Other studied metrics:
- Average path length (increase by 2):  
DF is 10% more resilient than DF

No. of	Health	Health	Health	Loss No.	Fatigue	Impairment	Resilience	Resilience
512	30%	-	40%	55%	35%	-	55%	60%
1024	25%	-	40%	55%	40%	50%	60%	65%
2048	20%	-	40%	55%	40%	55%	65%	65%
4096	15%	-	45%	55%	55%	60%	70%	70%
8192	10%	35%	45%	55%	60%	65%	75%	75%

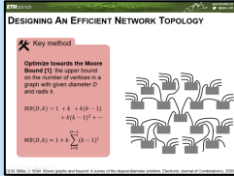
\*Missing values indicate the non-applicability of a measured metric, except for a given %.

## TAKE-AWAY MESSAGE



## A LOWEST-DIAMETER TOPOLOGY

- Approaching the Moore Bound
- Resilient



**STRUCTURE ANALYSIS**  
Resiliency\*

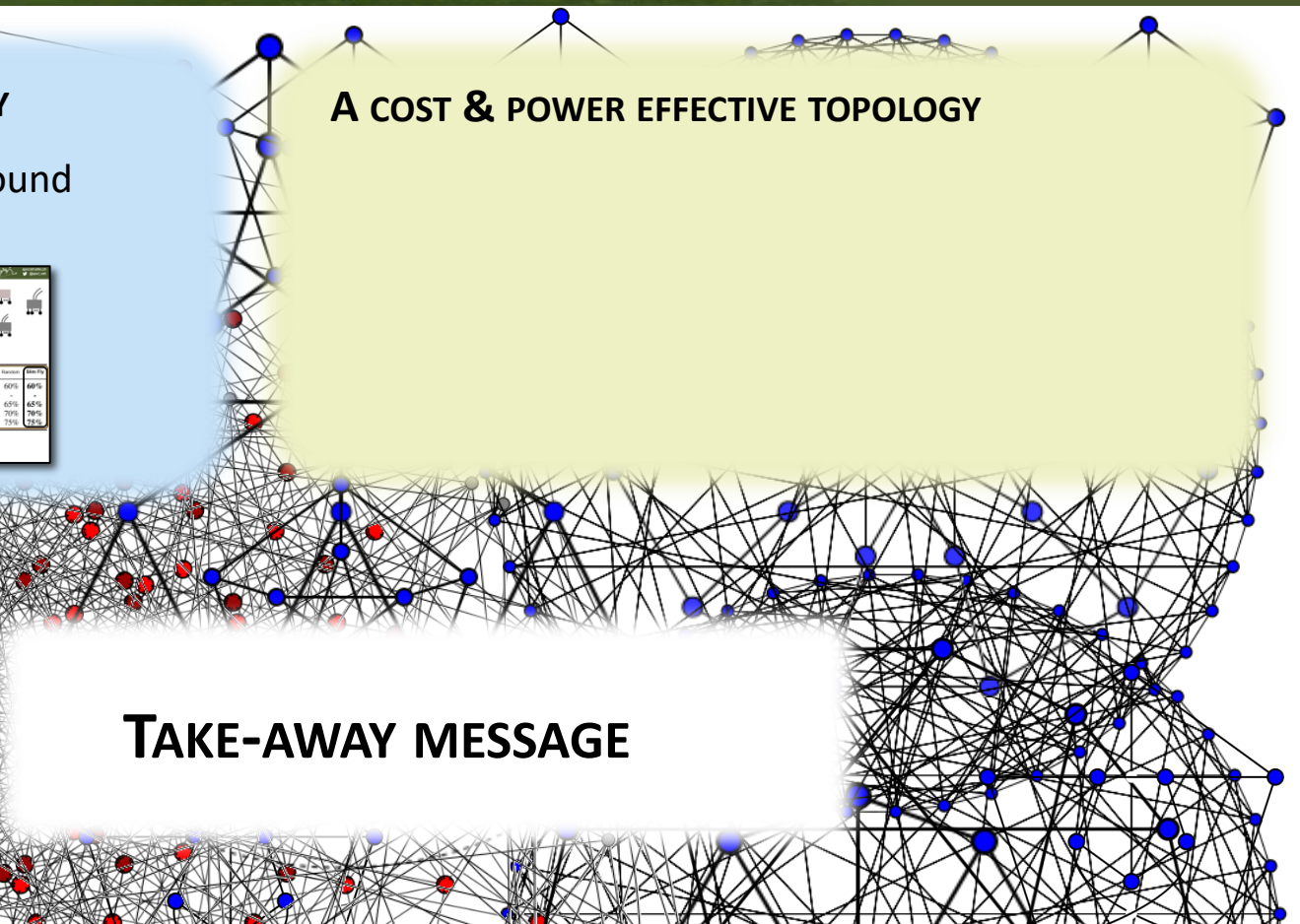
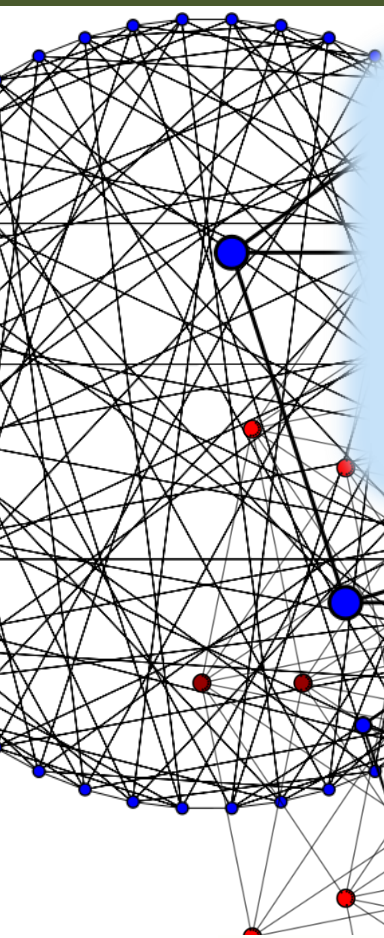
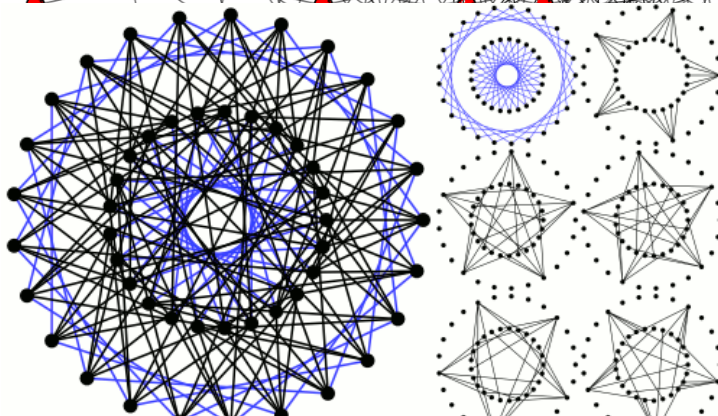
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- Other studied metrics:
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DF is 10% more resilient than DF

No. of Nodes	Health	Health	Health	Loss Rate	Fatigue	Disrupt	Ext. Industry	Resilient	Health
512	30%	40%	40%	55%	35%	-	55%	60%	60%
1024	25%	40%	40%	55%	40%	50%	60%	65%	65%
2048	20%	40%	55%	40%	55%	65%	65%	65%	65%
4096	15%	45%	55%	55%	60%	70%	70%	70%	70%
8192	10%	35%	45%	55%	60%	65%	70%	75%	75%

\*Missing values indicate the necessity of a backup topology variant for a given N.

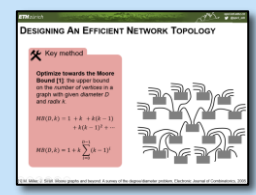
## A COST & POWER EFFECTIVE TOPOLOGY

# TAKE-AWAY MESSAGE



### A LOWEST-DIAMETER TOPOLOGY

- Approaching the Moore Bound
- Resilient



**STRUCTURE ANALYSIS**  
Resiliency\*

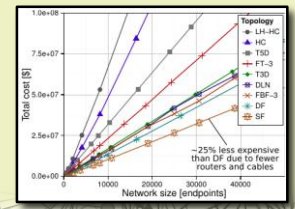
- Disconnection metrics\*
- Other studied metrics:
- Average path length (increase by 2); SF is 10% more resilient than DF

No. of Nodes	Health	Health	Health	Loss Rate	Disrupt	Resilience	Resilience	Resilience
512	30%	40%	55%	35%	55%	60%	65%	65%
1024	25%	40%	55%	40%	60%	65%	65%	65%
2048	20%	40%	55%	40%	65%	65%	65%	65%
4096	15%	45%	55%	40%	70%	70%	70%	70%
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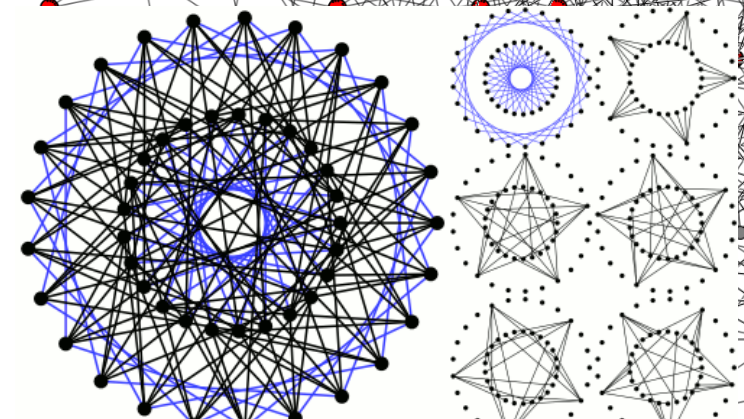
\*Missing values indicate the nonfeasibility of a balanced topology variant for a given N.

### A COST & POWER EFFECTIVE TOPOLOGY

- 25% less expensive than Dragonfly,
- 26% less power-hungry than Dragonfly

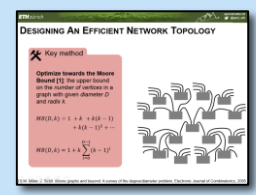


## TAKE-AWAY MESSAGE



### A LOWEST-DIAMETER TOPOLOGY

- Approaching the Moore Bound
- Resilient



**STRUCTURE ANALYSIS**  
Resiliency\*

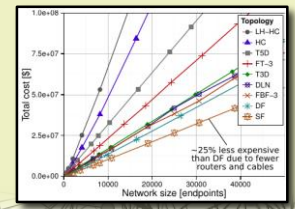
- Disconnection metrics\*
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- Average path length (increase by 2); SF is 10% more resilient than DF

No. N	Health	Health	Health	Link No.	Fatness	Disaggr.	Exp. Industry	Resiliency	Best %
512	30%	-	40%	55%	35%	-	55%	60%	60%
1024	25%	40%	40%	55%	40%	50%	60%	65%	65%
2048	20%	-	40%	55%	40%	55%	65%	65%	65%
4096	15%	-	45%	55%	55%	60%	70%	70%	70%
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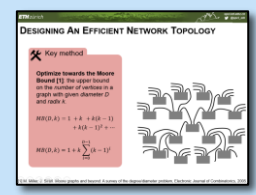


## TAKE-AWAY MESSAGE

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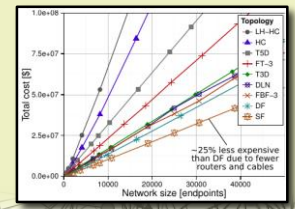
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N	Health	Health	Health	Loss	Loss	Loss	Loss	Loss	Loss	Loss	Loss
512	30%	40%	55%	35%	45%	60%	65%	70%	75%	80%	85%
1024	25%	35%	50%	40%	50%	65%	70%	75%	80%	85%	90%
2048	20%	30%	45%	45%	55%	65%	70%	75%	80%	85%	90%
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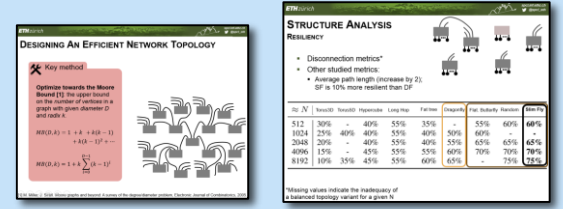
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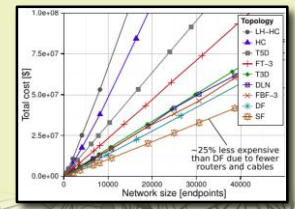
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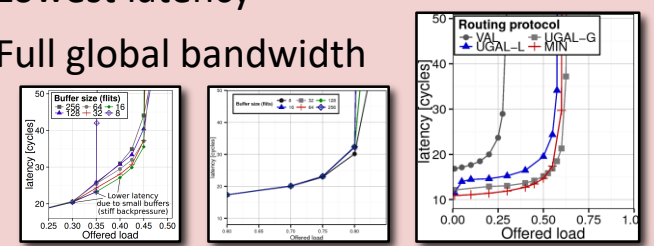
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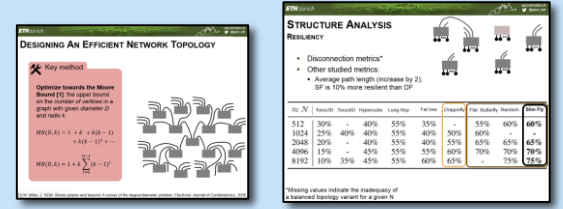
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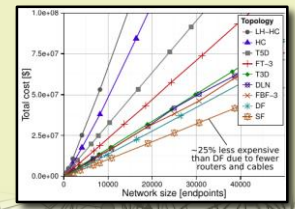
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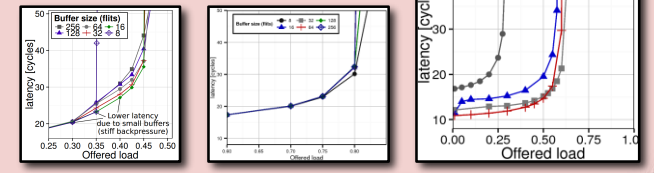
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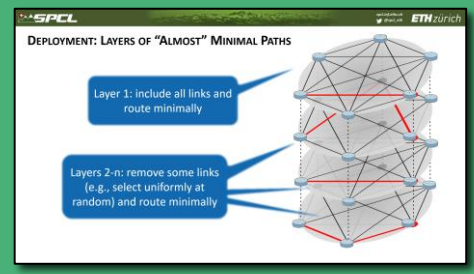
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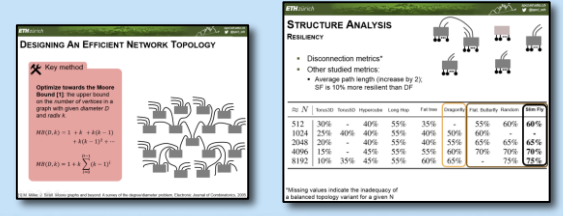


### POTENTIAL FOR NOVEL SOLUTIONS



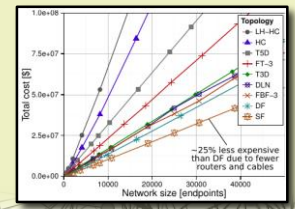
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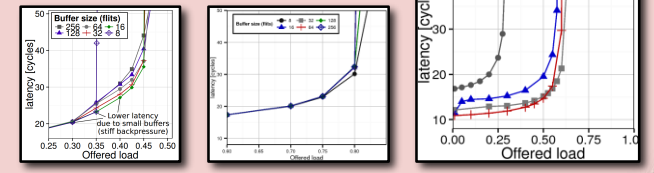
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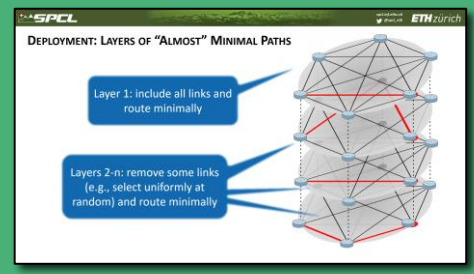
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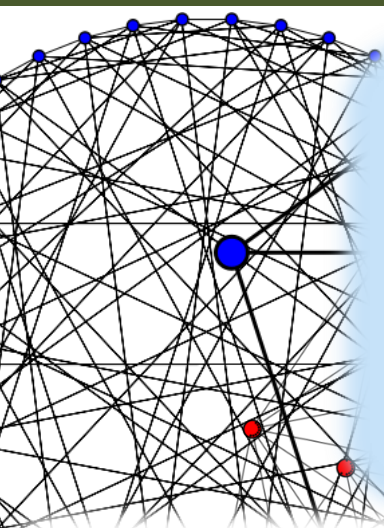
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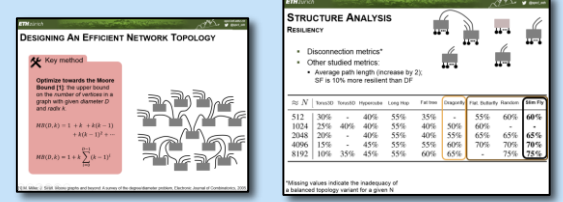
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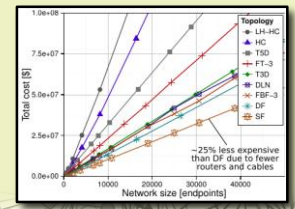
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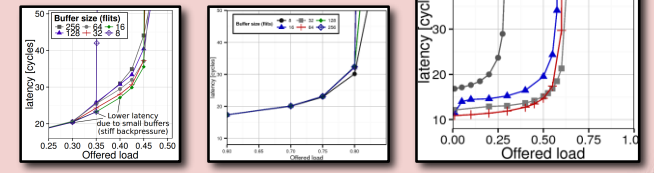
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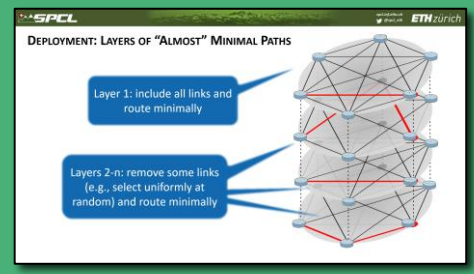
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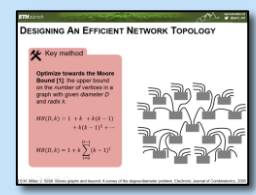


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**STRUCTURE ANALYSIS Resiliency**

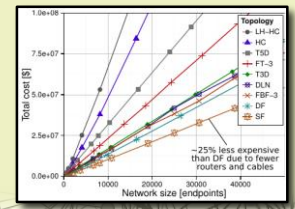
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1024	25%	40%	55%	40%	50%	60%	-	-
2048	20%	40%	55%	40%	55%	65%	65%	65%
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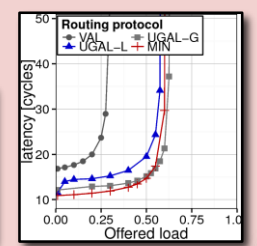
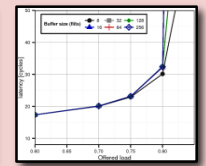
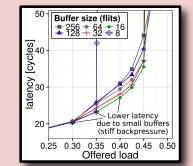
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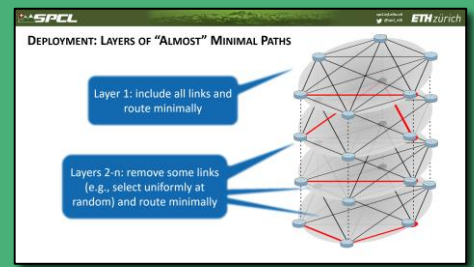
Thank you  
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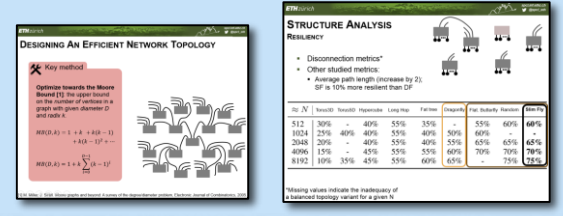


## POTENTIAL FOR NOVEL SOLUTIONS



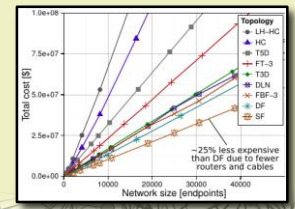
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[http://spcl.inf.ethz.ch/Research/Scalable\\_Networking/SlimFly](http://spcl.inf.ethz.ch/Research/Scalable_Networking/SlimFly)



# Thank you for your attention

**Scalable Parallel Computing Lab**

**Slim Fly: A Cost Effective Low-Diameter Network Topology**

**The key idea in a single sentence**  
 "It's ALL about the diameter!": Optimize your topology for low diameter to not only reduce the latency (due to shorter paths) but also cost and power consumption as packets will traverse and thus require fewer routers and cables.

**Key motivation**  
 Interconnection networks play an important role in today's large-scale computing systems. An ideal topology should ensure: high bandwidth, low total system cost and energy consumption, low endpoint-to-endpoint latency, and resiliency to link failures. We develop the Slim Fly topology that achieves all these goals by lowering the diameter.

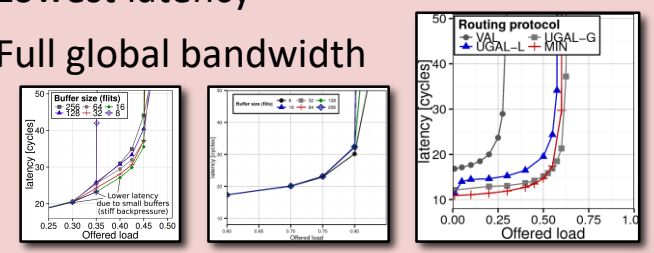
**Key idea:**  
Lower diameter and thus average path length: fewer cables and routers necessary.

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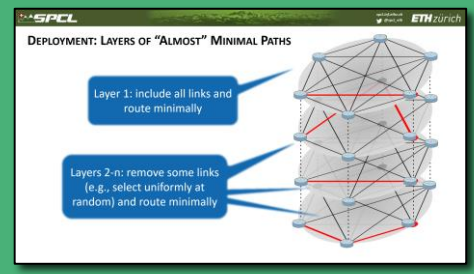
The key idea and motivation: On the left, a topology has a diameter of 3; two communicating endpoints require 3 hops. On the right, the topology has been rearranged so that the diameter is 2. Thus, packets require fewer routers and cables.

## A HIGH-PERFORMANCE TOPOLOGY

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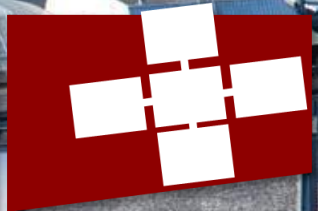


## POTENTIAL FOR NOVEL SOLUTIONS



# Lowering Diameter Enables Cost-Effective and High-Performance Networks

MACIEJ BESTA, ERIK HENRIKSSON, TORSTEN HOEFLER



# DIAMETER-2 SLIM FLY

1 *Select a prime power  $q$*



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$$q = 4w + \delta;$$

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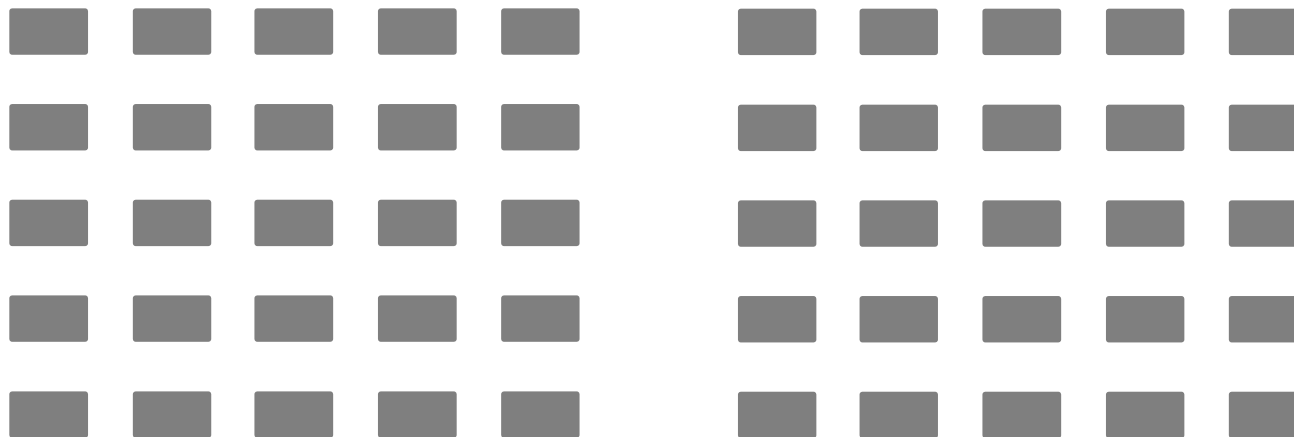
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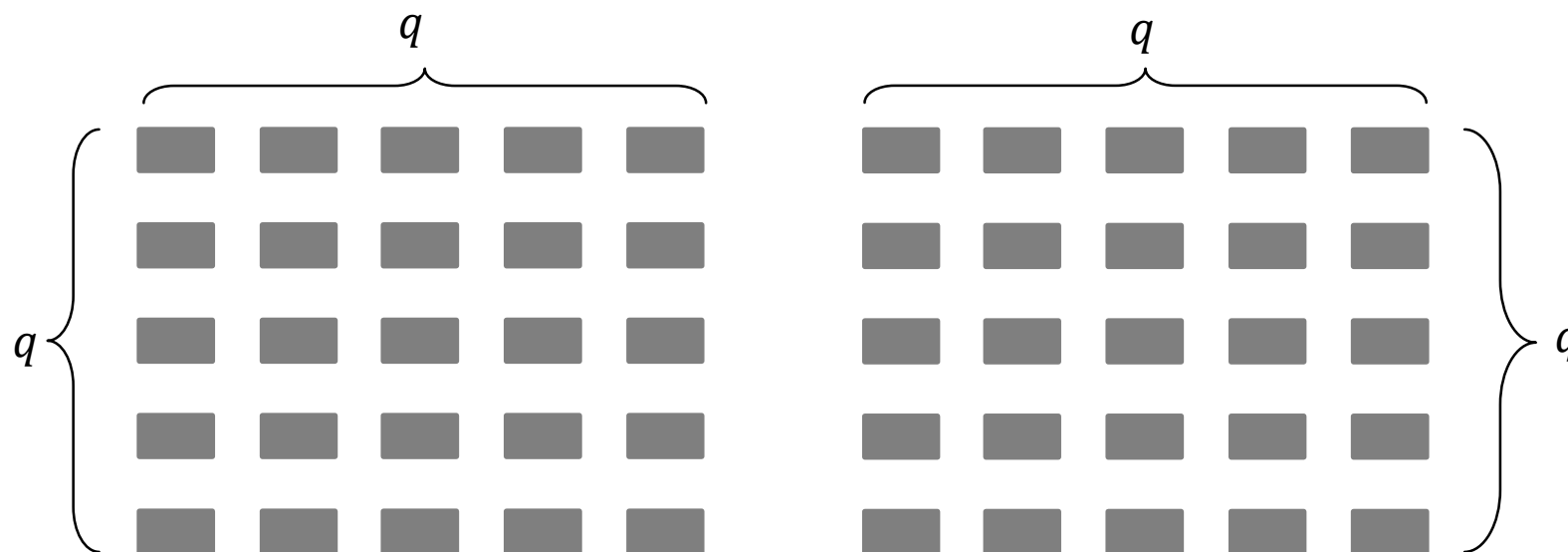
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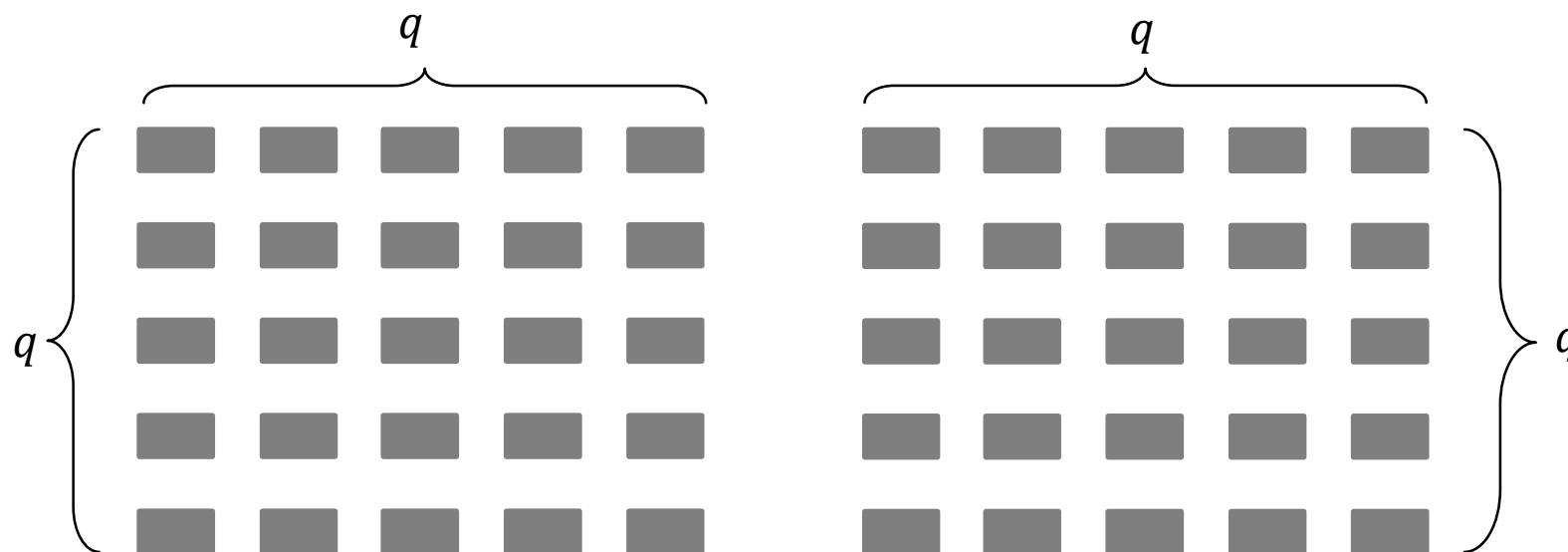
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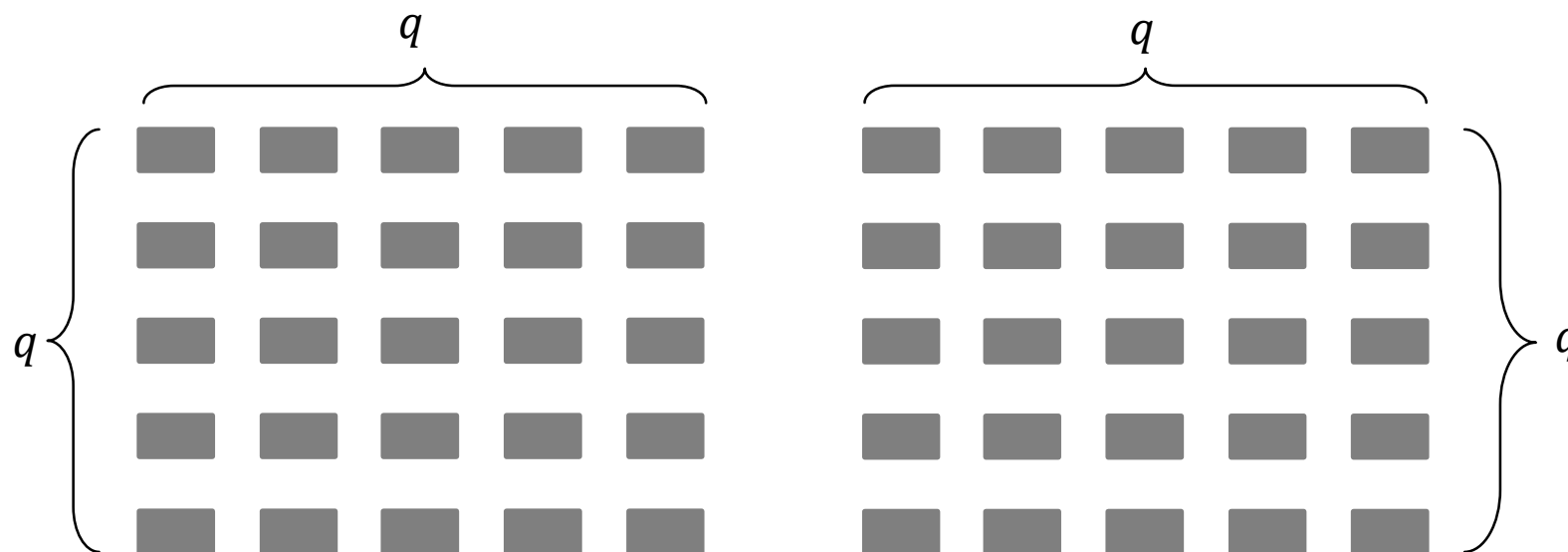
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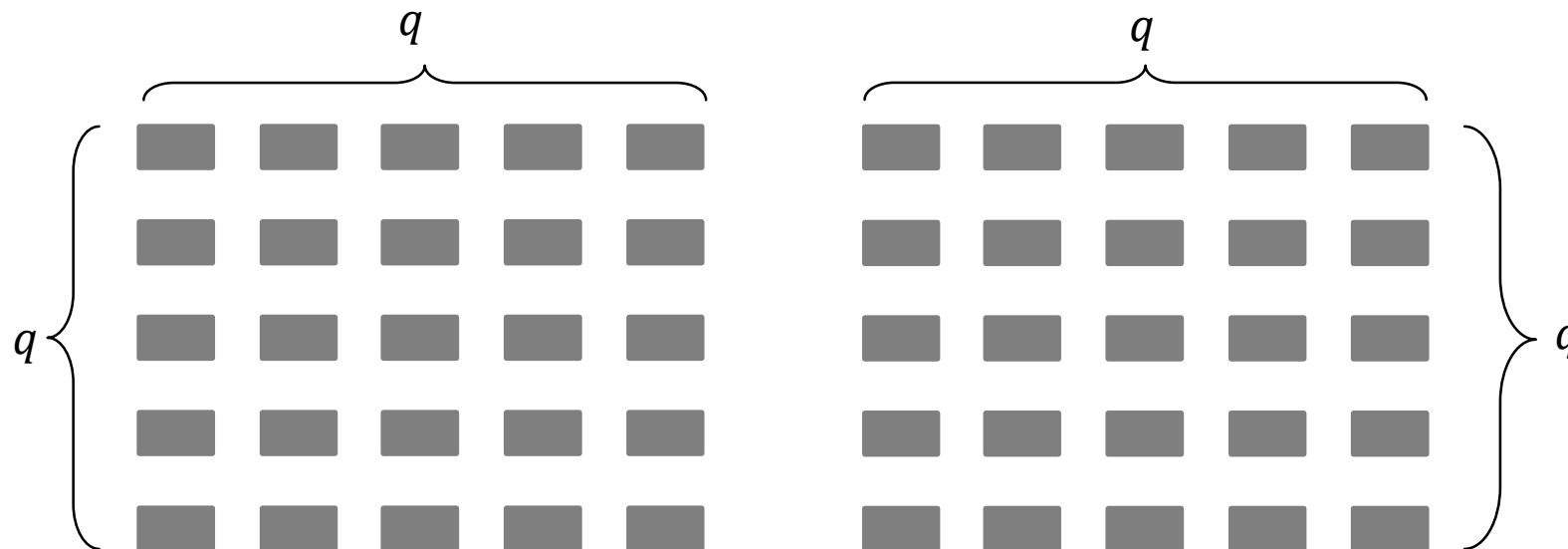
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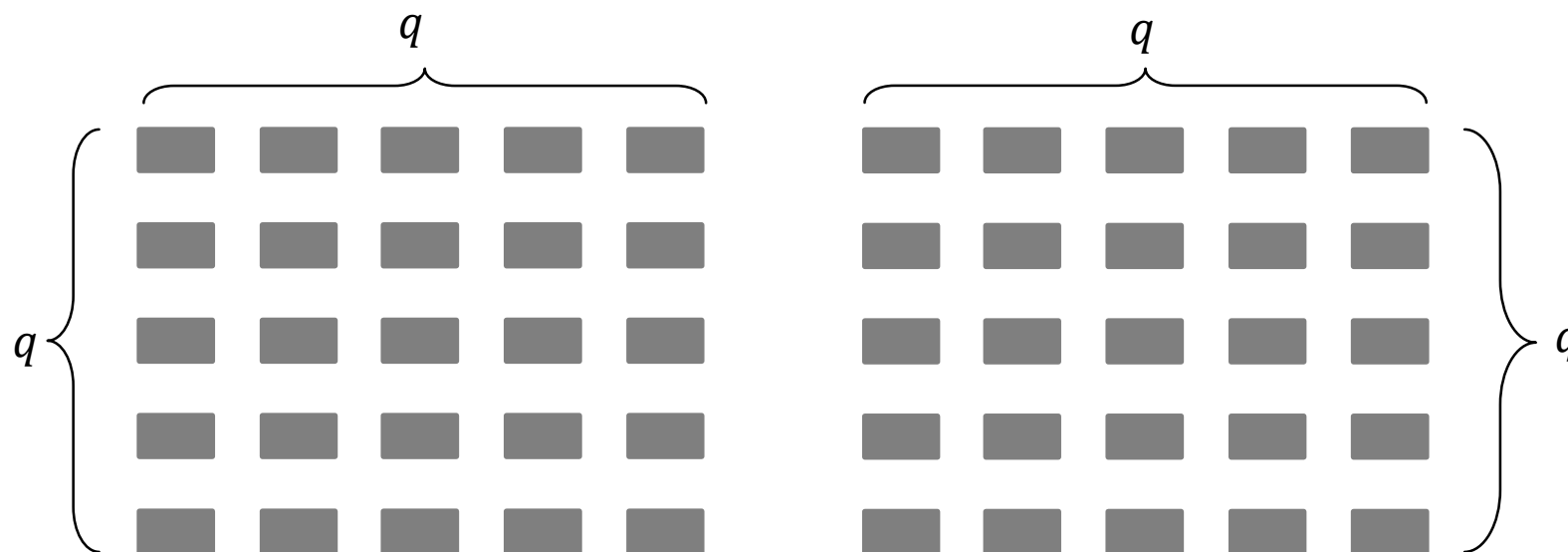
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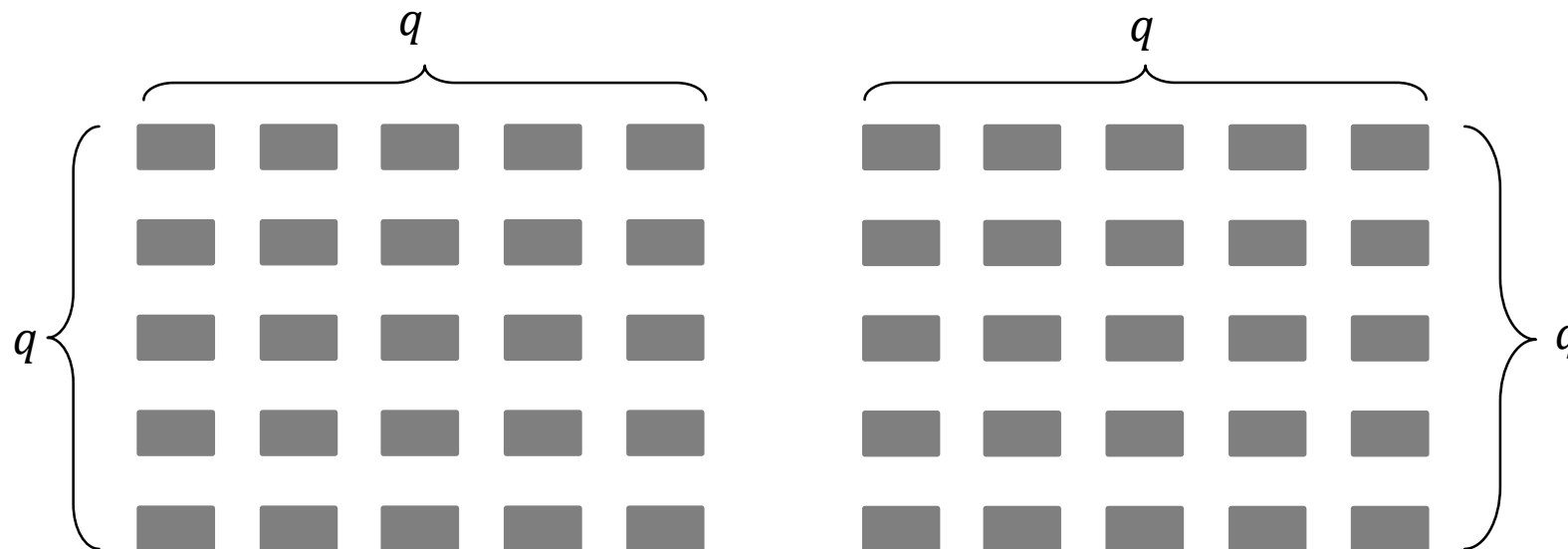
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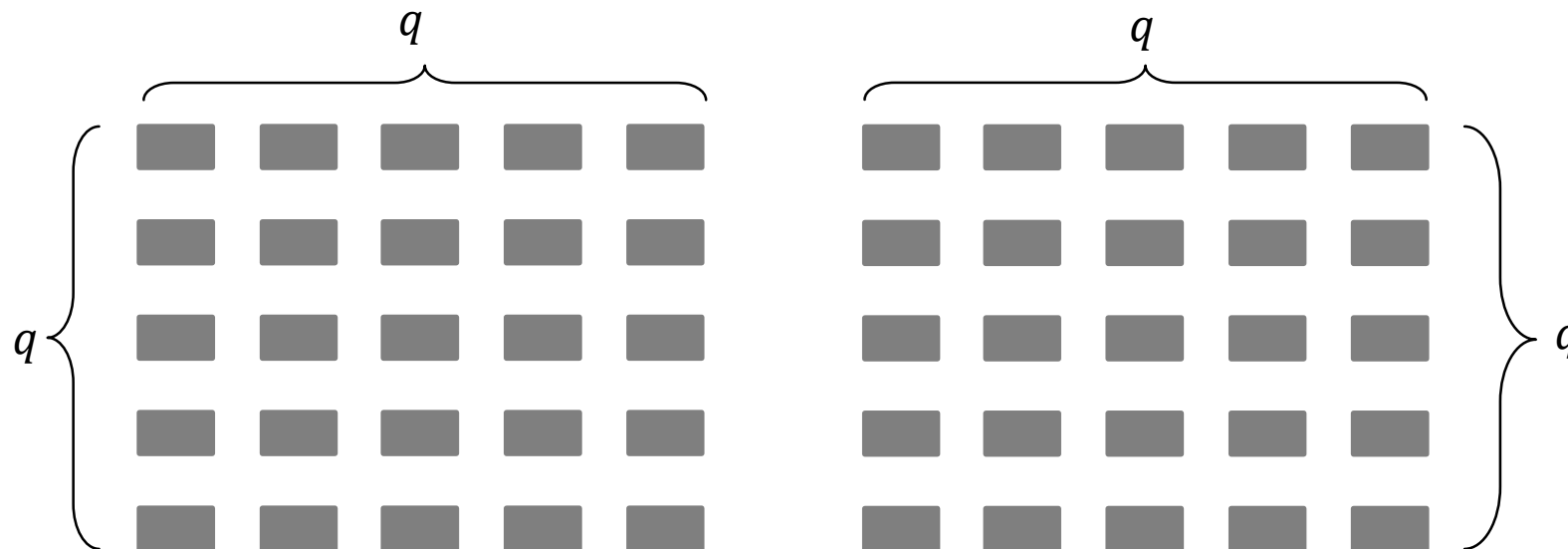
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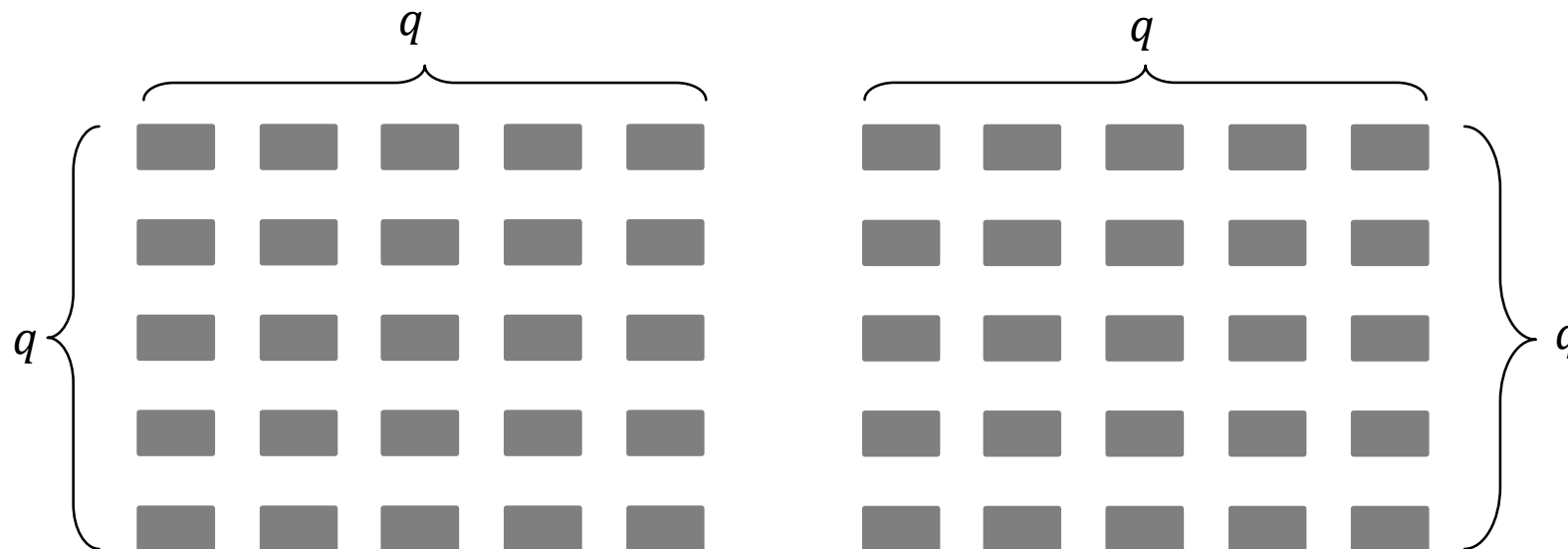
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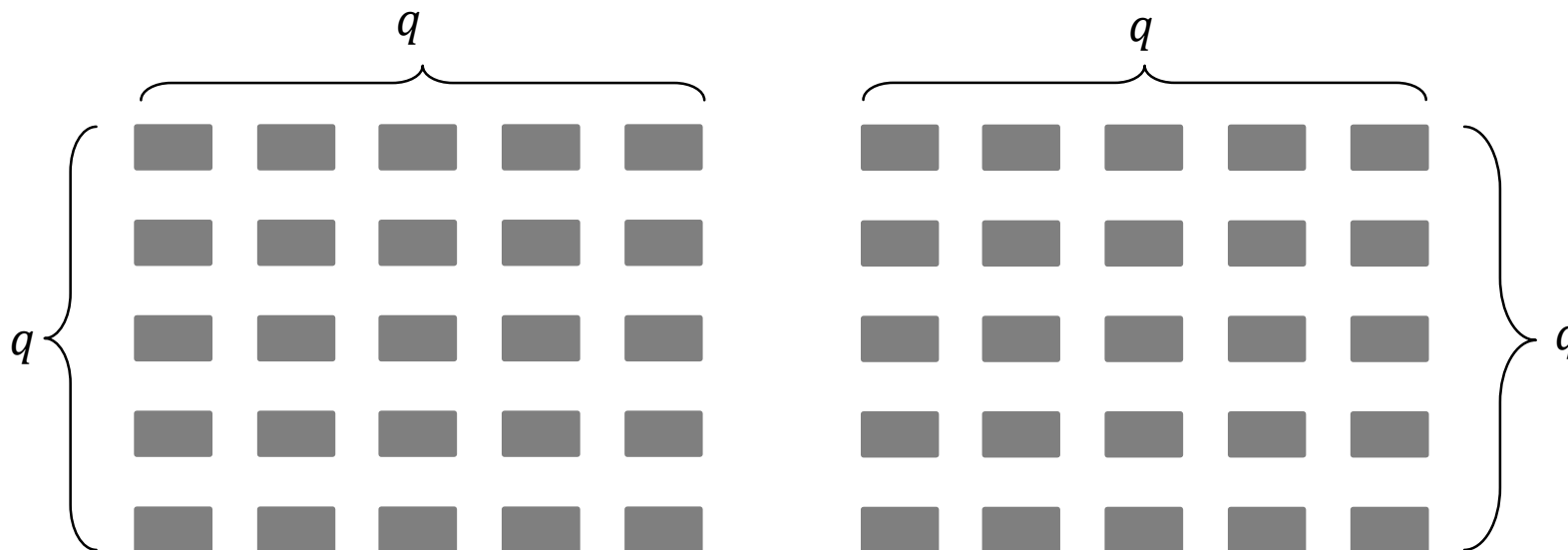
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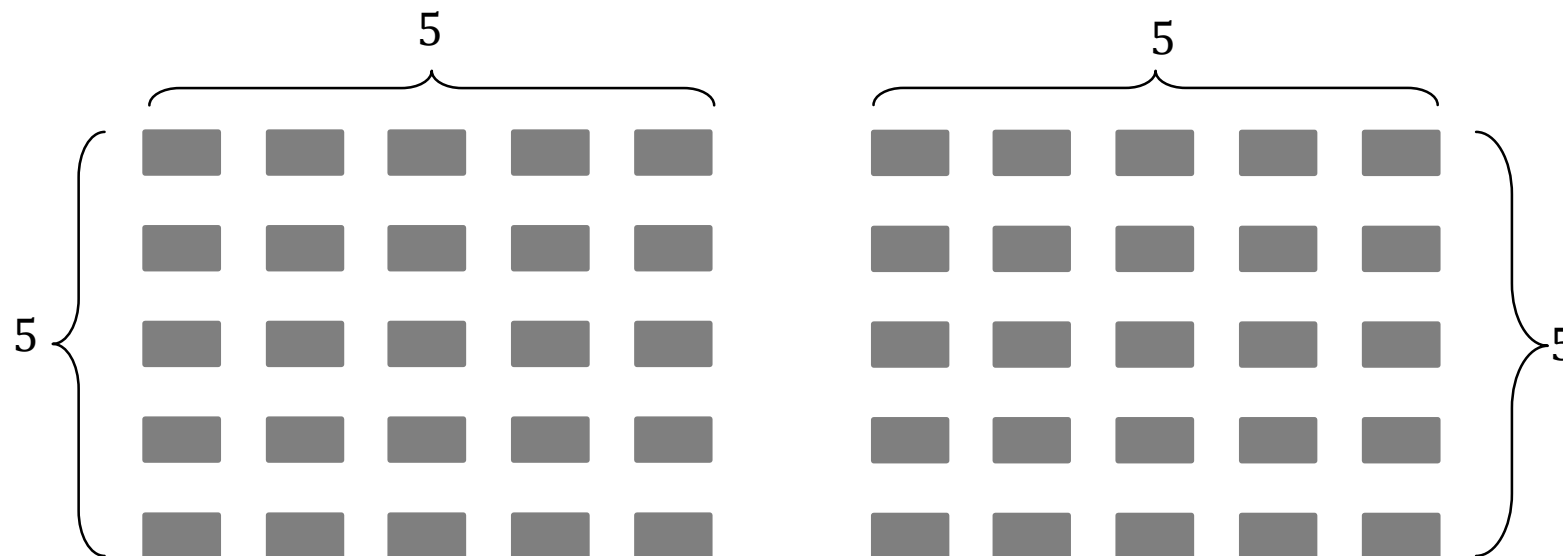
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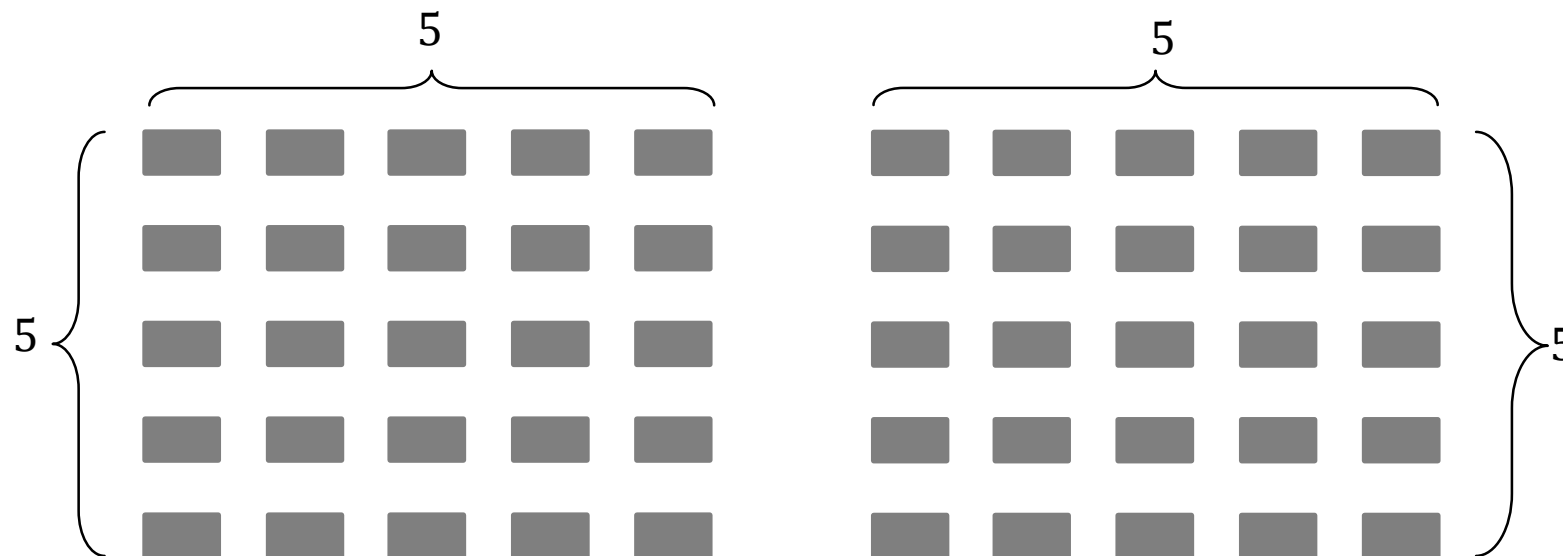
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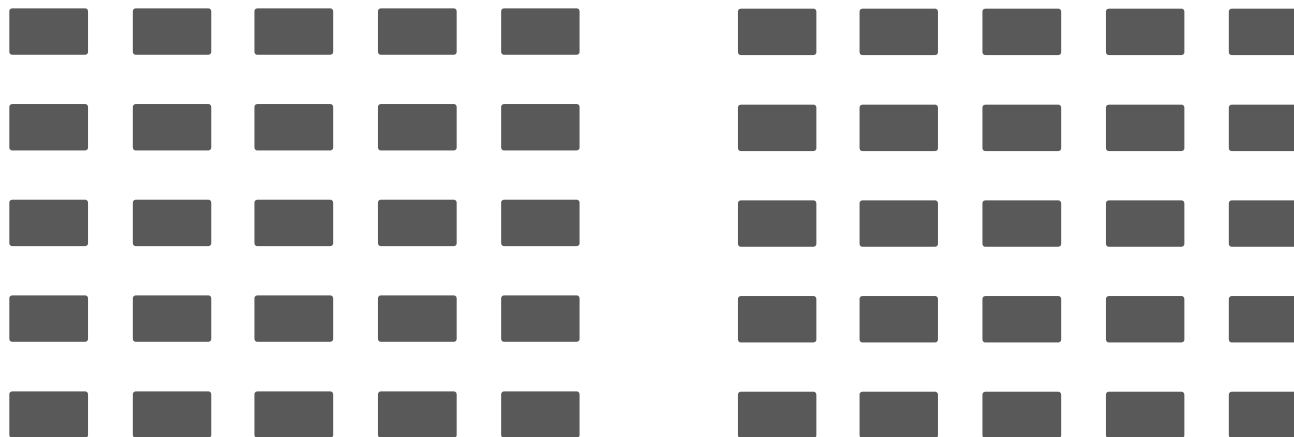
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$$\mathcal{F}_5 = \{0, 1, 2, 3, 4\}$$



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**3** *Label the routers*



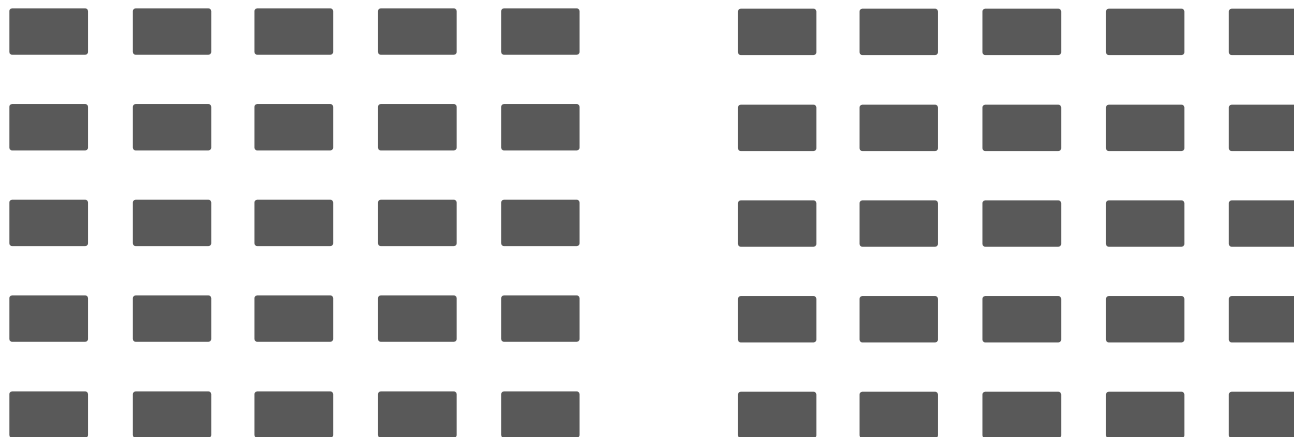


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## 3 Label the routers

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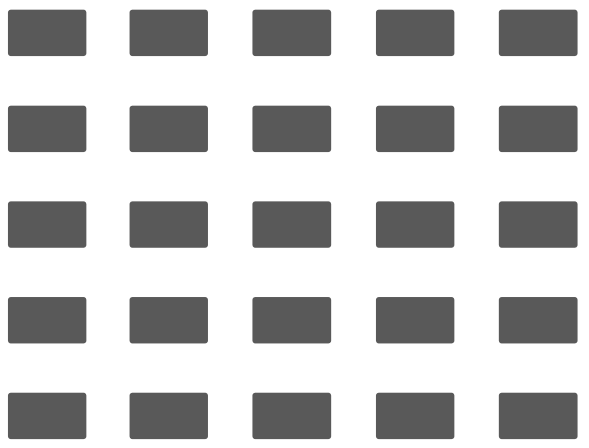
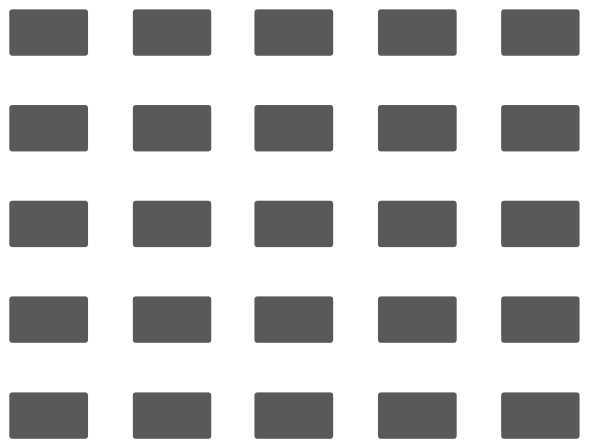
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...



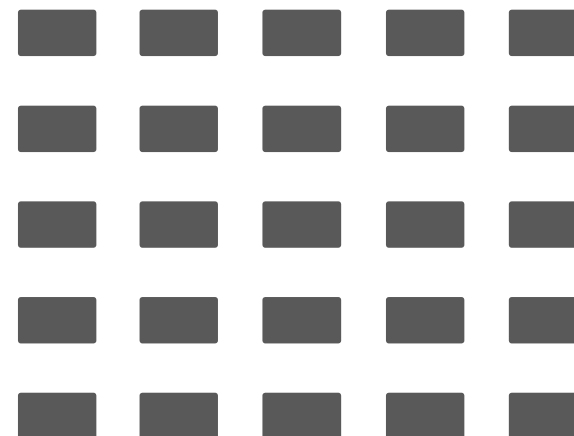
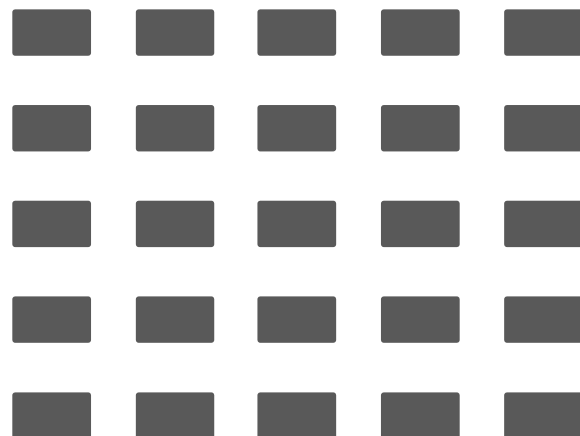
# DIAMETER-2 SLIM FLY

**3** Label the routers

Set of routers:  
 $\{0,1\} \times \mathcal{F}_q \times \mathcal{F}_q$

**E** Example:  $q = 5$

...



# DIAMETER-2 SLIM FLY

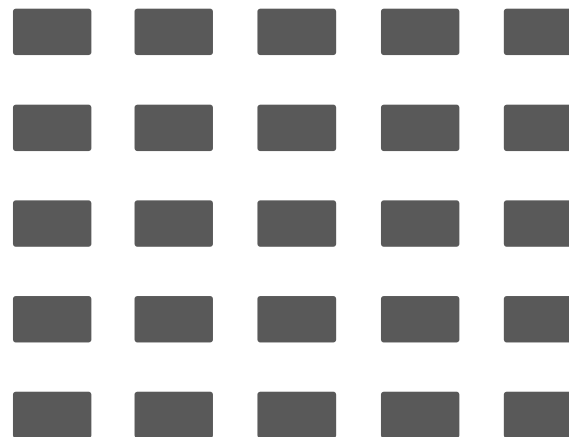
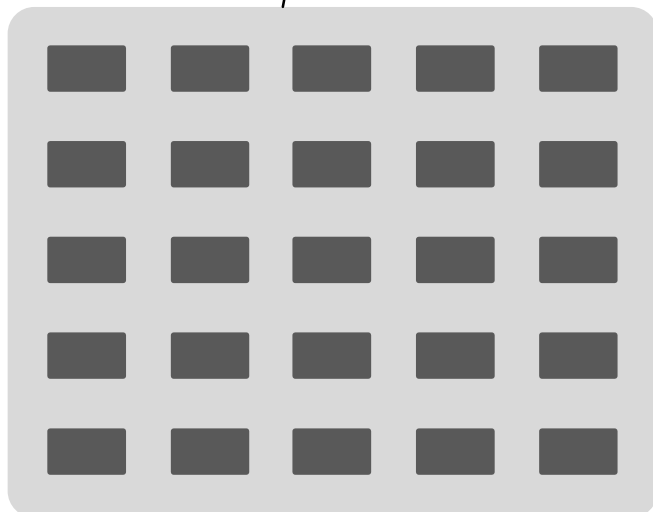
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Routers (0,..)



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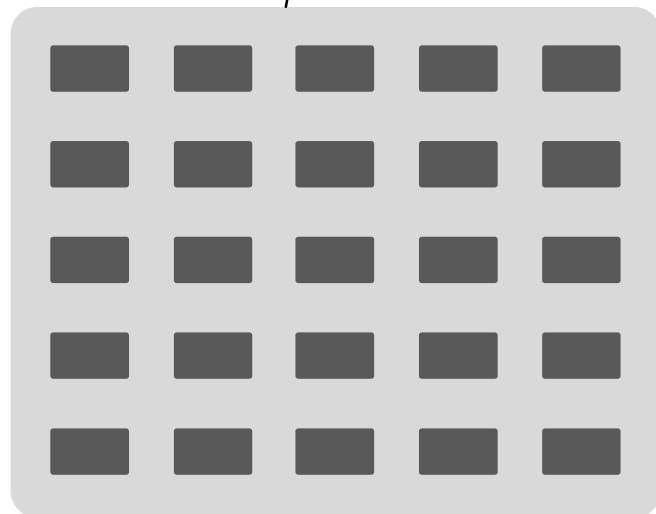
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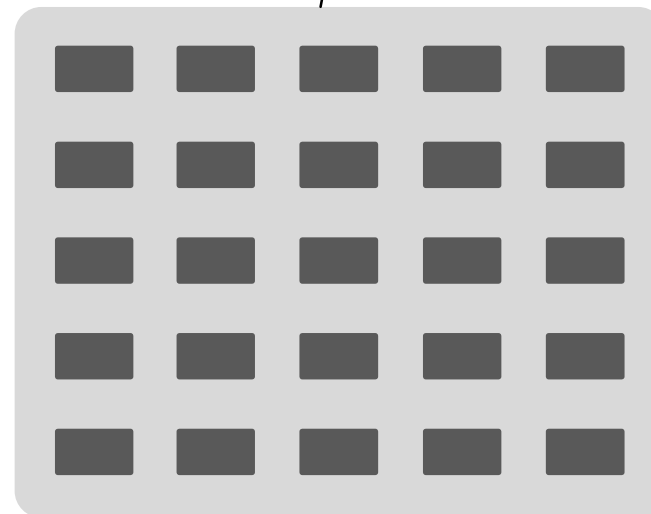
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Routers (0,..)



Routers (1,..)



# DIAMETER-2 SLIM FLY

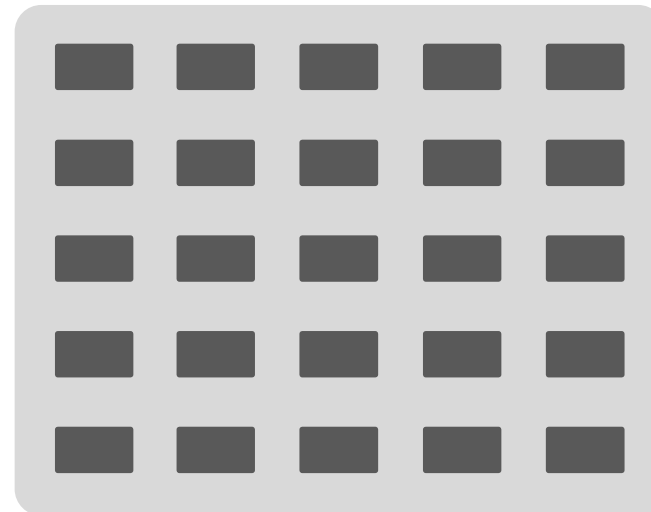
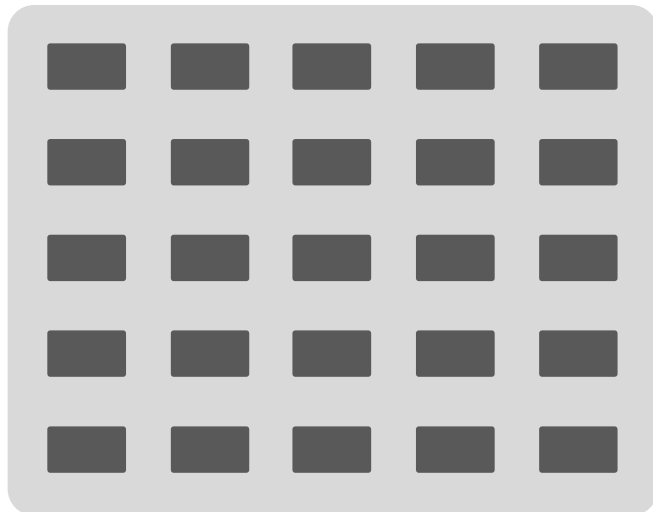
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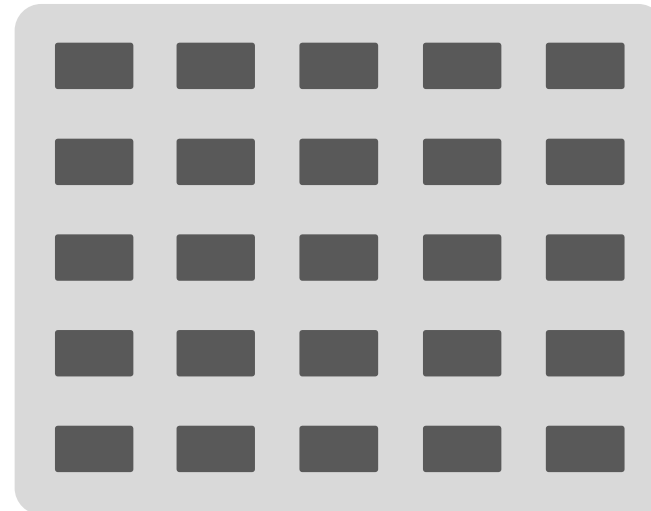
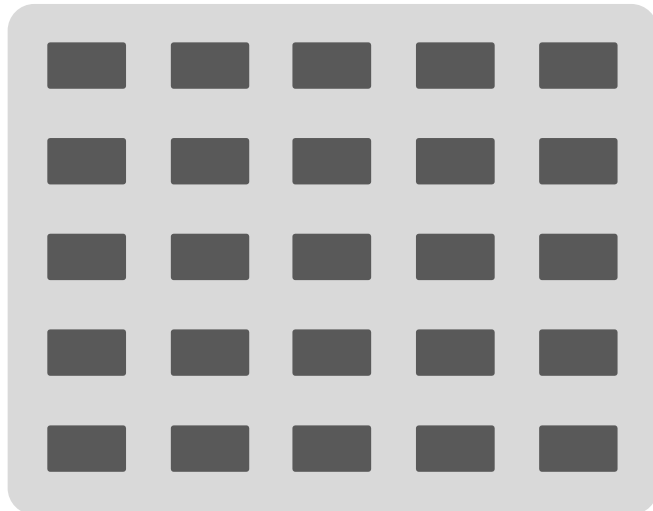
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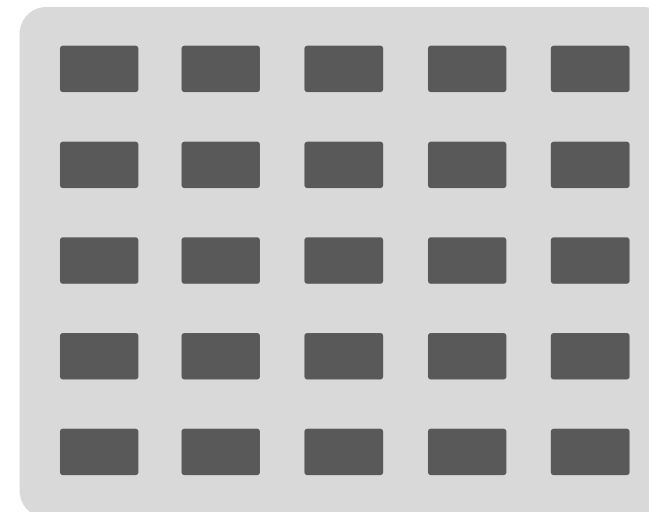
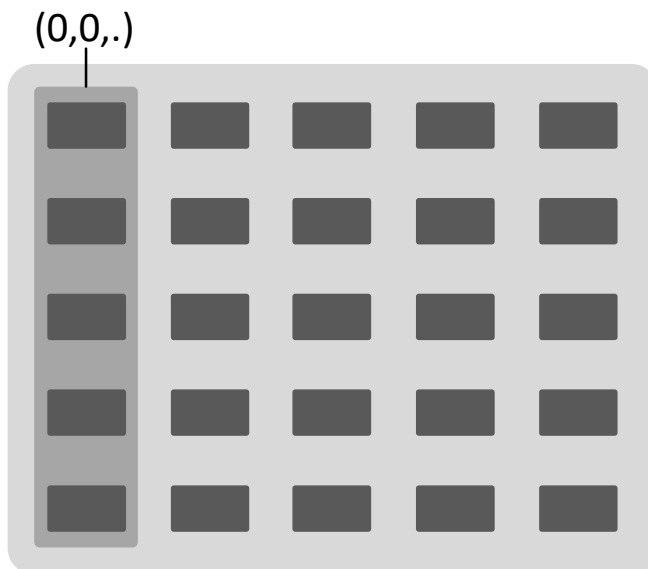
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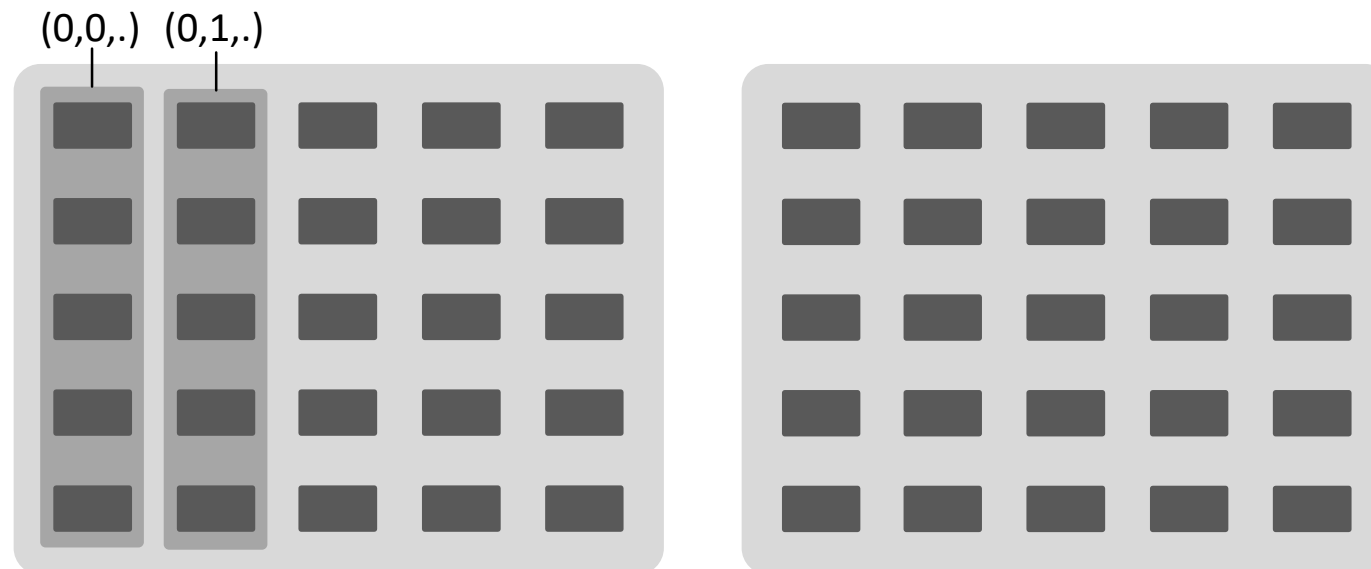
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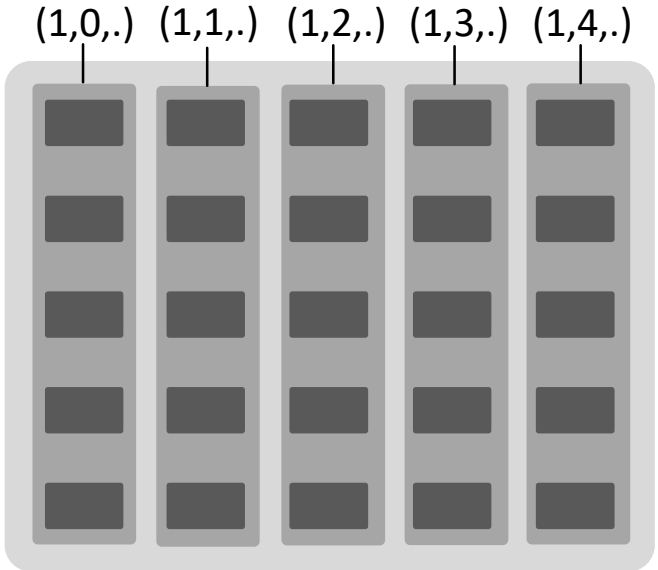
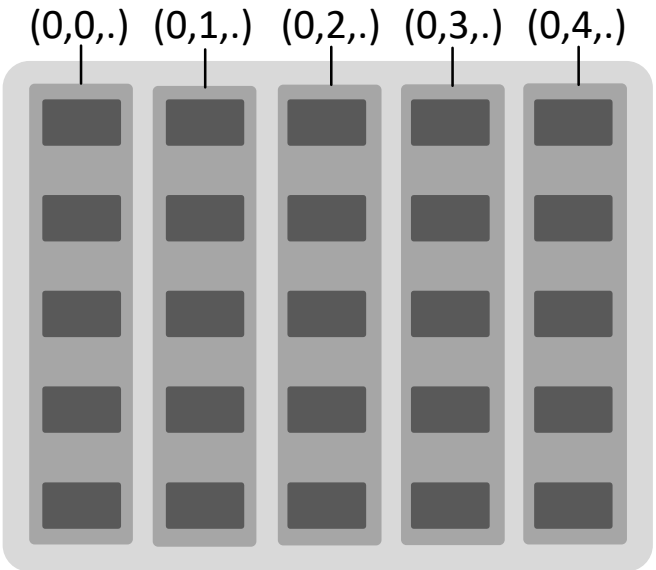
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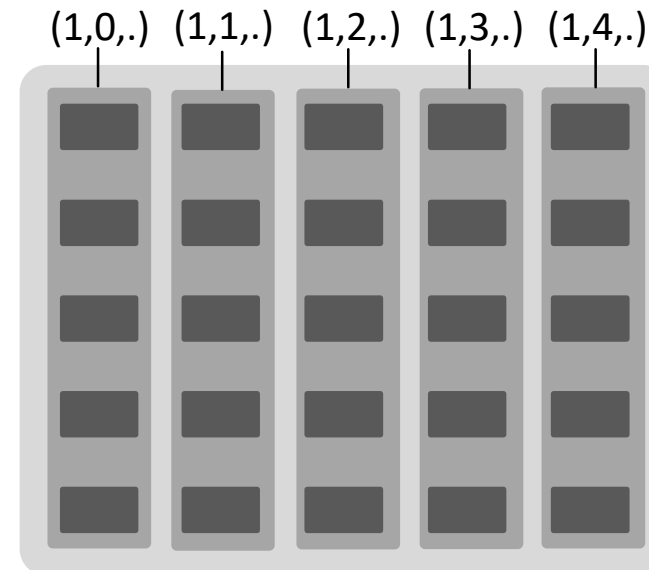
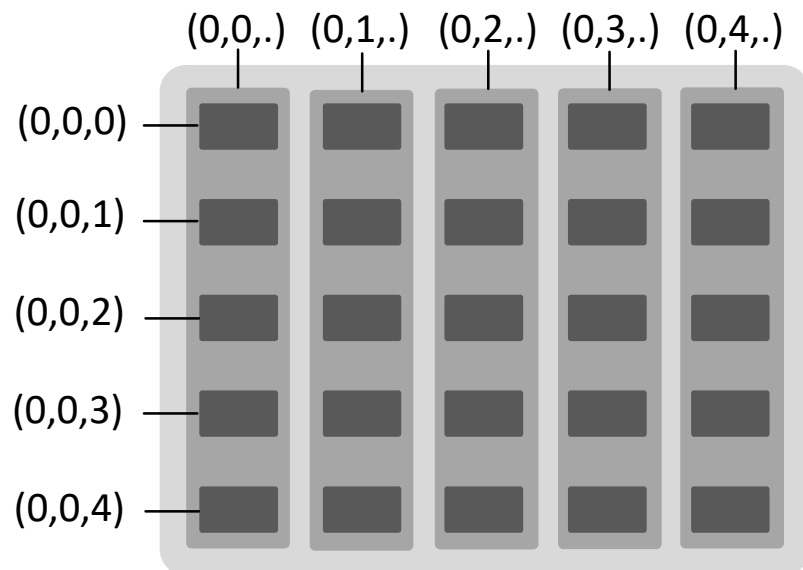
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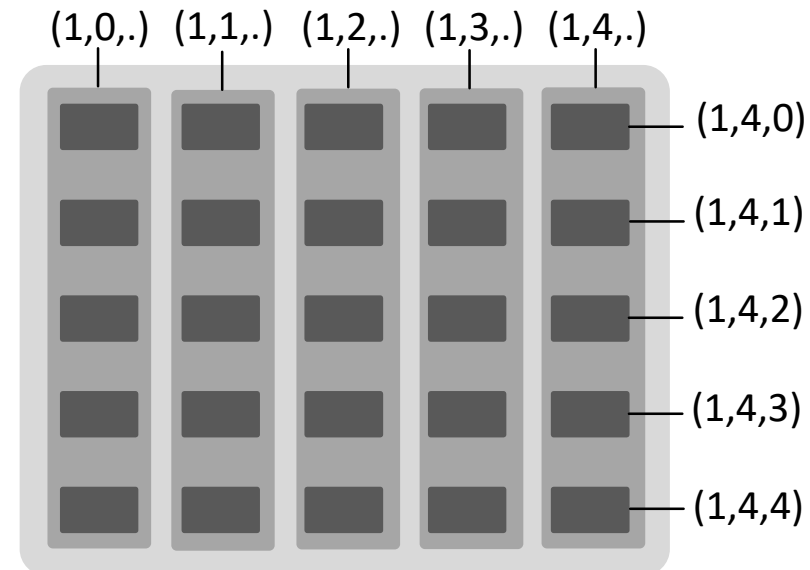
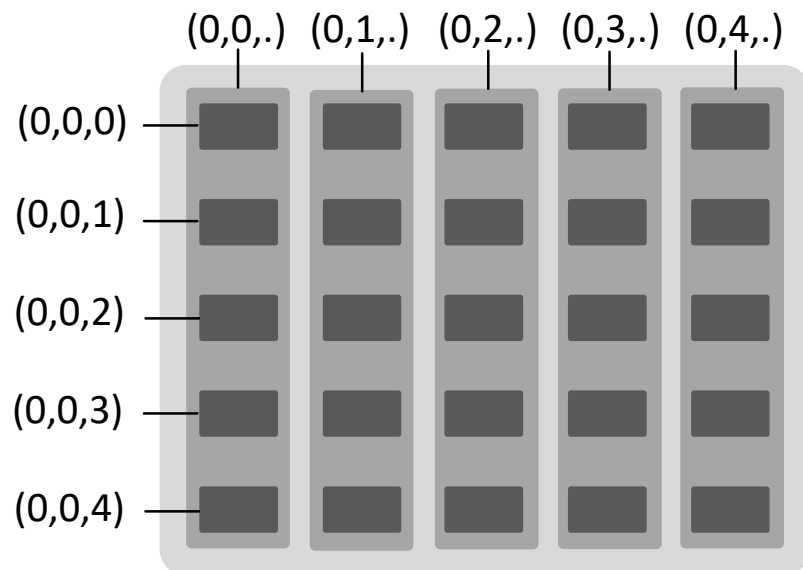
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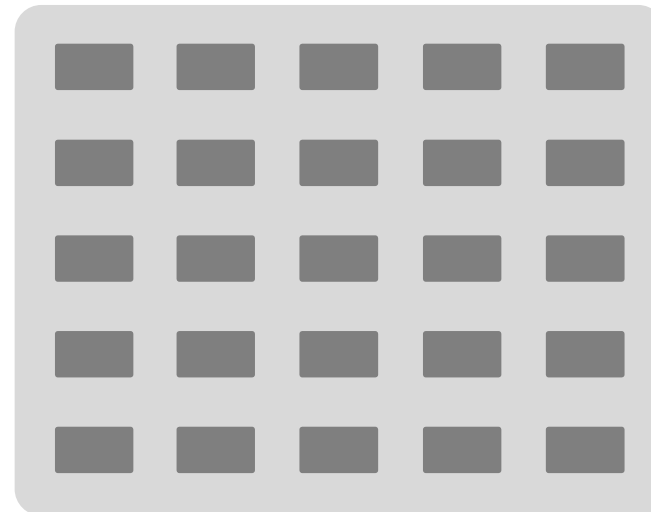
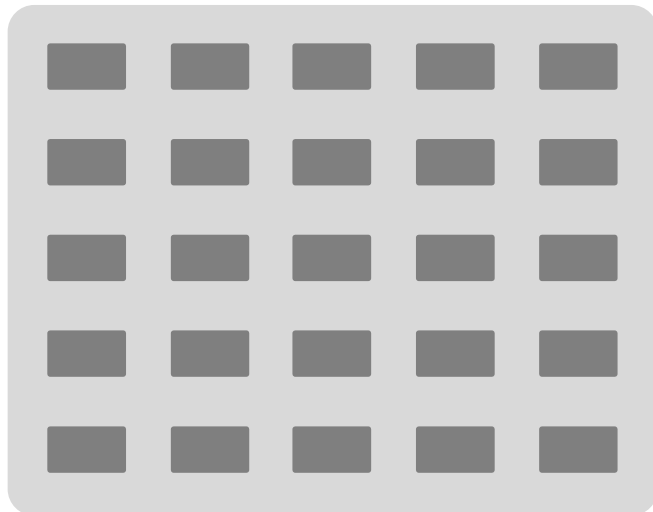
E Example:  $q = 5$

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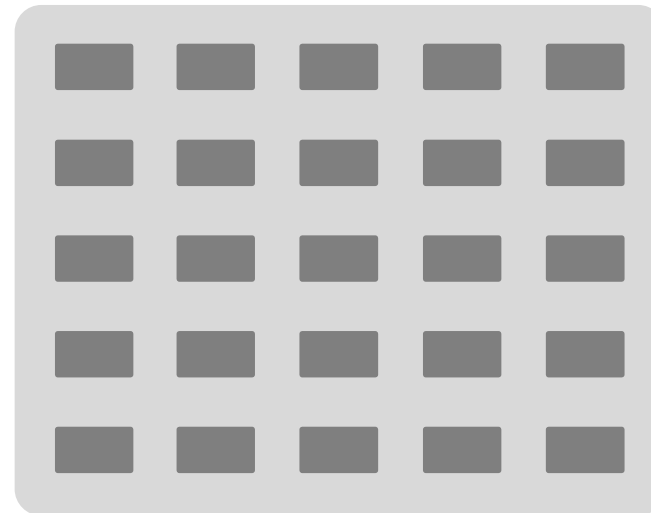
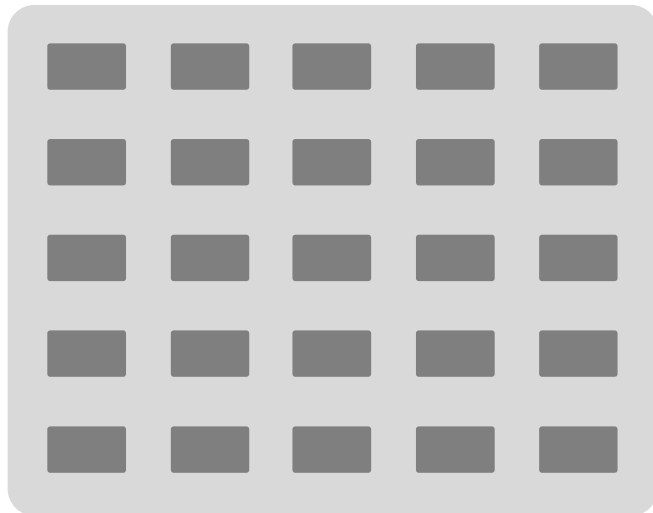
4 Find primitive element  $\xi$



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$\xi \in \mathcal{F}_q$  generates  $\mathcal{F}_q$ :

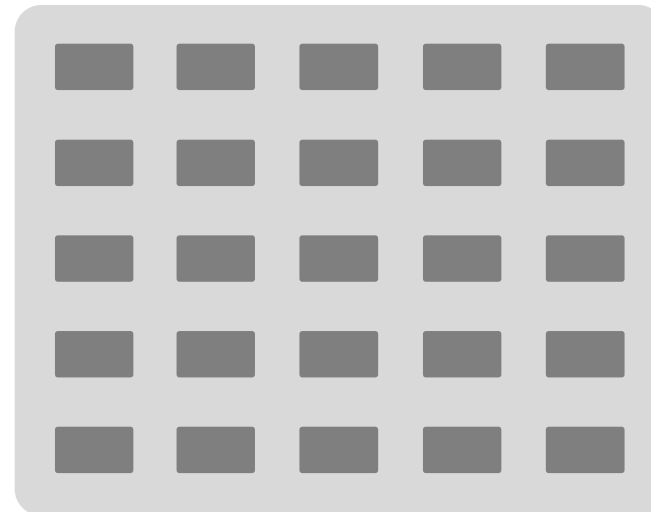
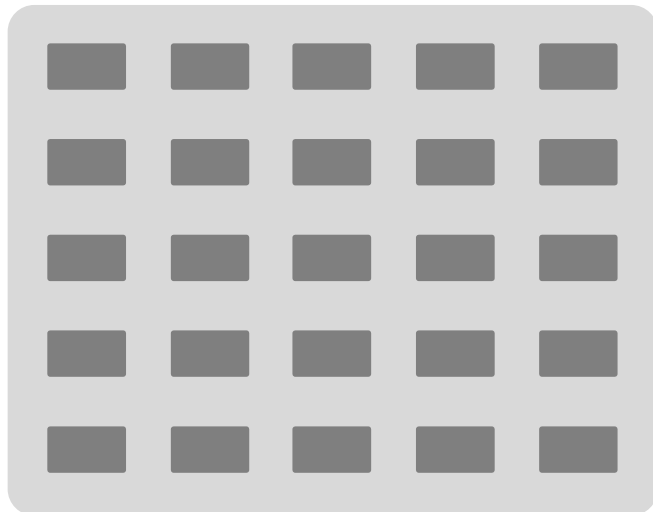


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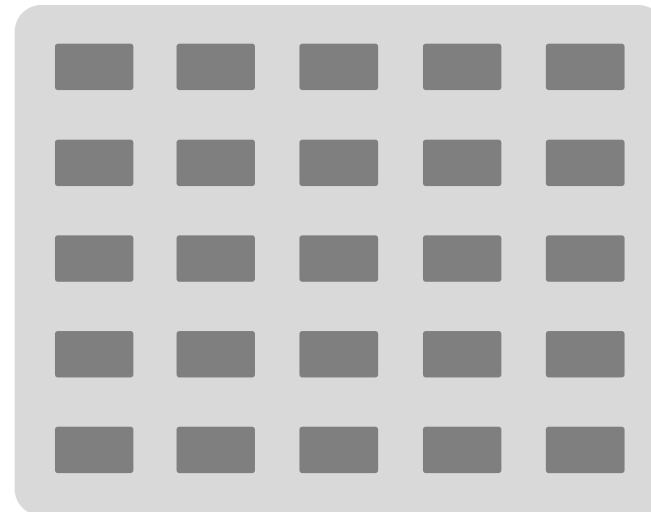
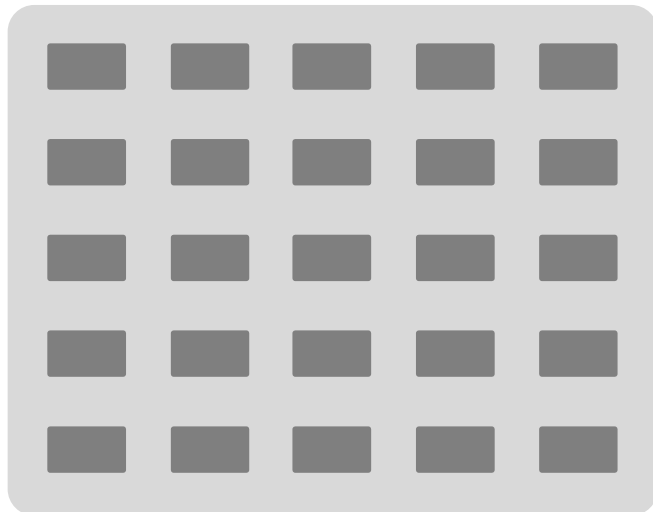
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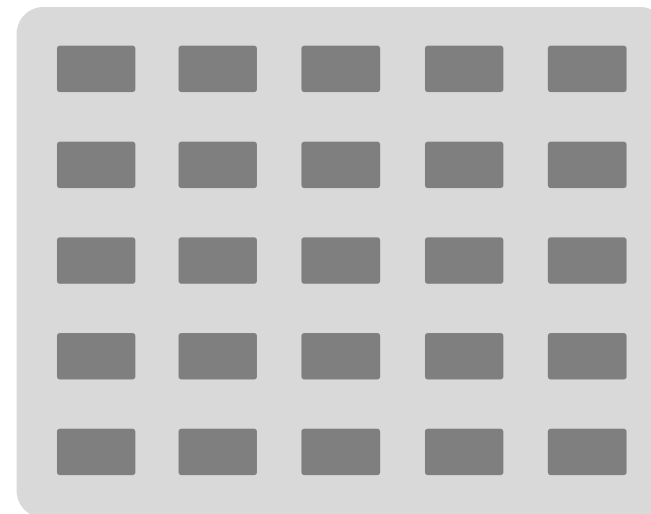
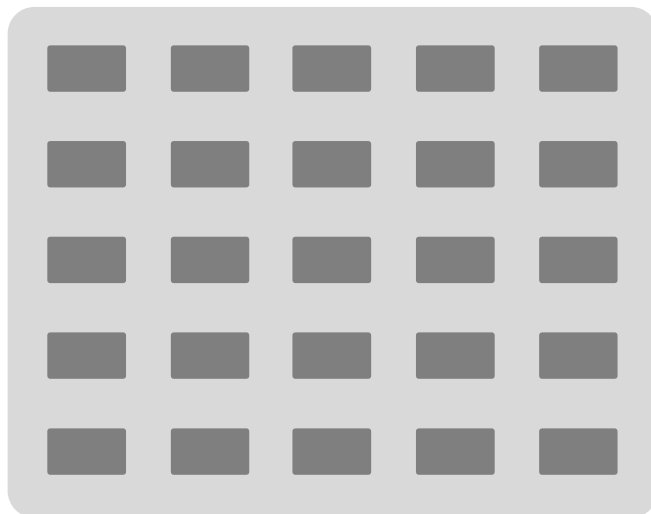
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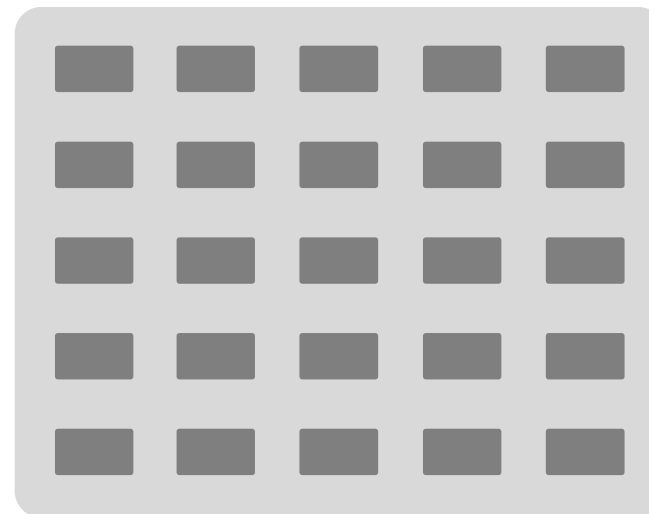
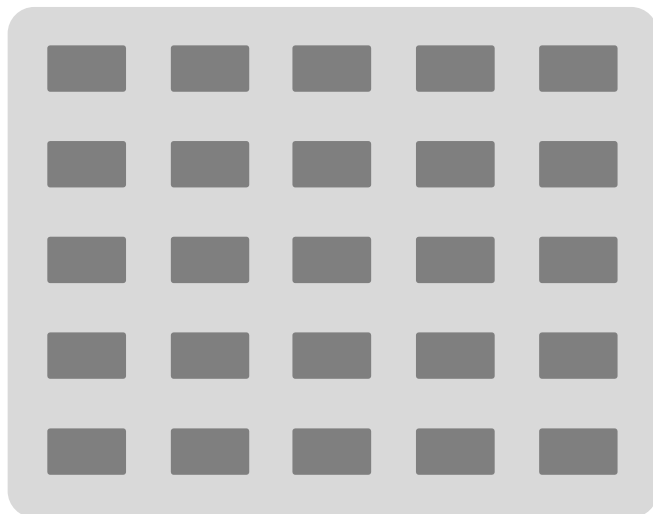
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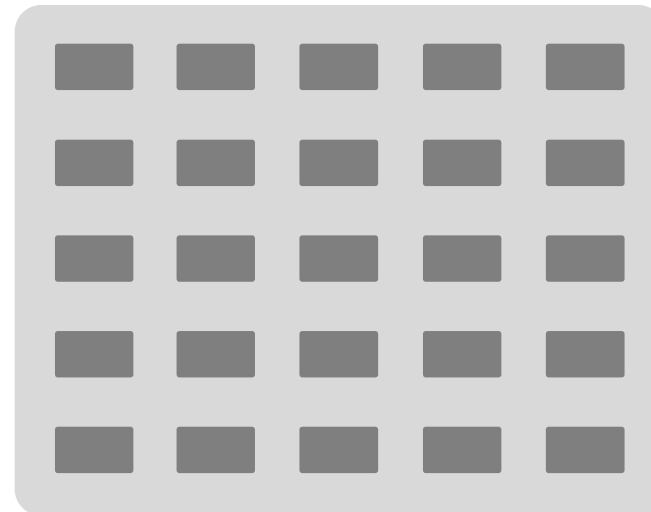
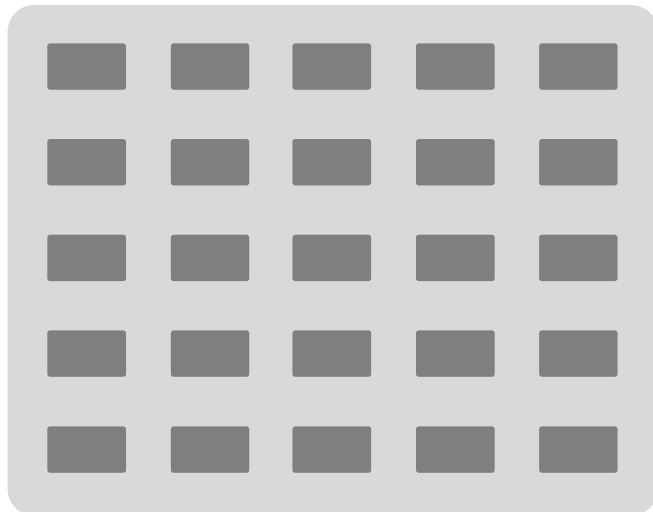
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$$1 = \xi^4 \bmod 5 =$$

$$2^4 \bmod 5 = 16 \bmod 5$$



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## 5 Build Generator Sets

$$X = \{1, \xi^2, \dots, \xi^{q-3}\}$$

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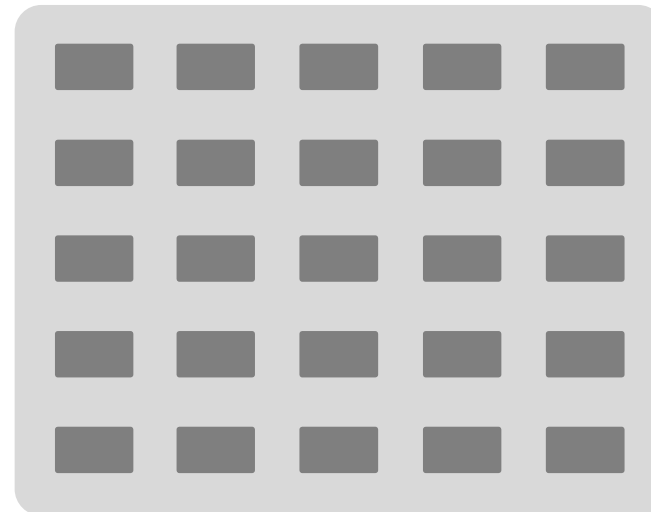
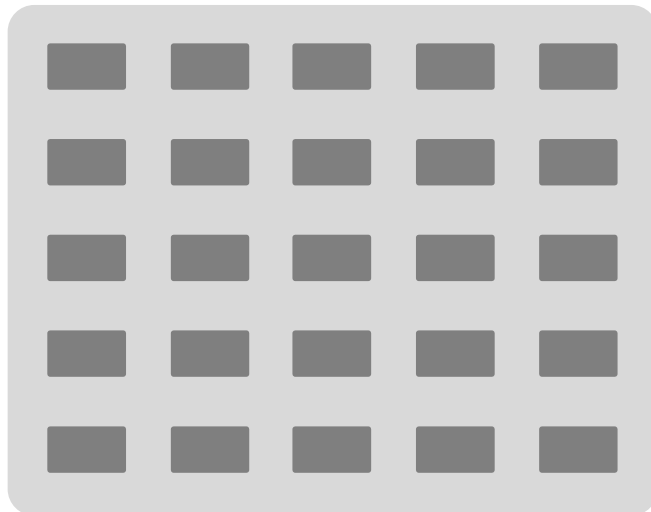
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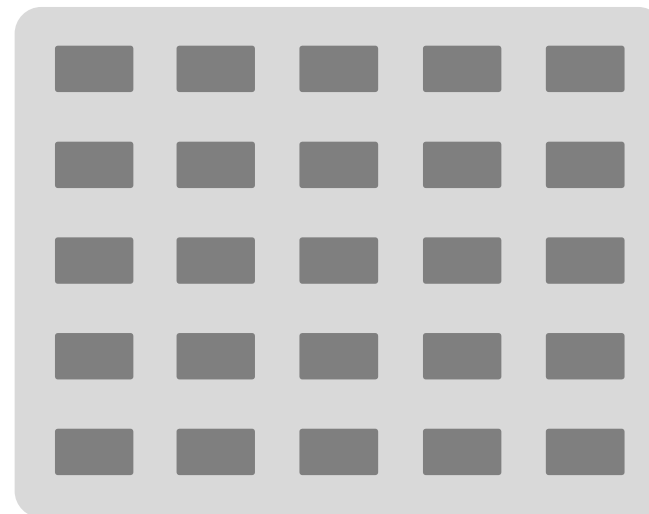
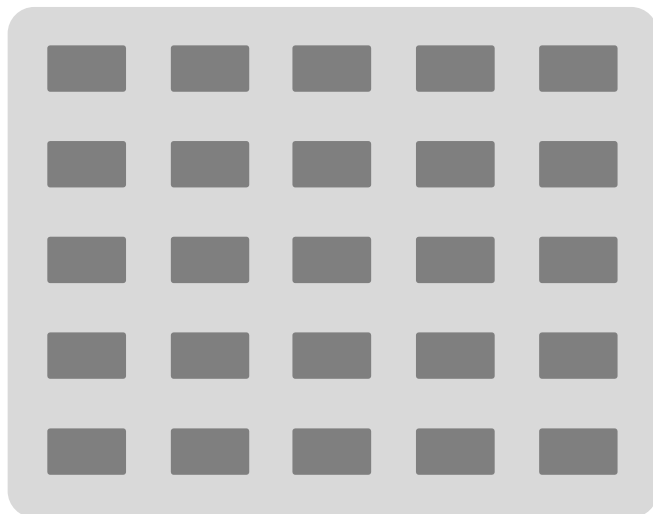
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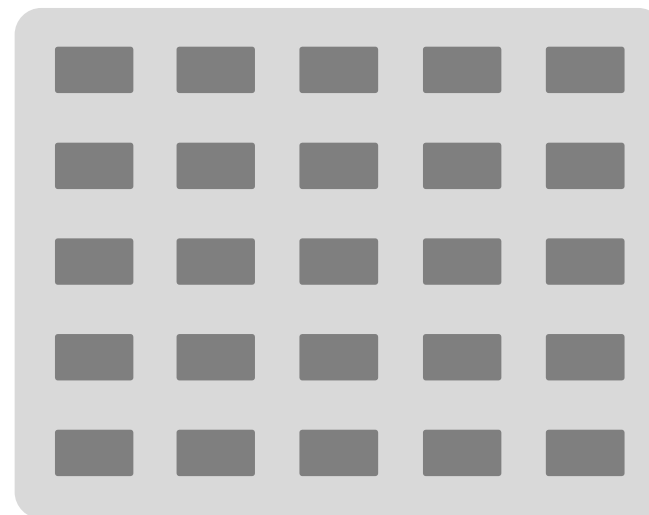
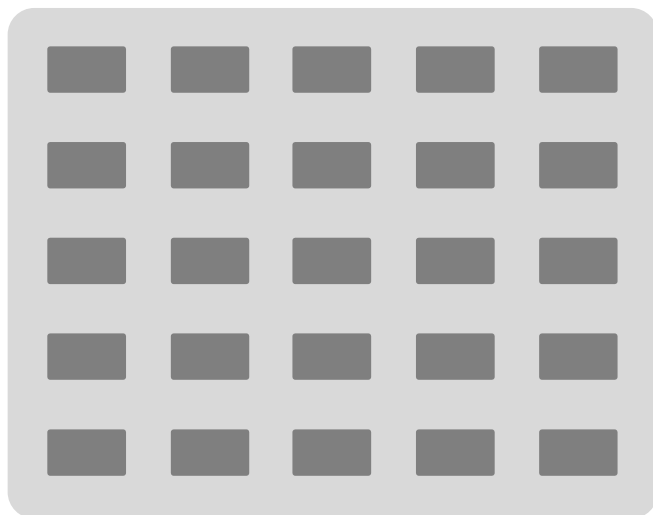
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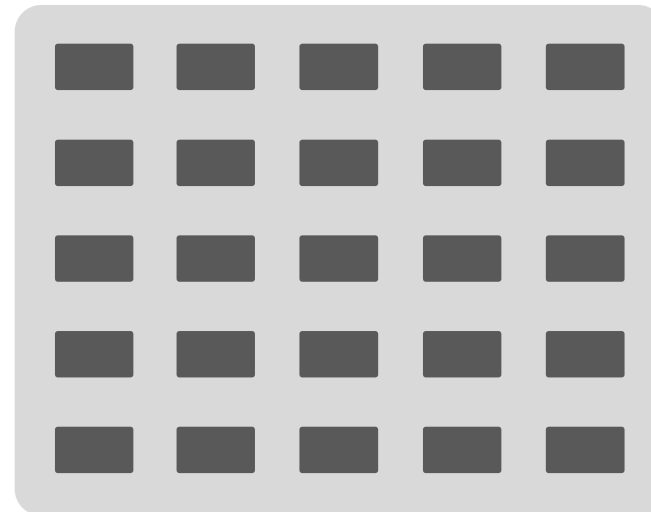
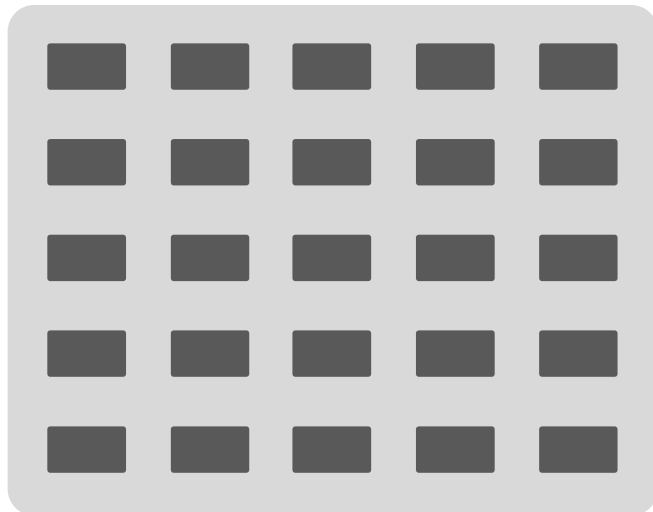
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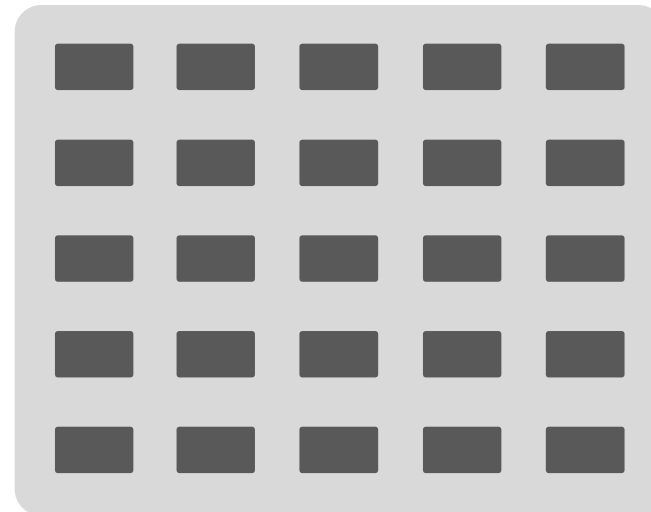
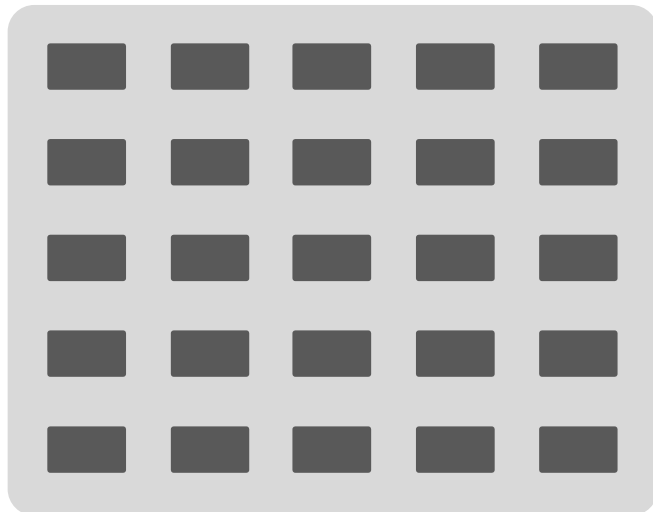


# DIAMETER-2 SLIM FLY



# DIAMETER-2 SLIM FLY

## 6 *Intra-group connections*

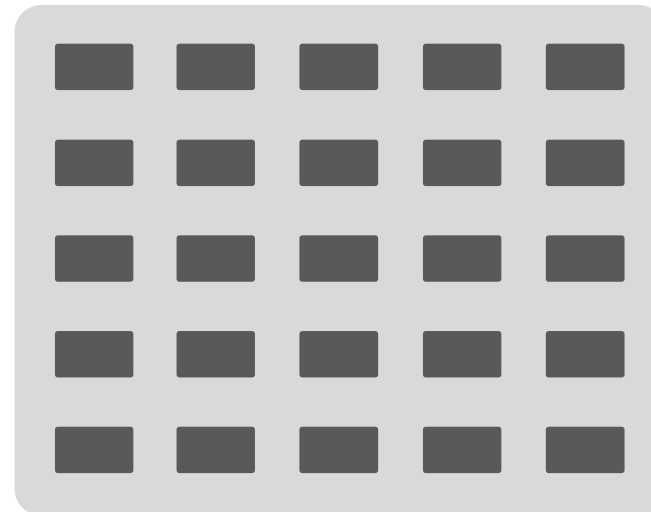
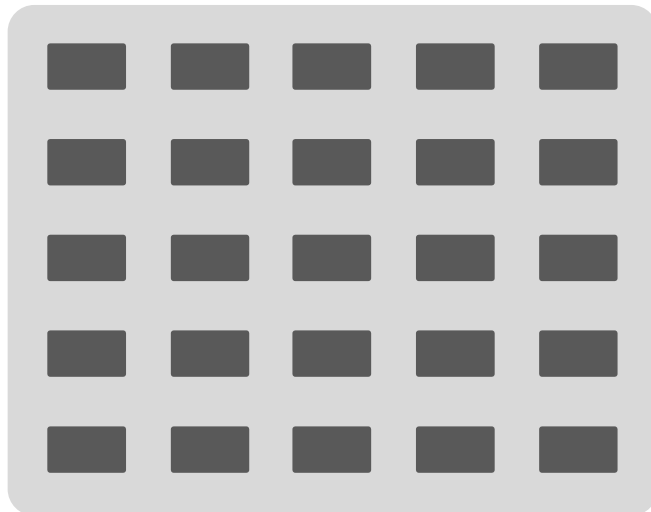




# DIAMETER-2 SLIM FLY

## 6 *Intra-group connections*

Two routers in one group are connected iff their “vertical Manhattan distance” is an element from:

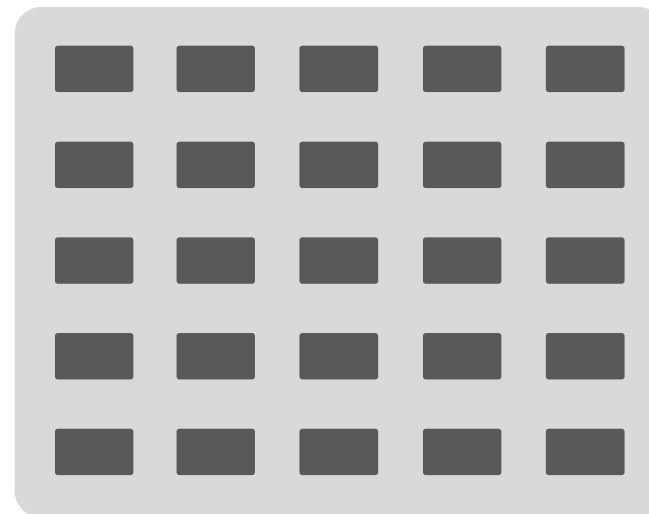
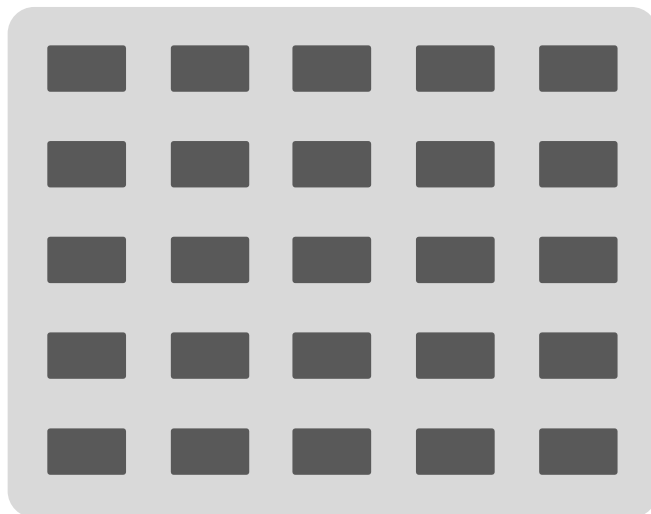


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# DIAMETER-2 SLIM FLY

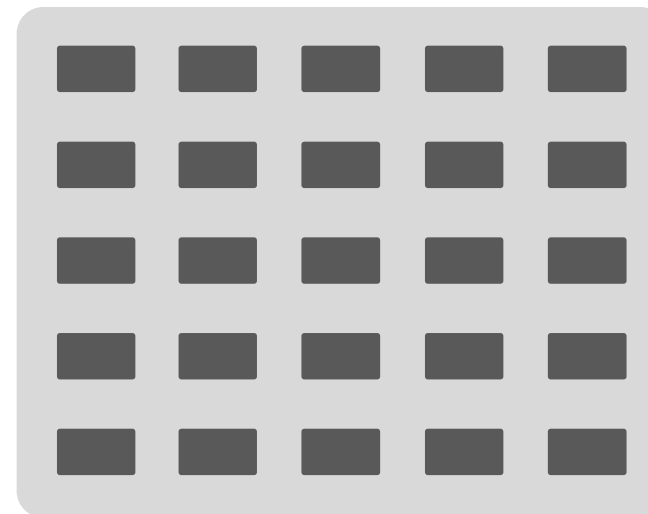
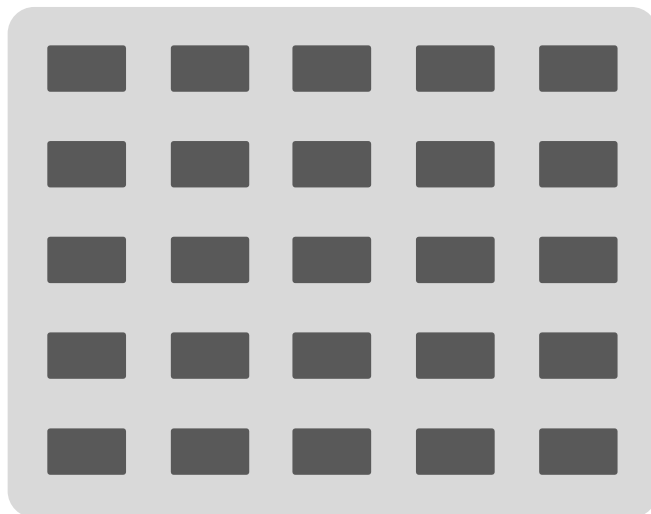
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E Example:  $q = 5$



# DIAMETER-2 SLIM FLY

## 6 Intra-group connections

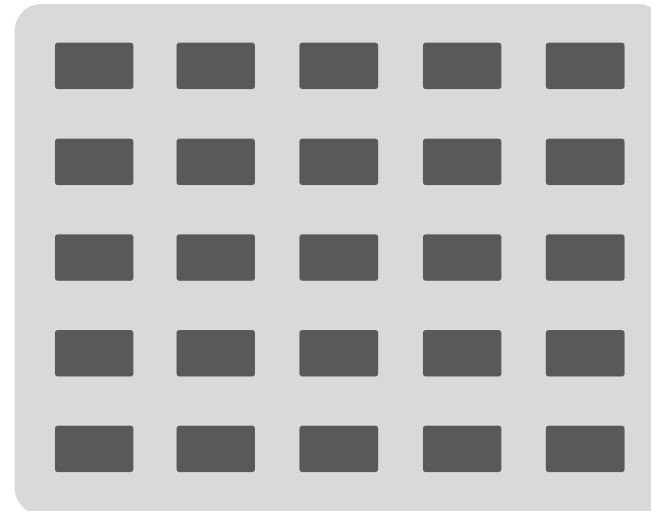
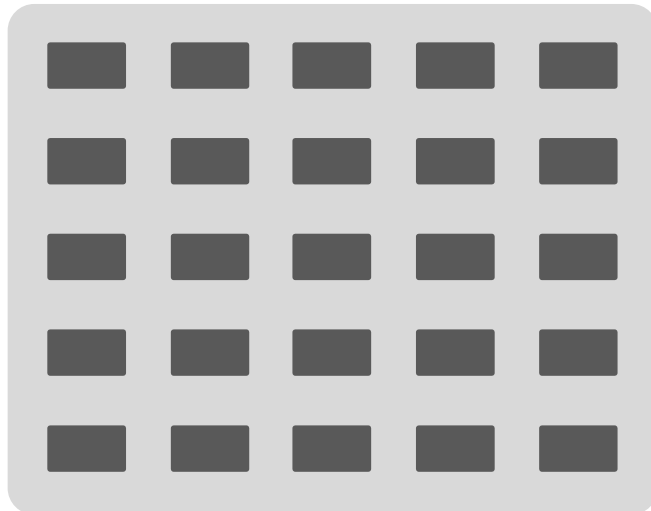
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**E** Example:  $q = 5$

Take Routers  $(0, 0, \cdot)$



# DIAMETER-2 SLIM FLY

## 6 Intra-group connections

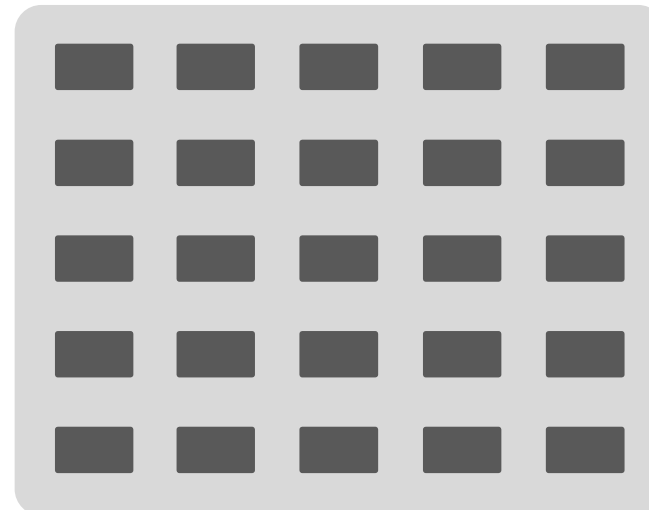
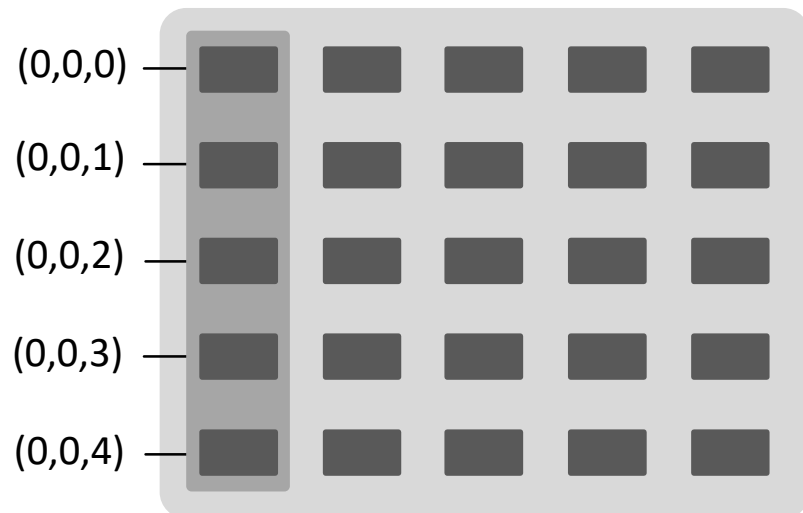
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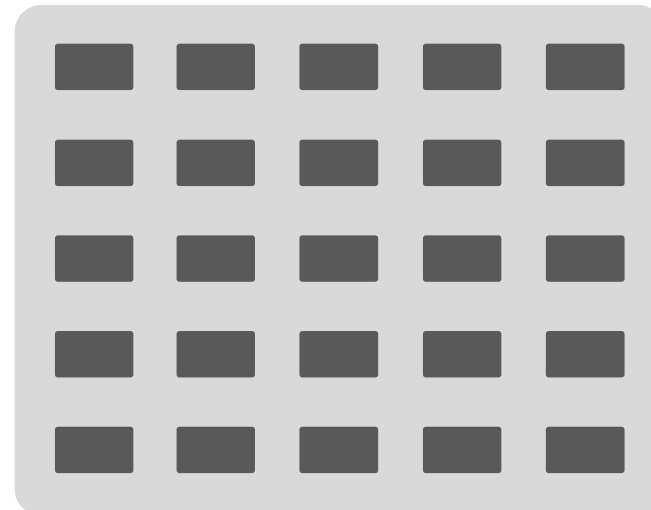
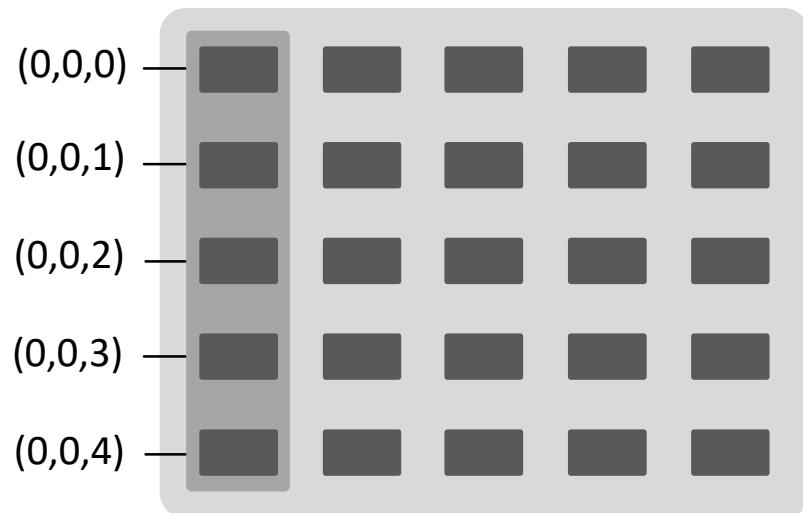
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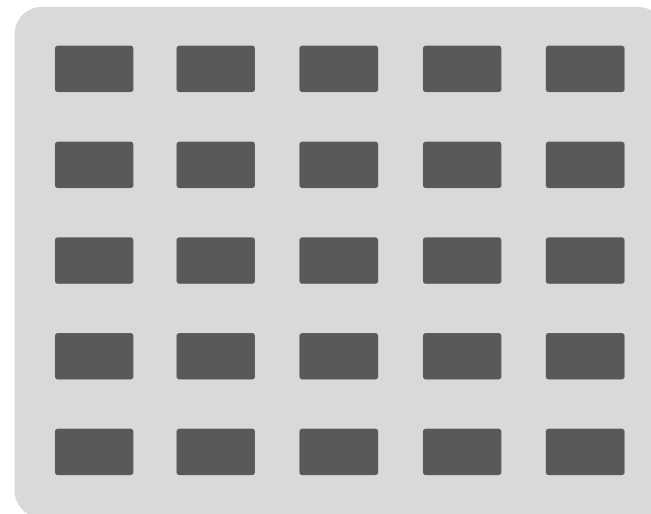
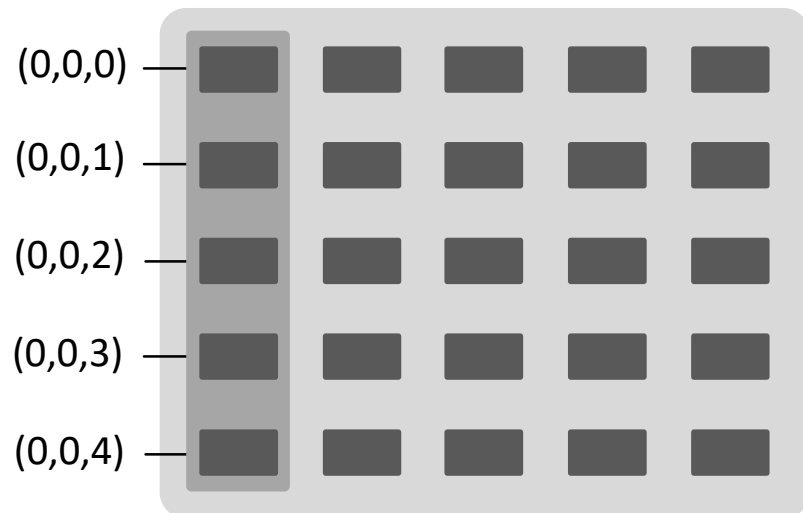
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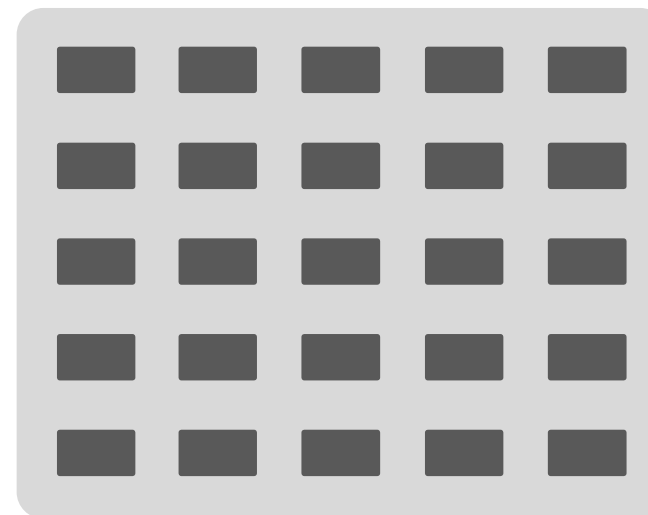
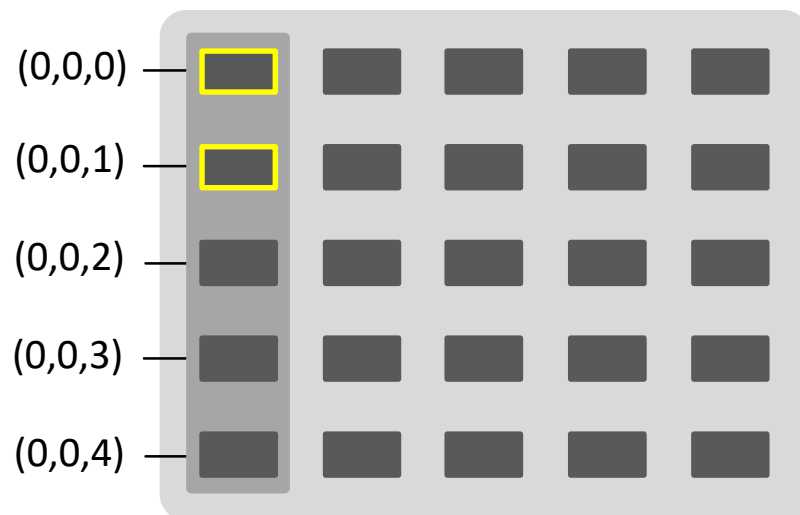
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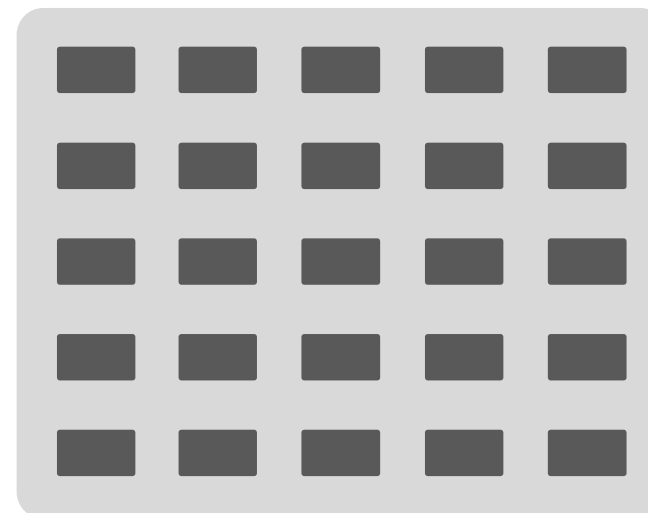
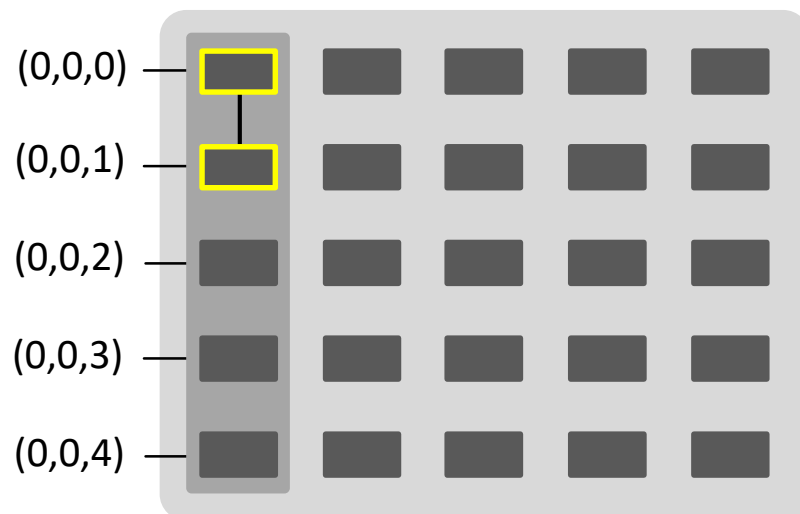
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**E** Example:  $q = 5$

Take Routers  $(0,0,.)$

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# DIAMETER-2 SLIM FLY

## 6 Intra-group connections

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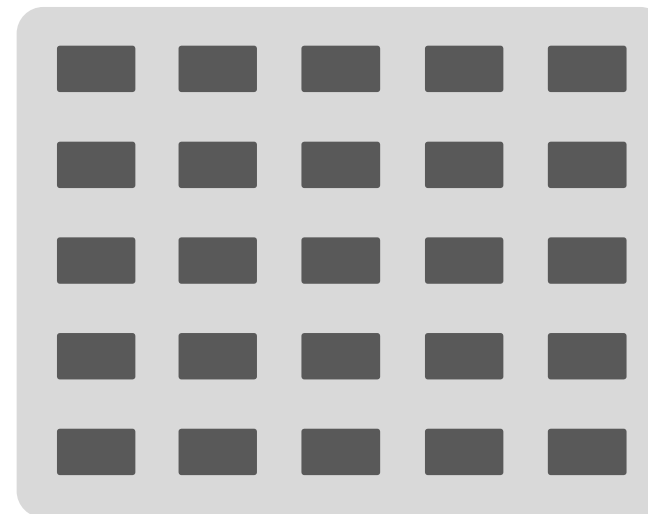
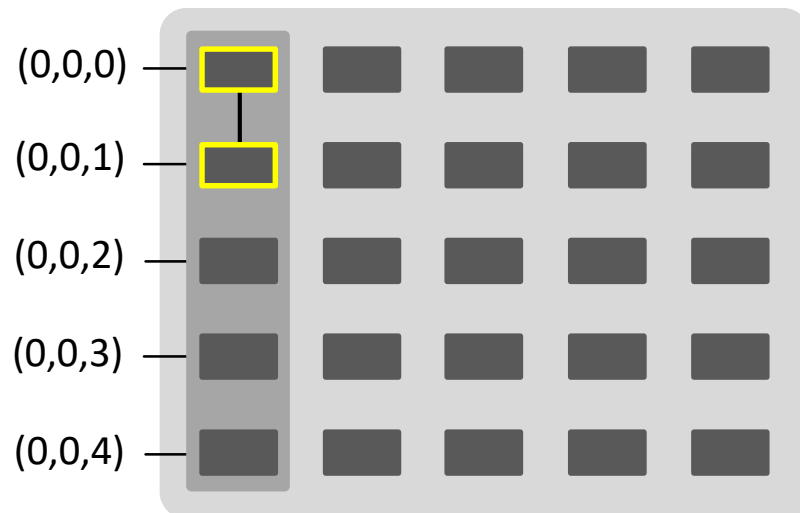
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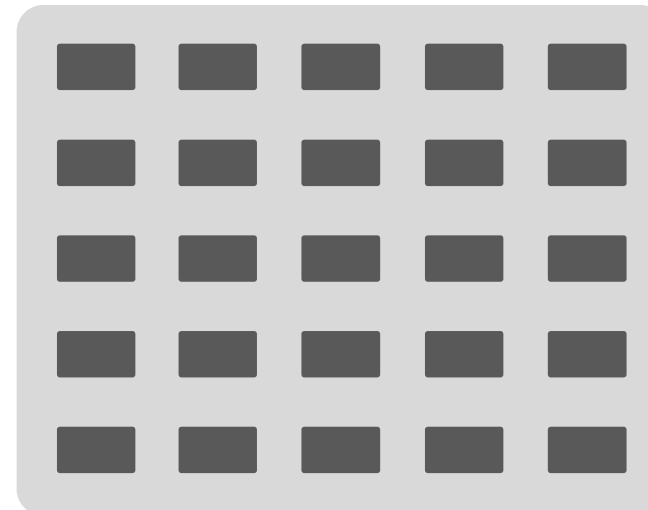
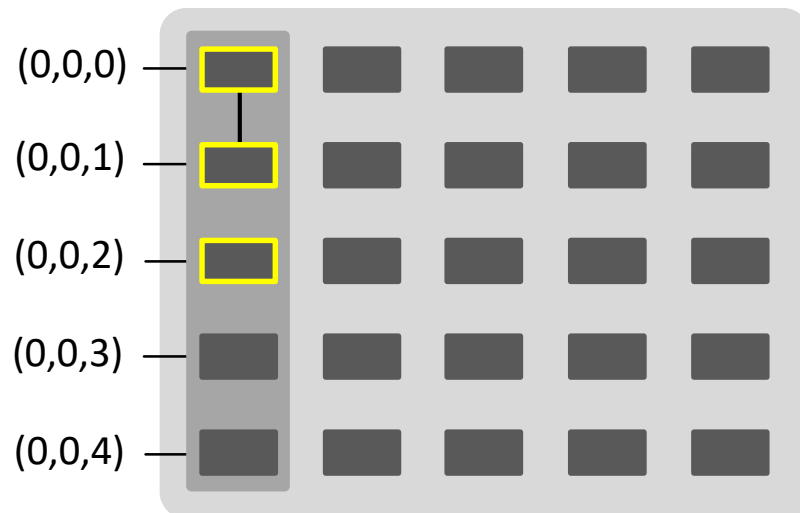
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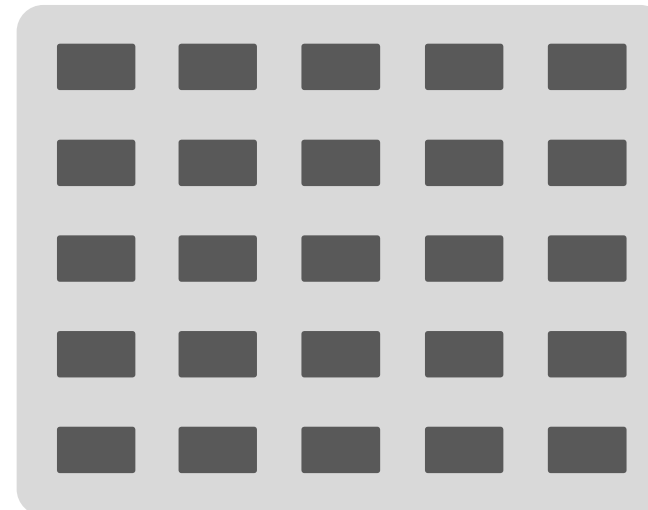
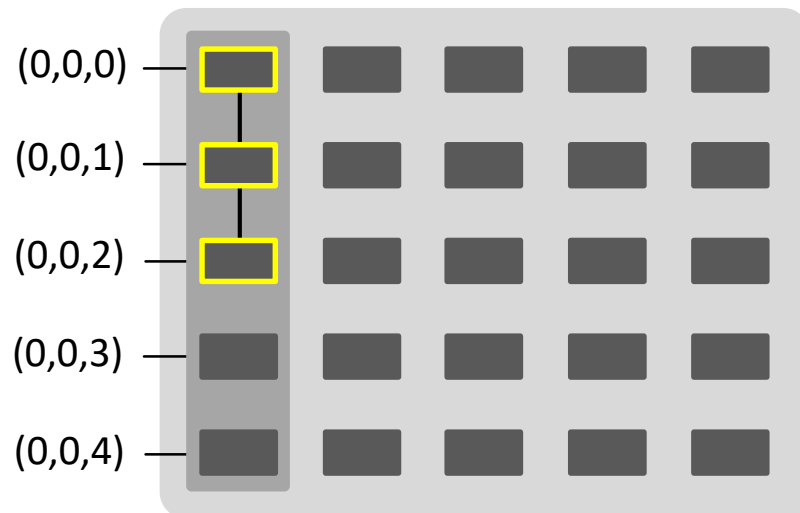
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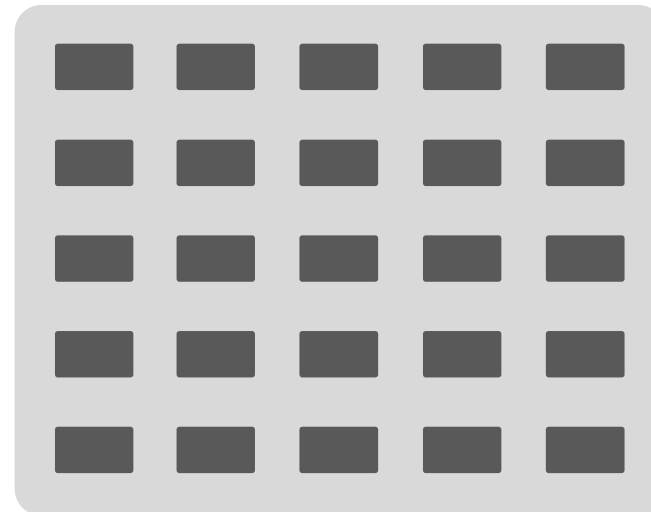
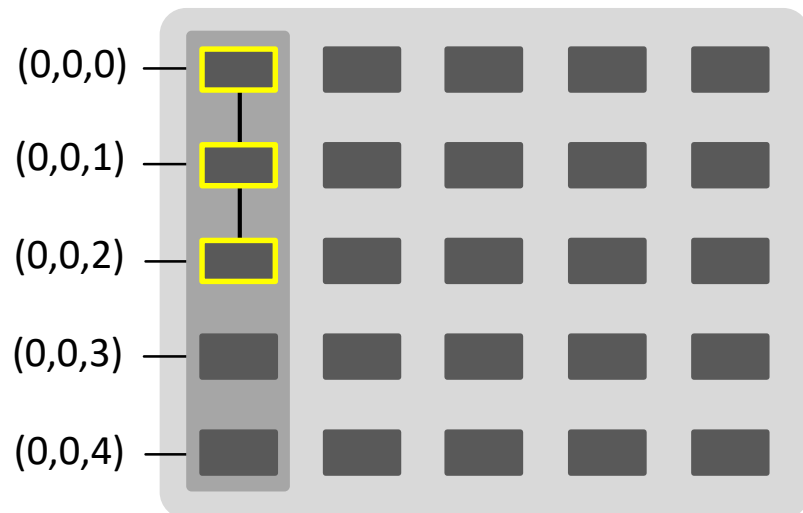
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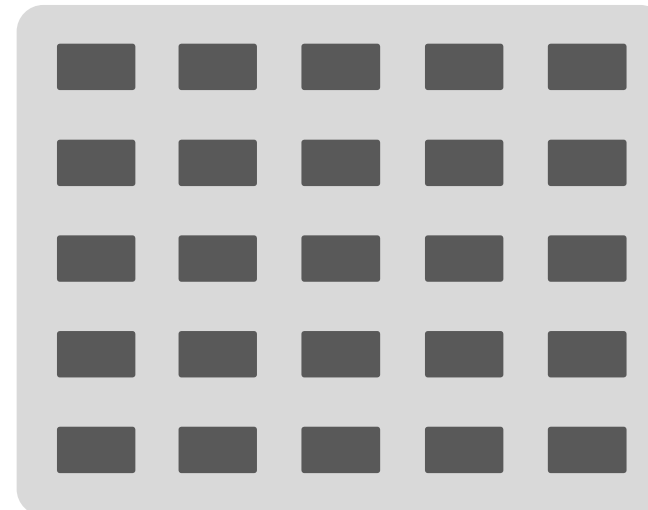
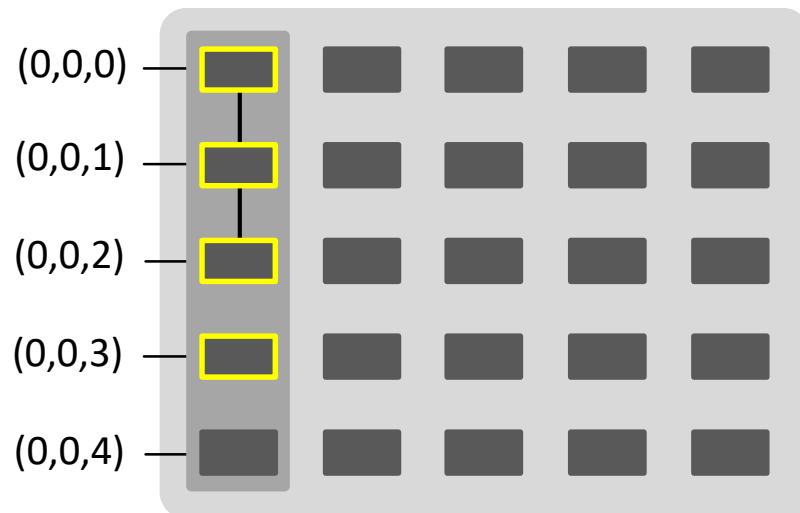
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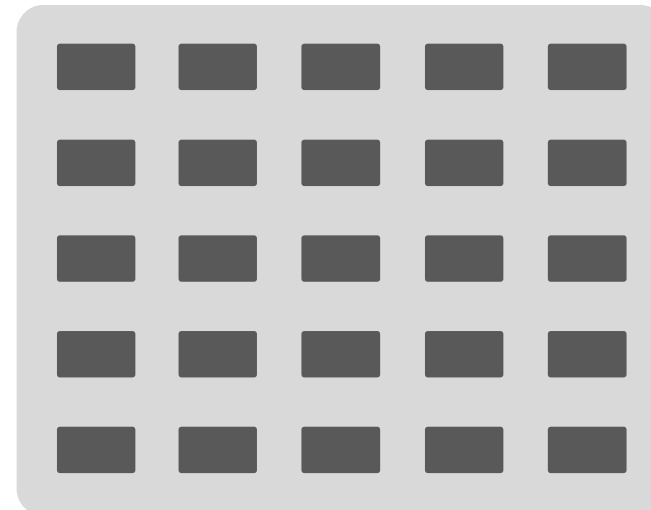
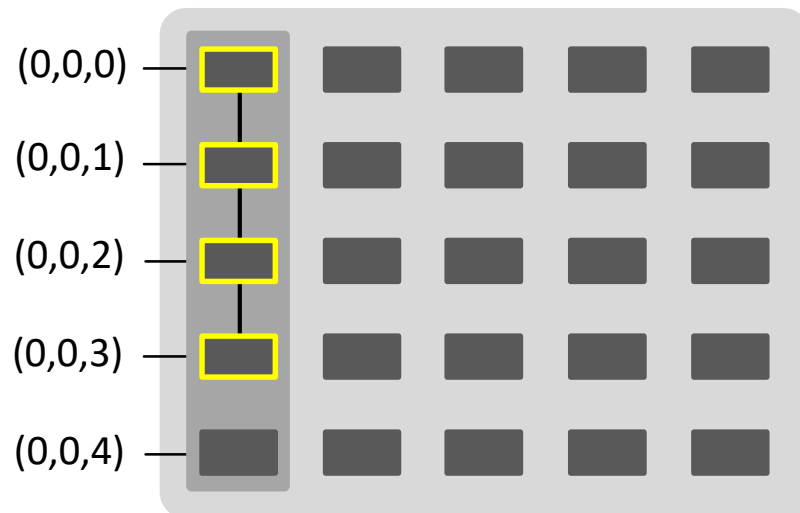
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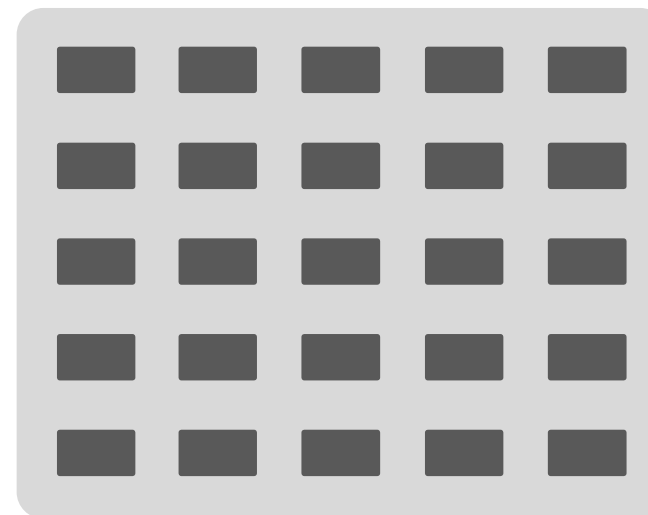
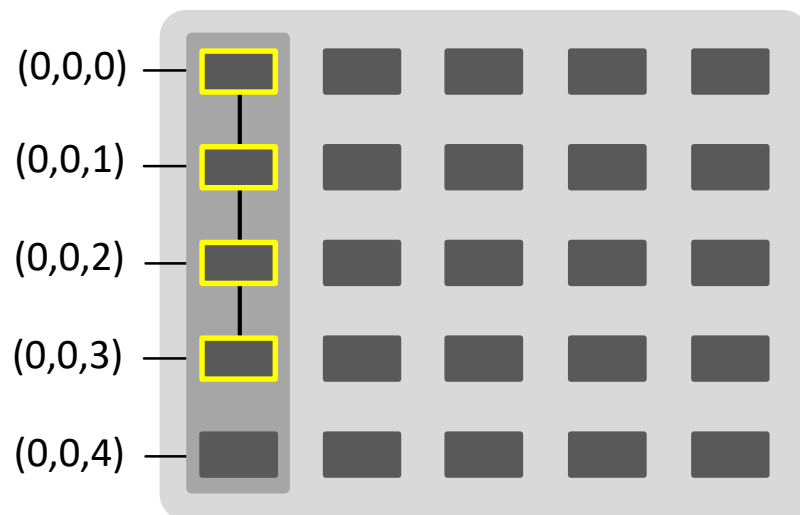
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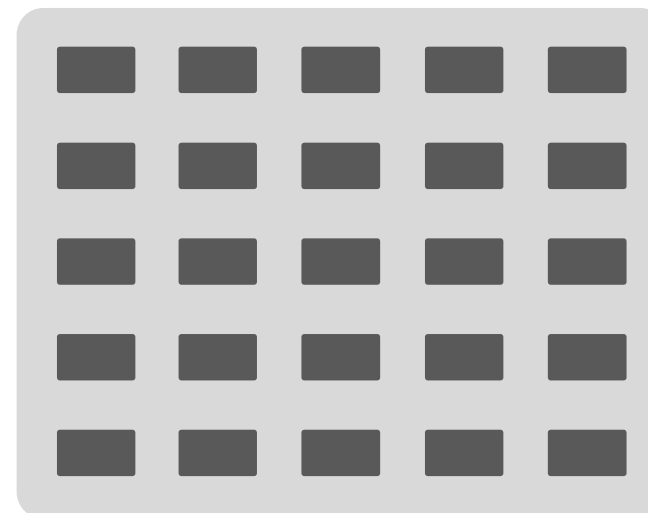
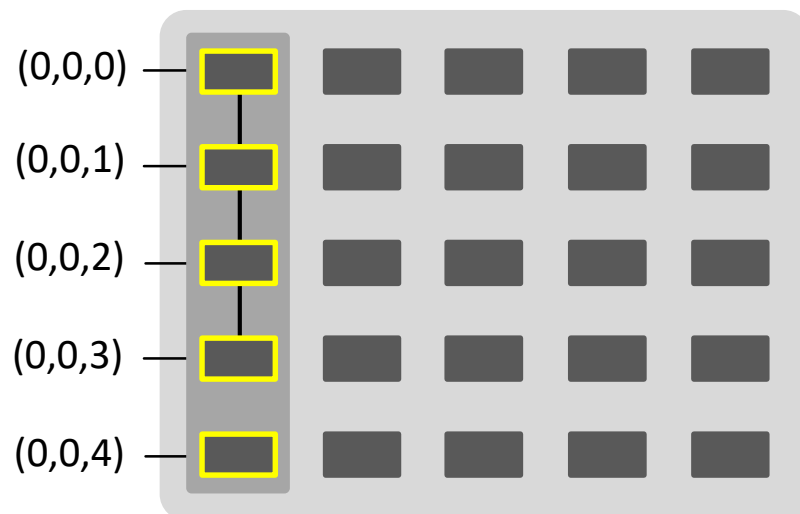
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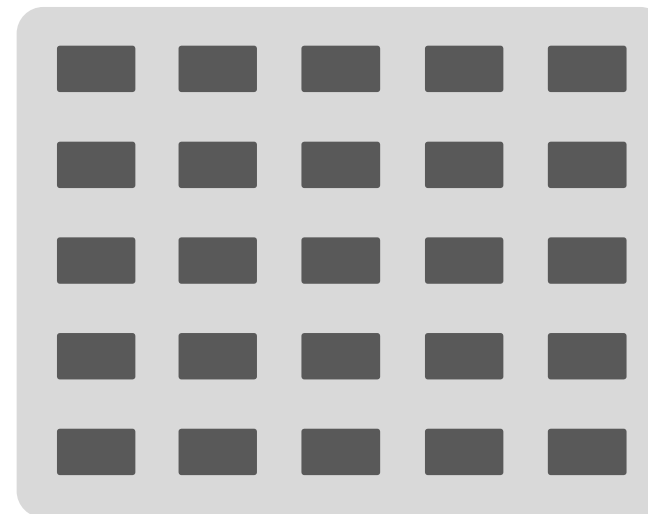
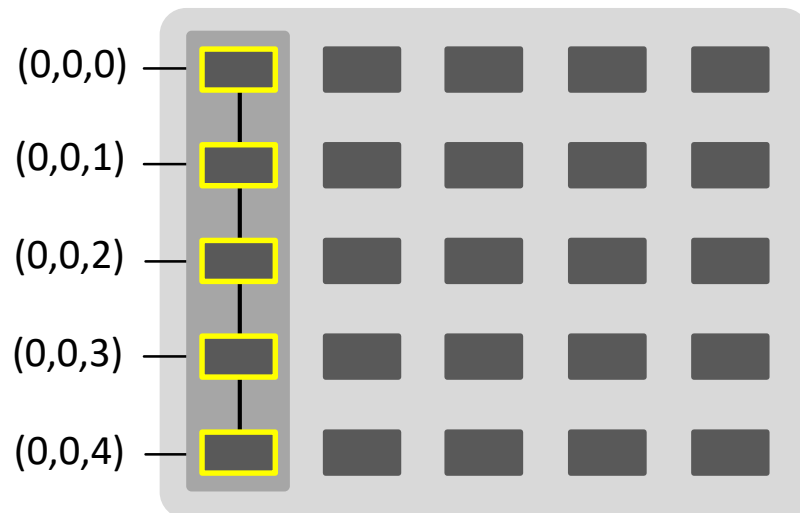
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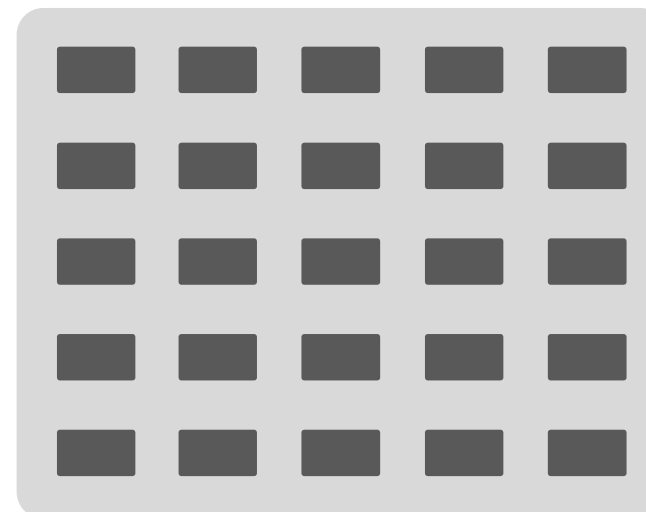
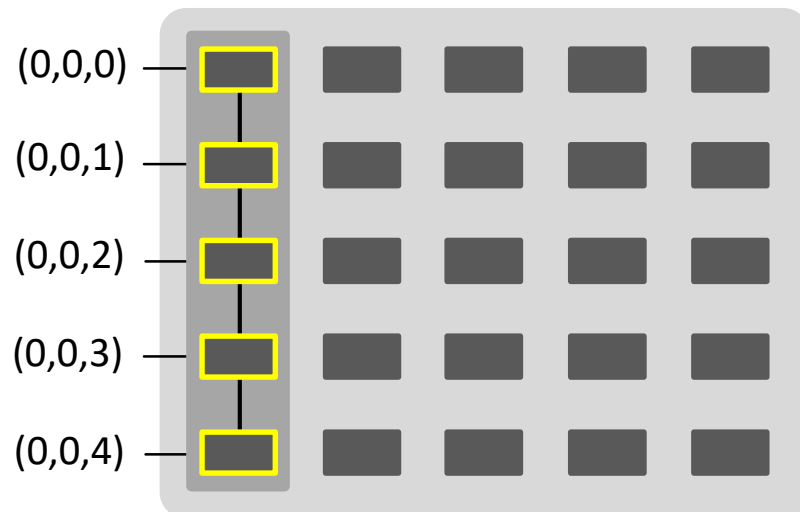
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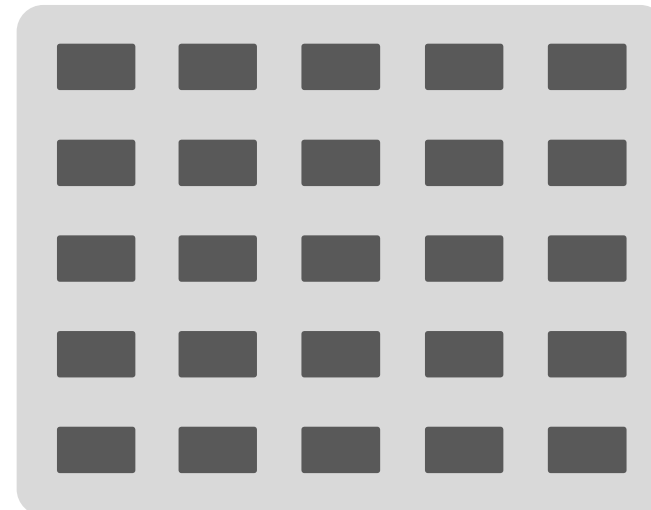
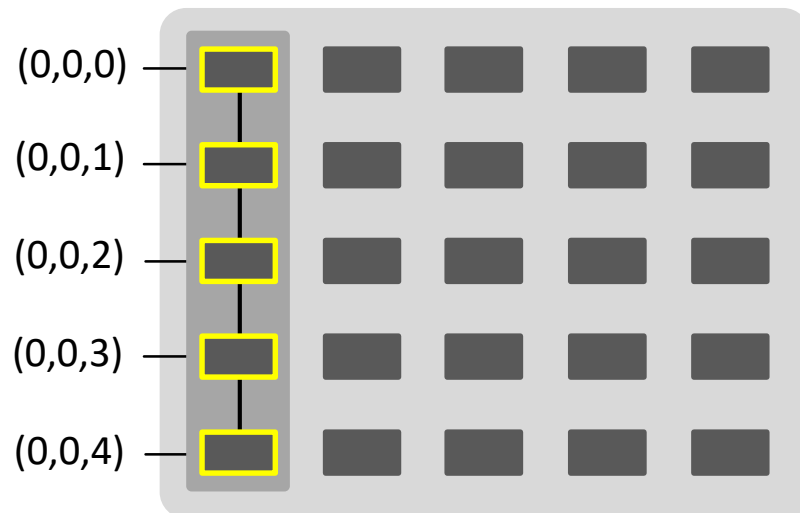
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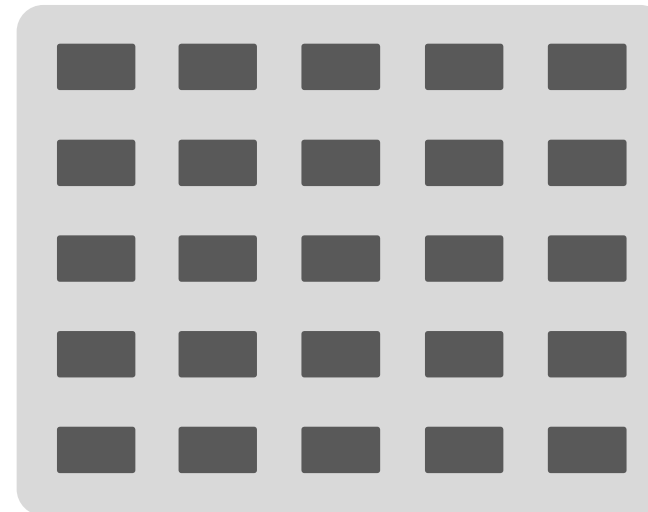
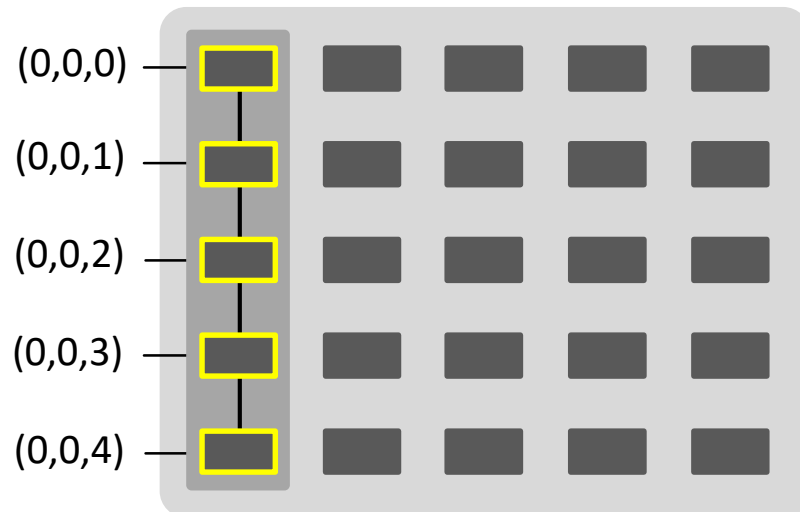
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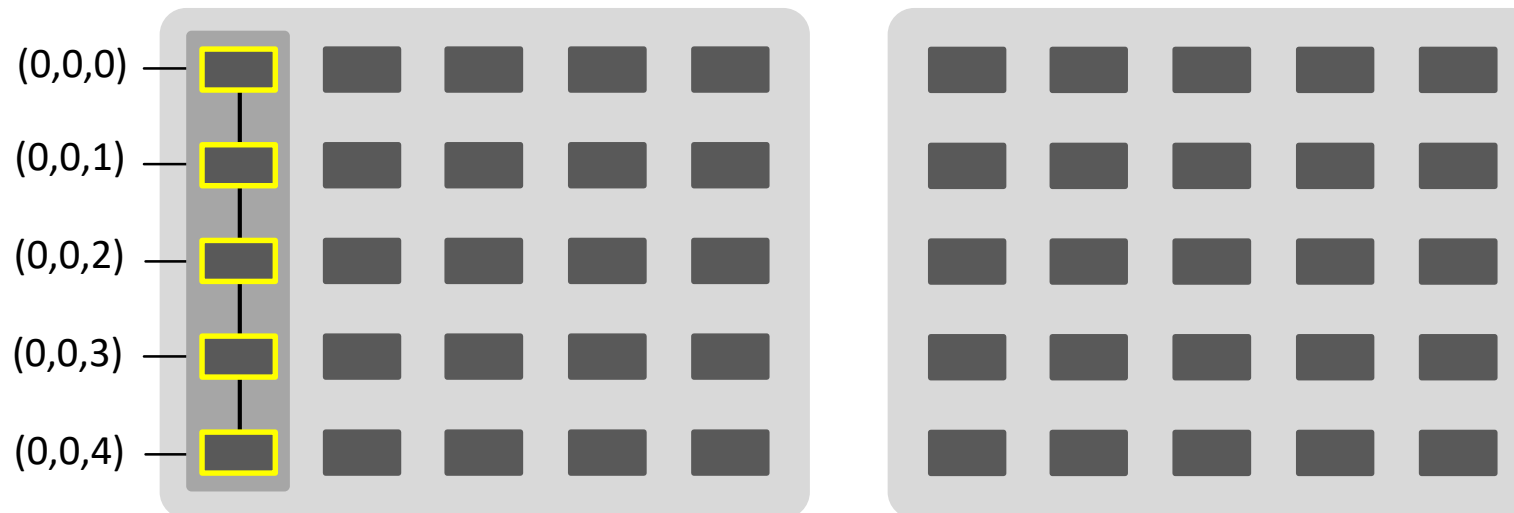
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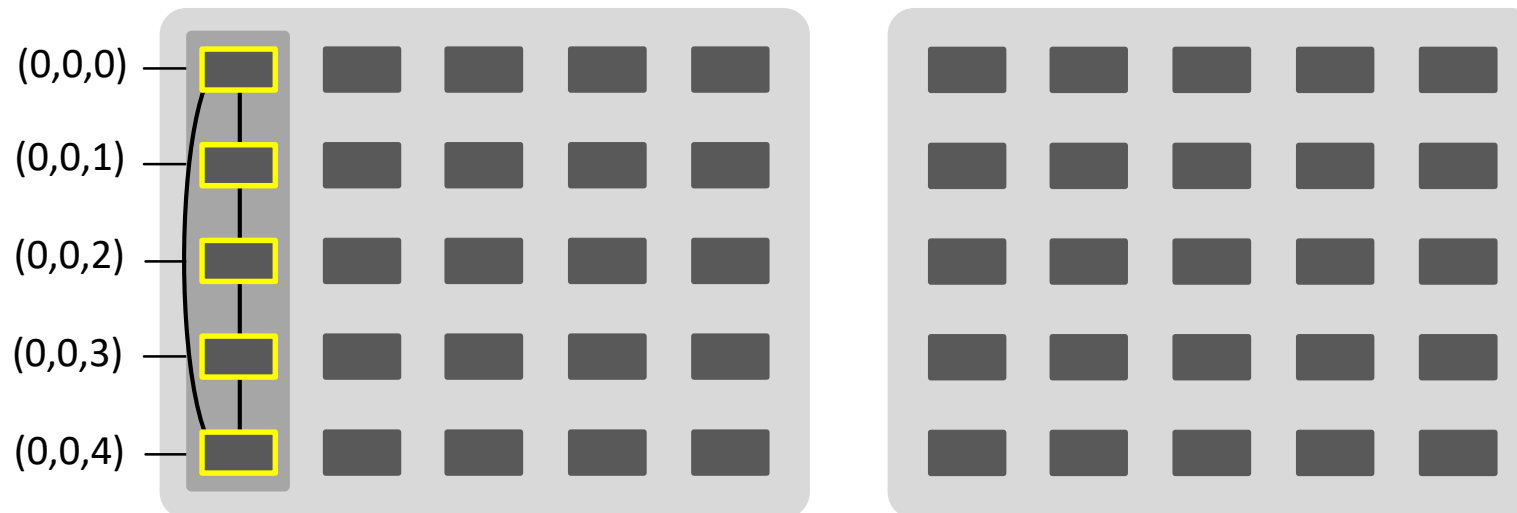
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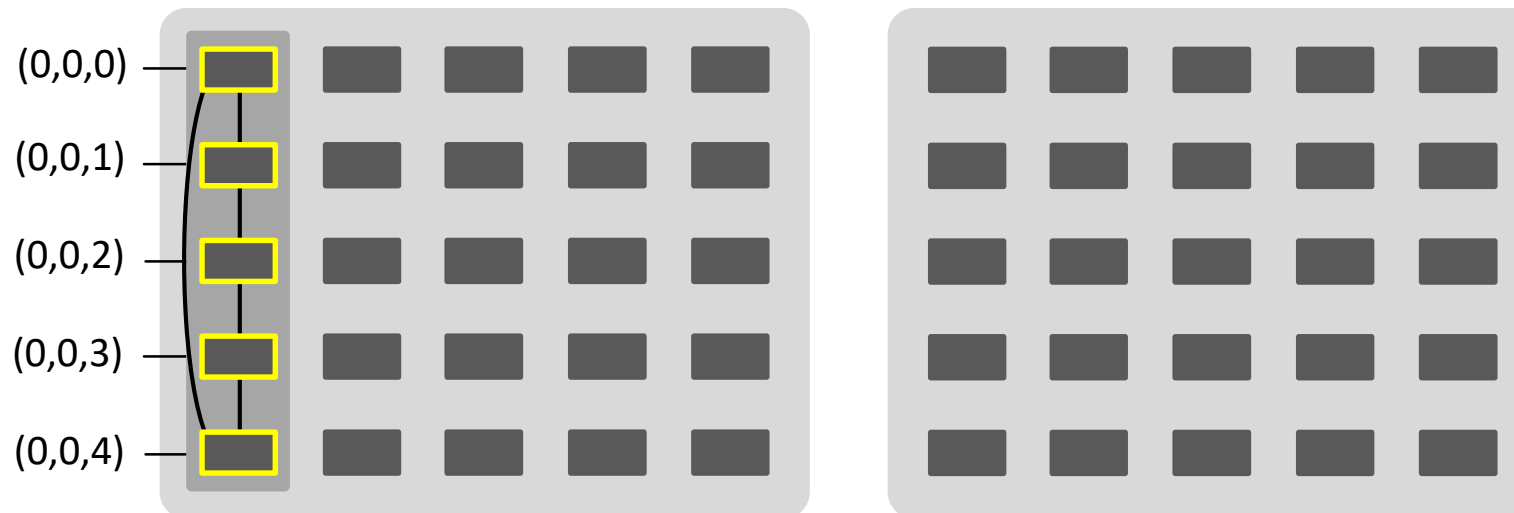
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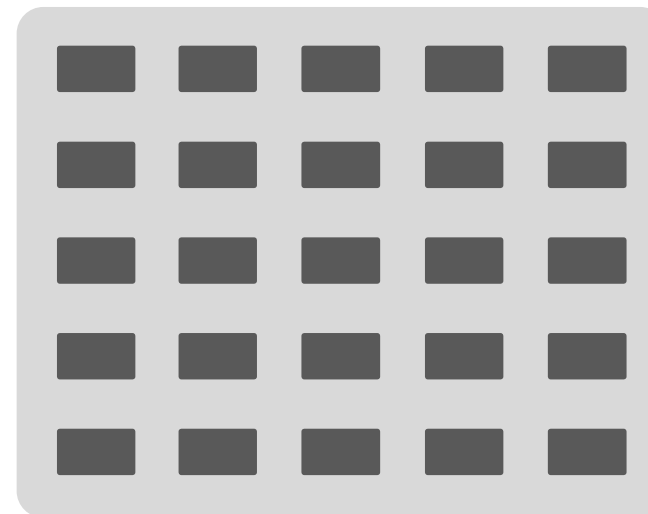
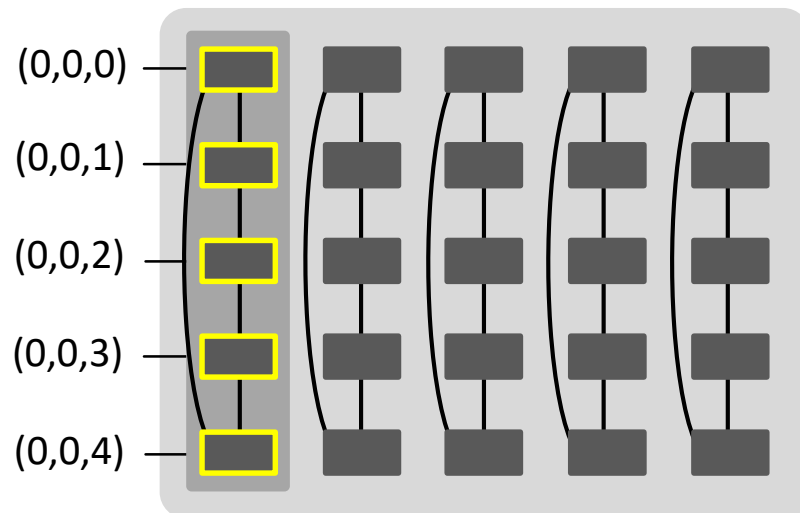
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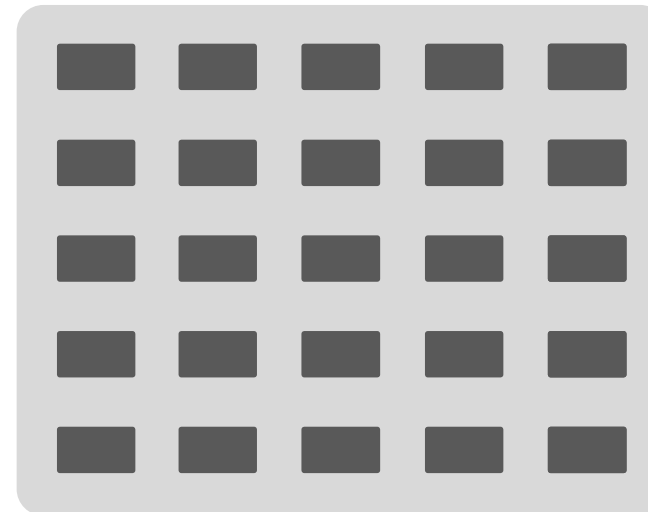
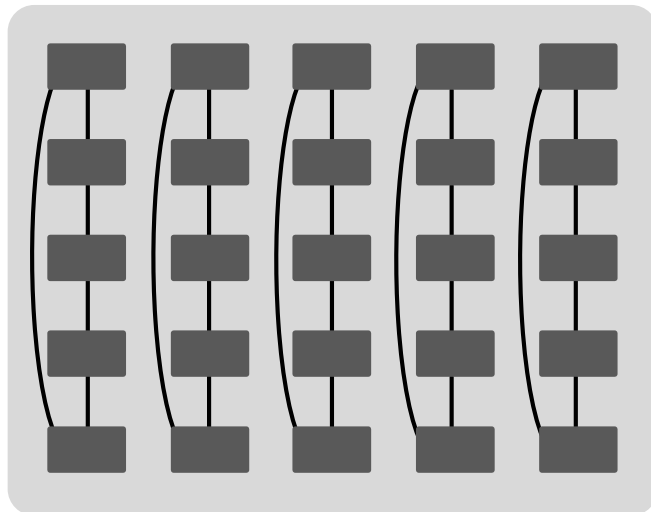
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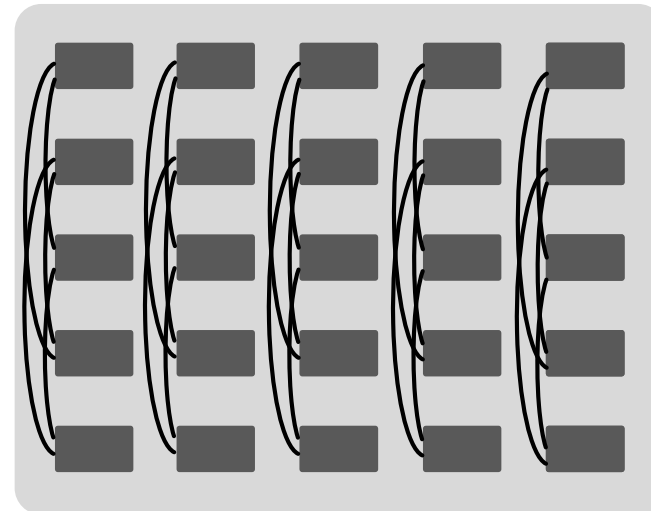
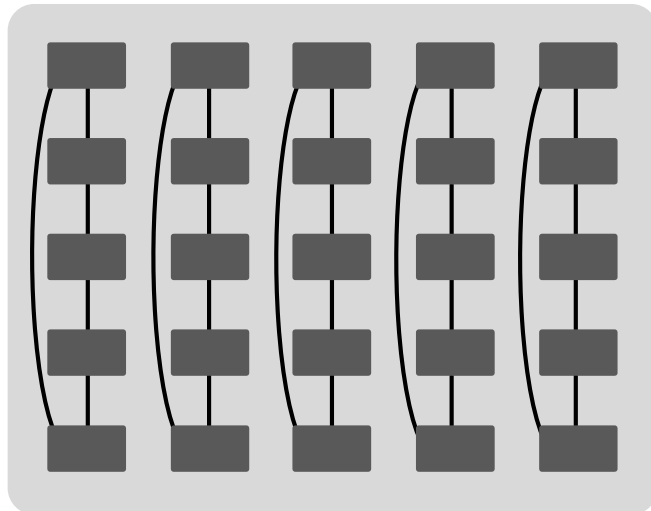
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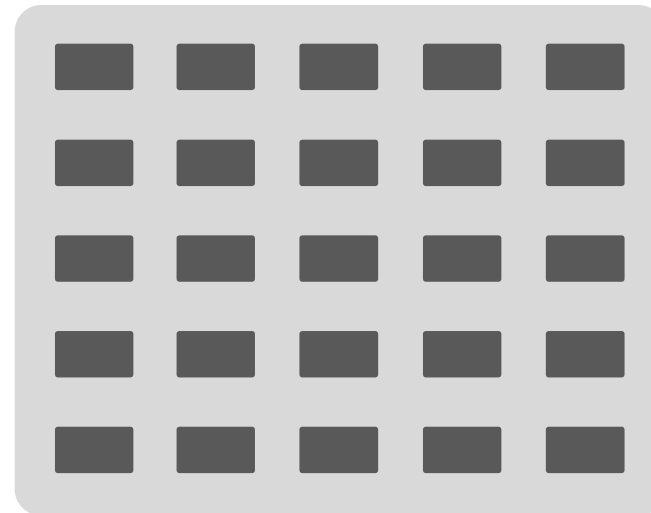
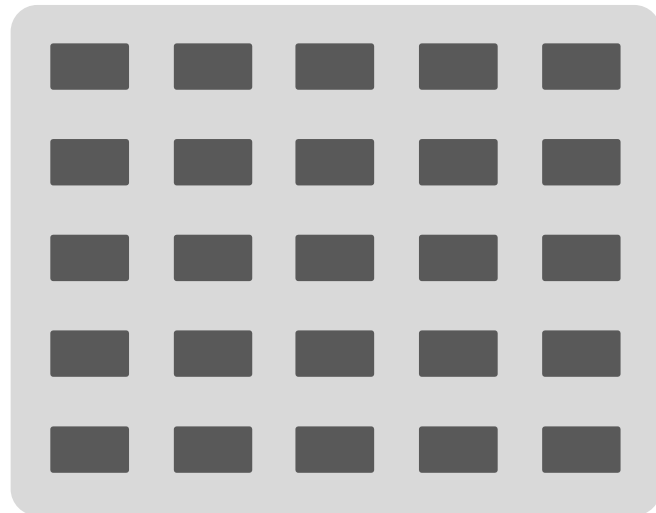
**E** Example:  $q = 5$

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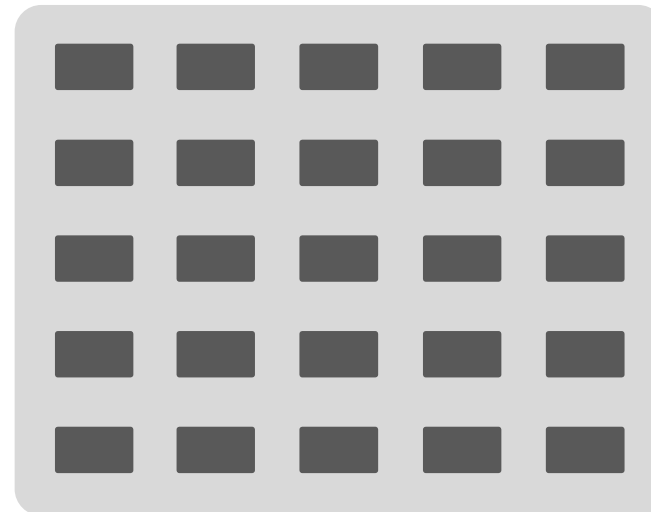
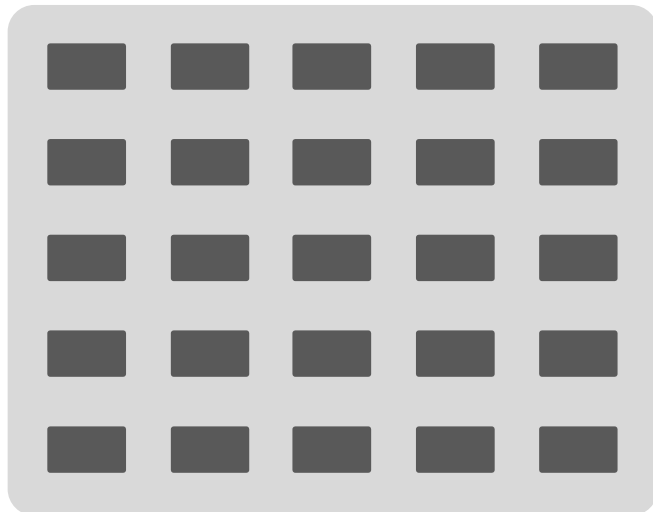


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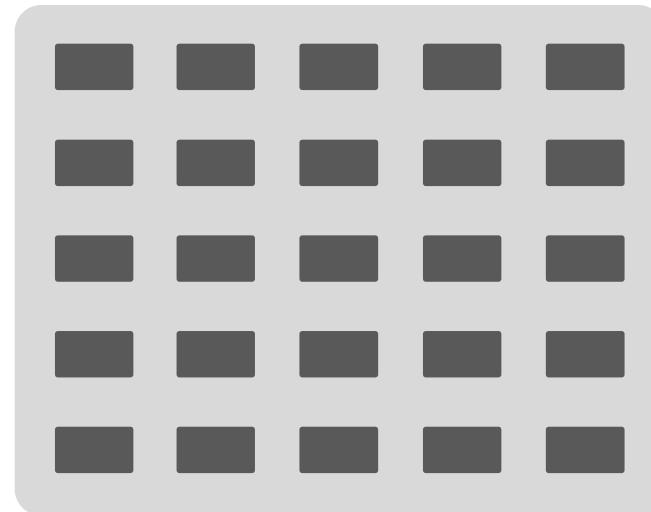
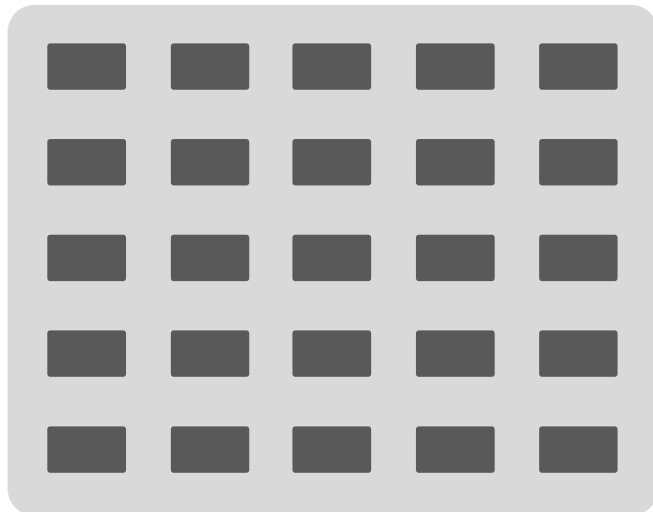
# DIAMETER-2 SLIM FLY

## 7 *Inter-group connections*



# DIAMETER-2 SLIM FLY

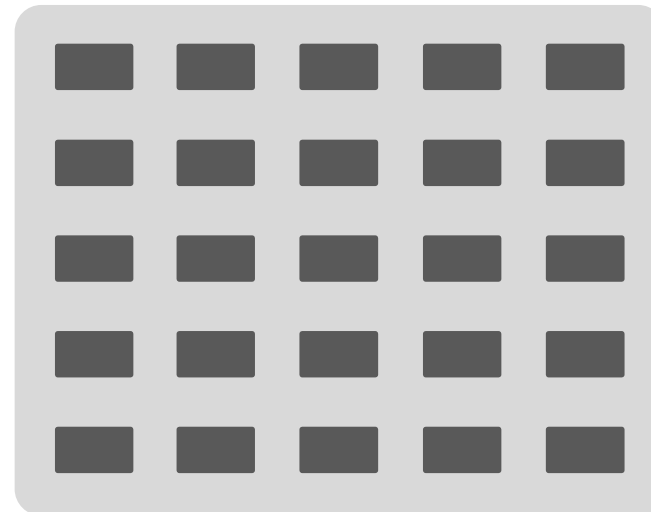
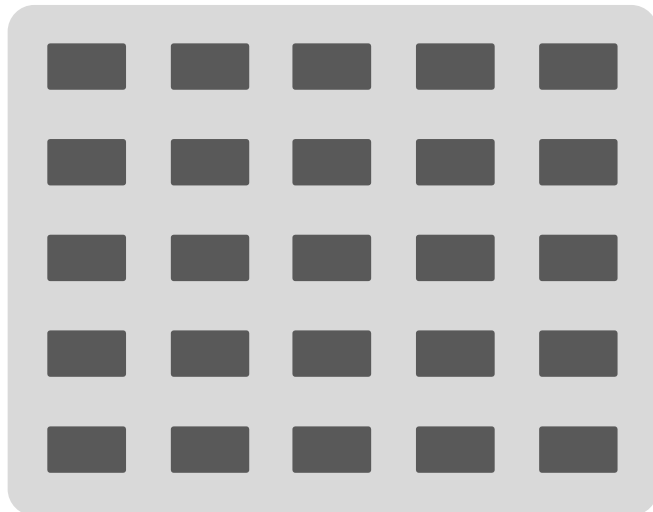
## 7 Inter-group connections



# DIAMETER-2 SLIM FLY

## 7 Inter-group connections

Router  $(0, x, y) \leftrightarrow (1, m, c)$

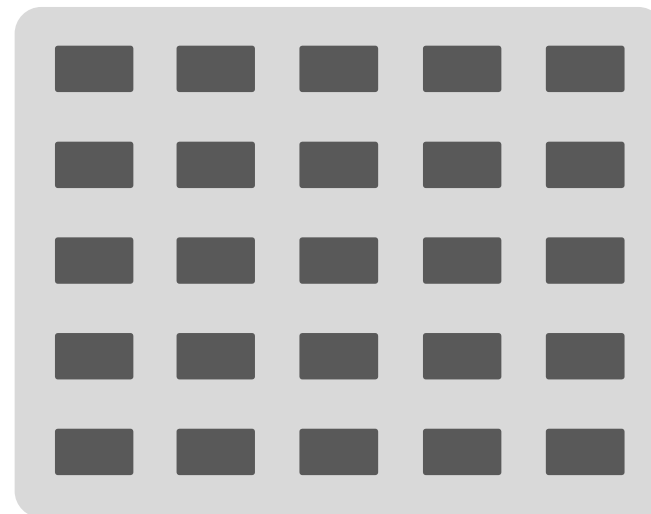
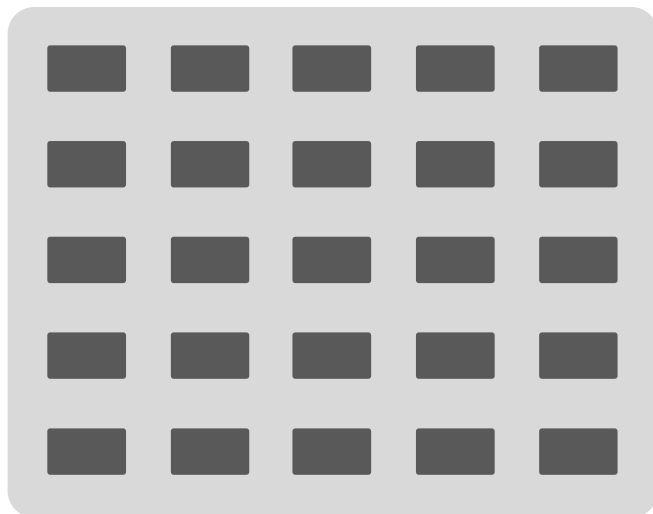


# DIAMETER-2 SLIM FLY

## 7 Inter-group connections

Router  $(0, x, y) \leftrightarrow (1, m, c)$

$$\text{iff } y = mx + c$$





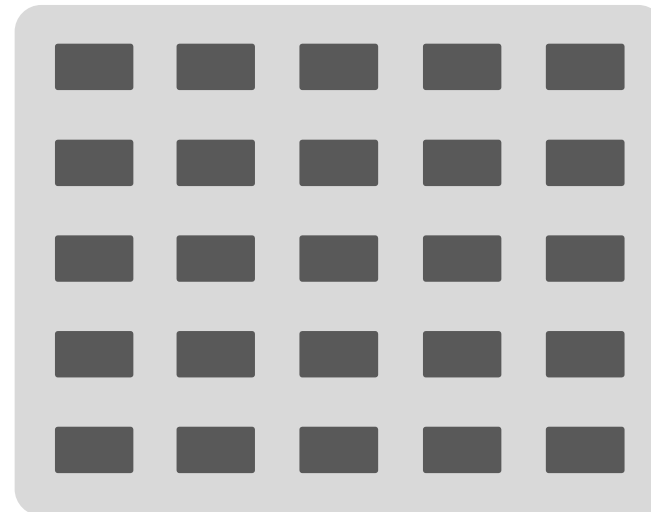
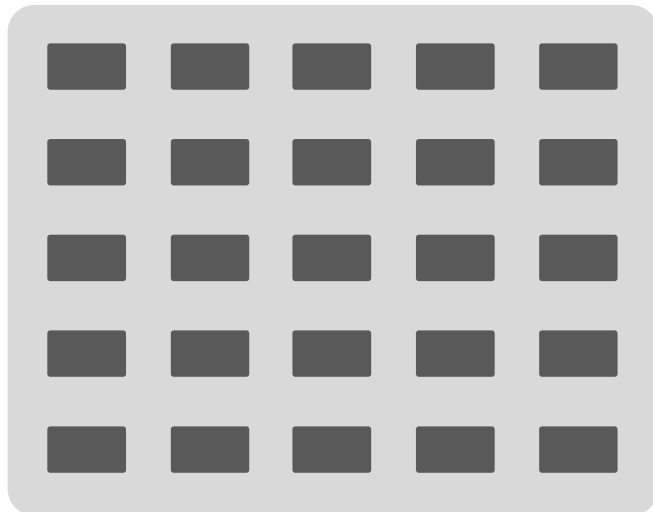
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E Example:  $q = 5$



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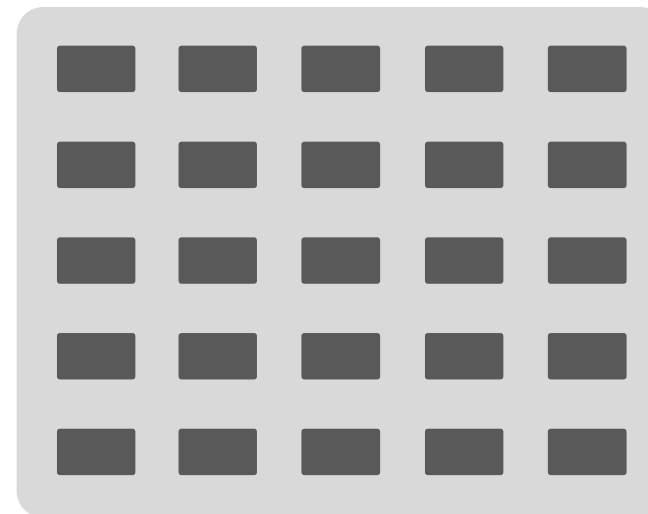
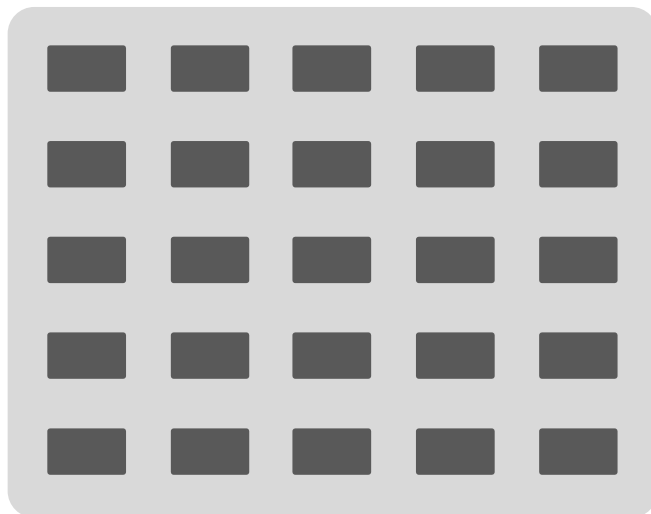
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**E** Example:  $q = 5$

Take Router  $(1, 0, 0)$



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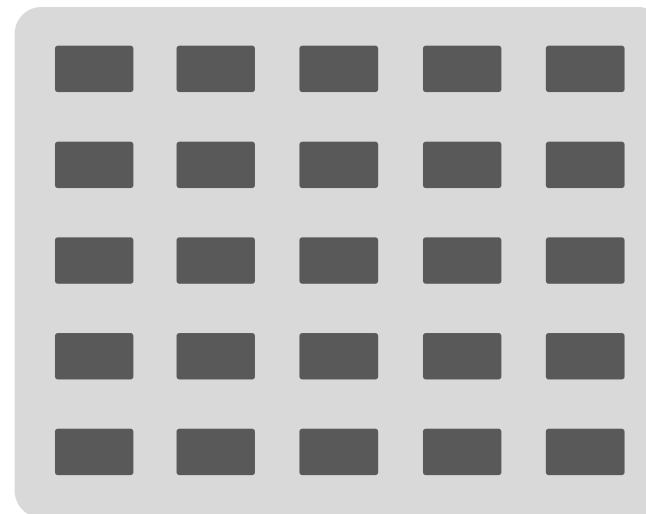
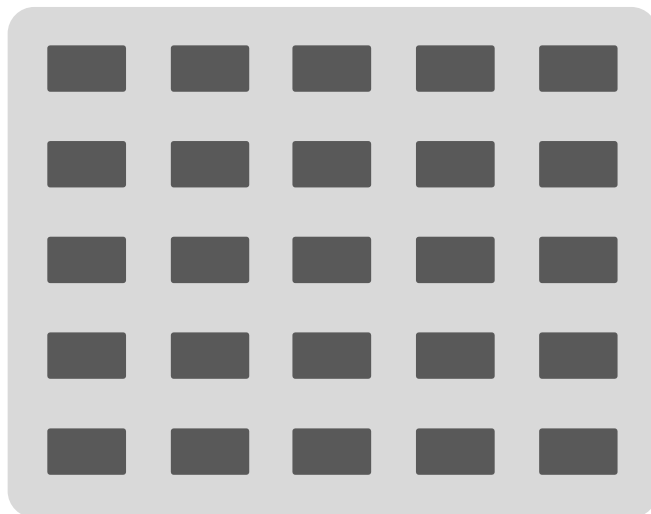
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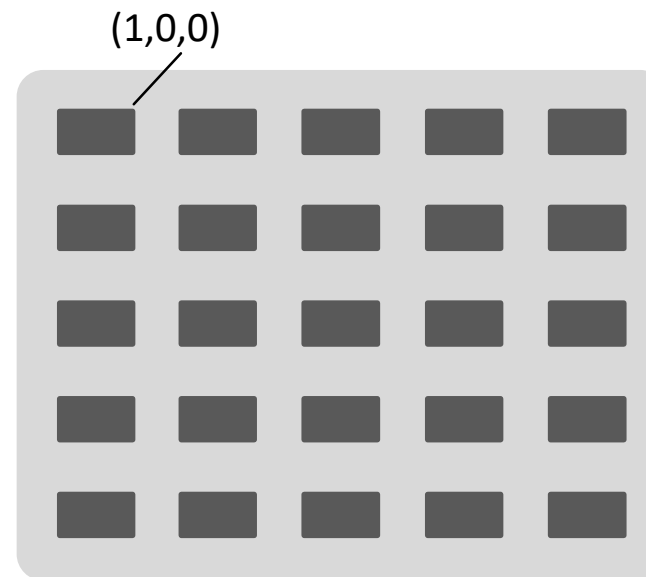
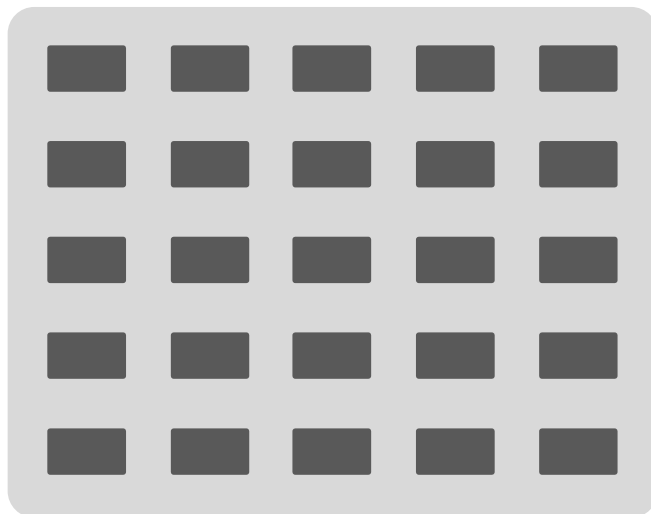
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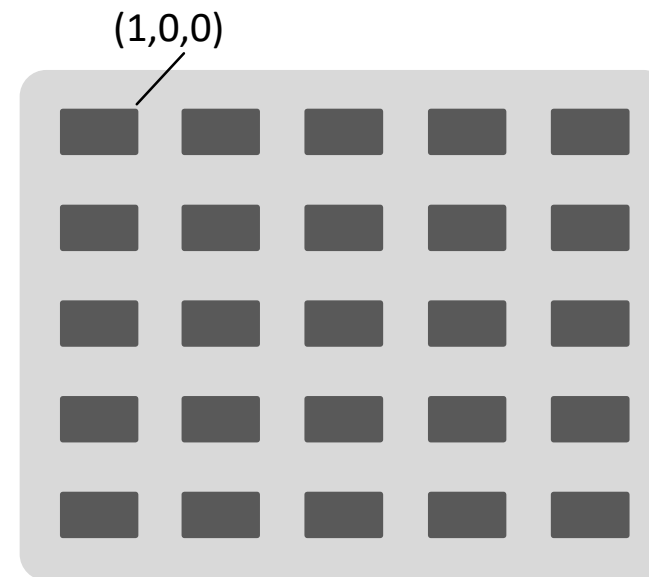
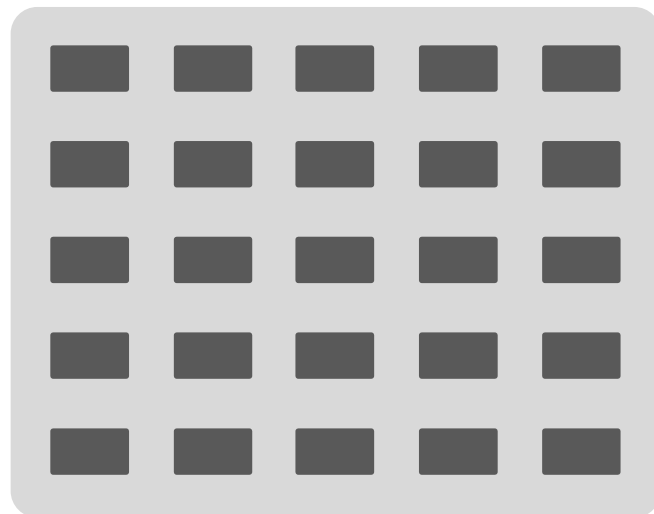
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Take Router  $(1, 0, 0)$   $m = 0, c = 0$



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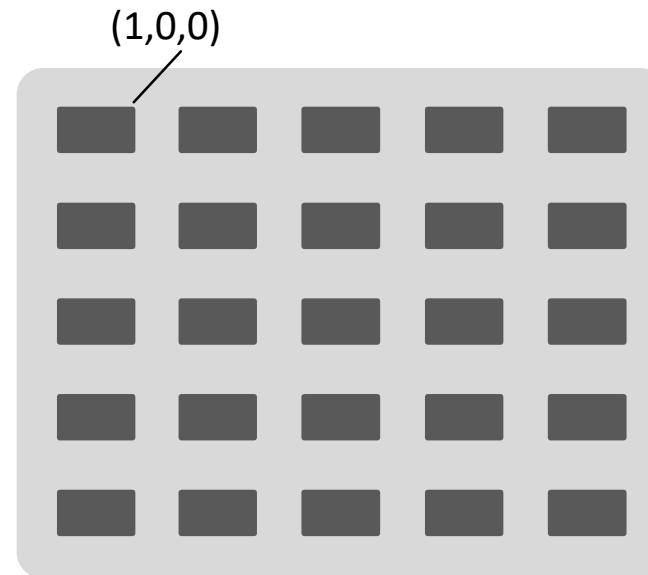
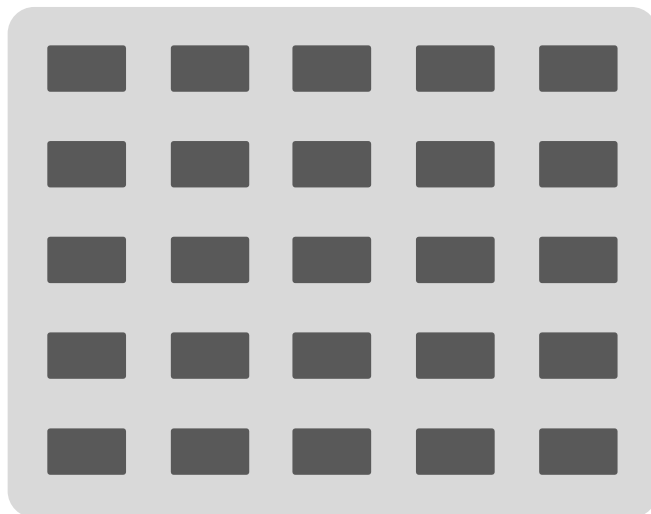
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# DIAMETER-2 SLIM FLY

## 7 Inter-group connections

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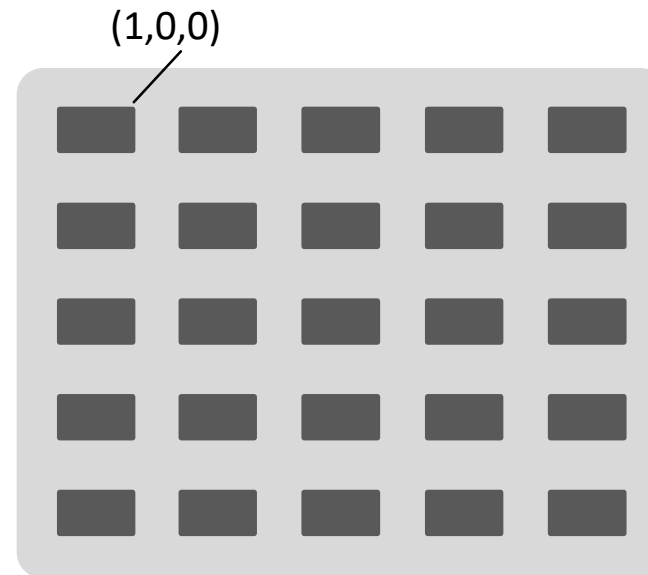
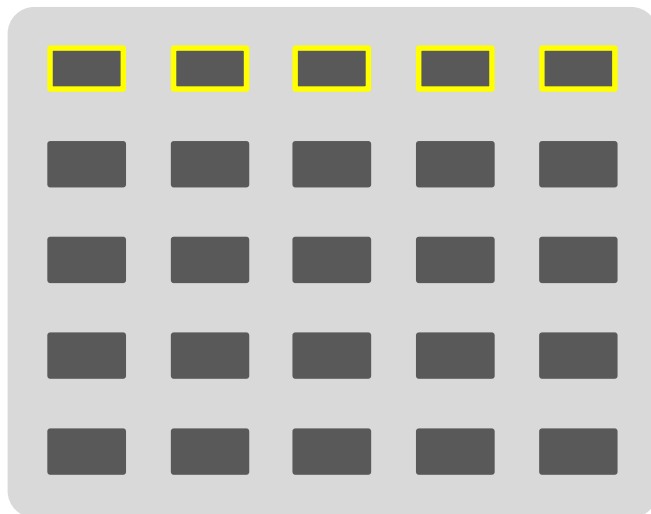
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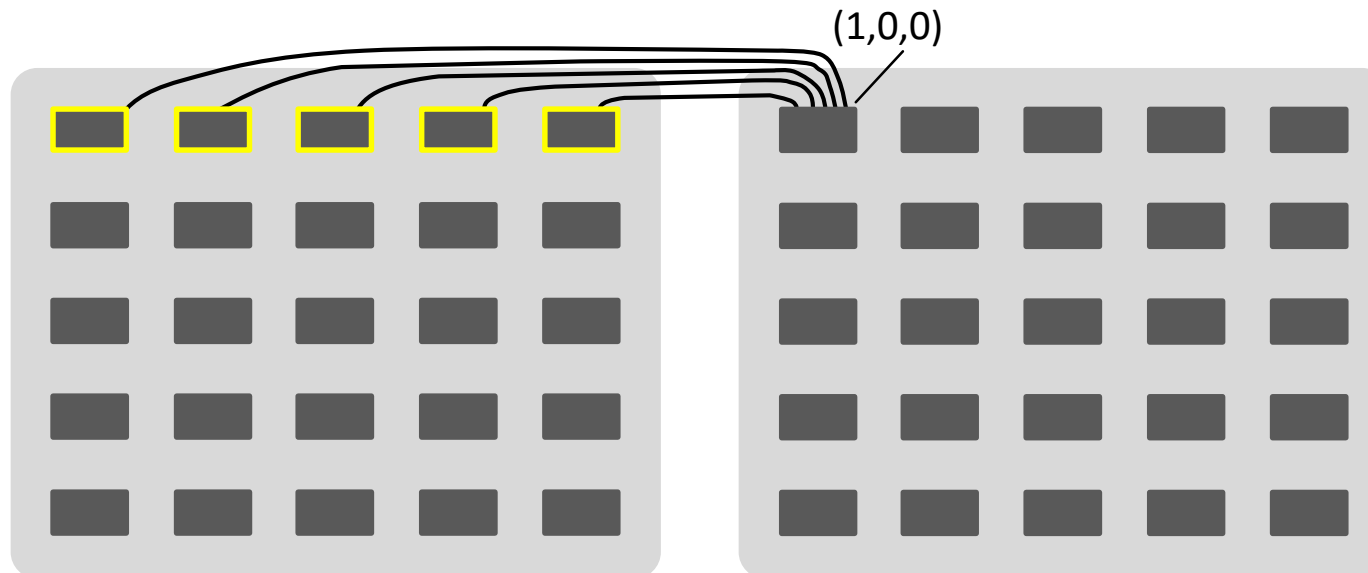
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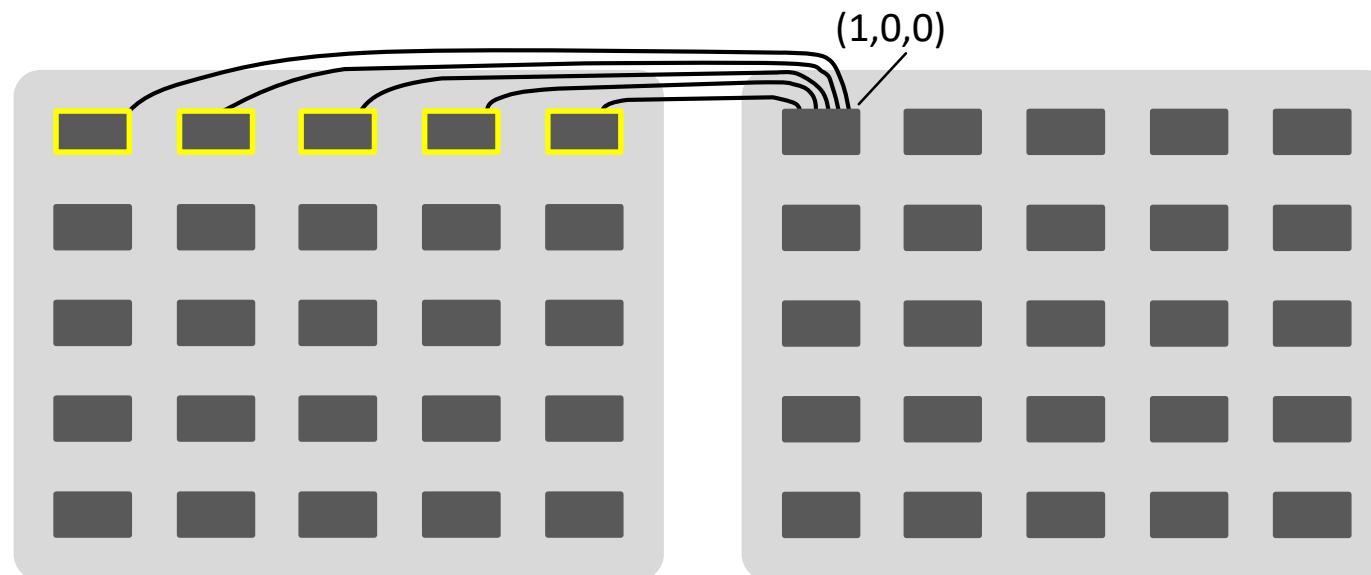
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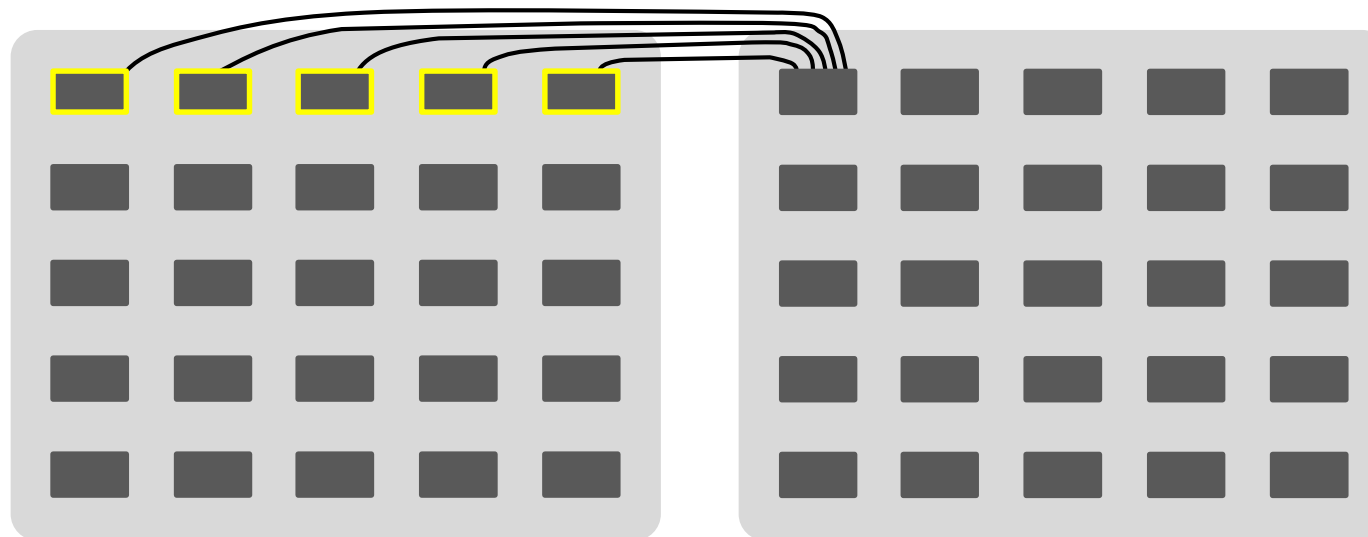
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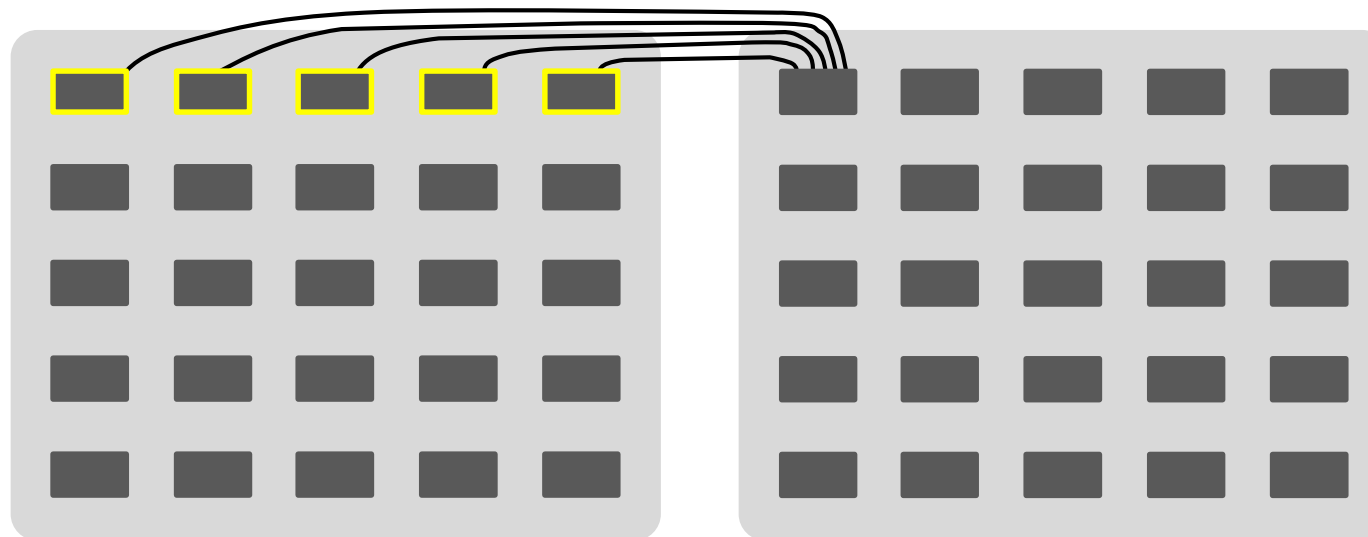
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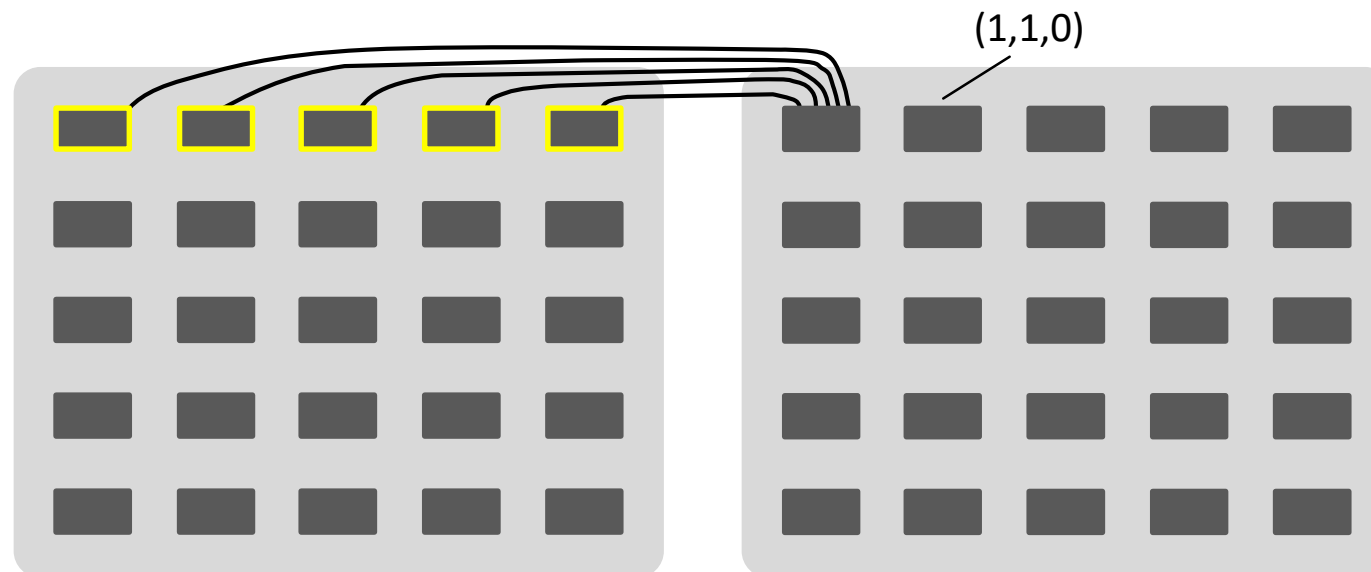
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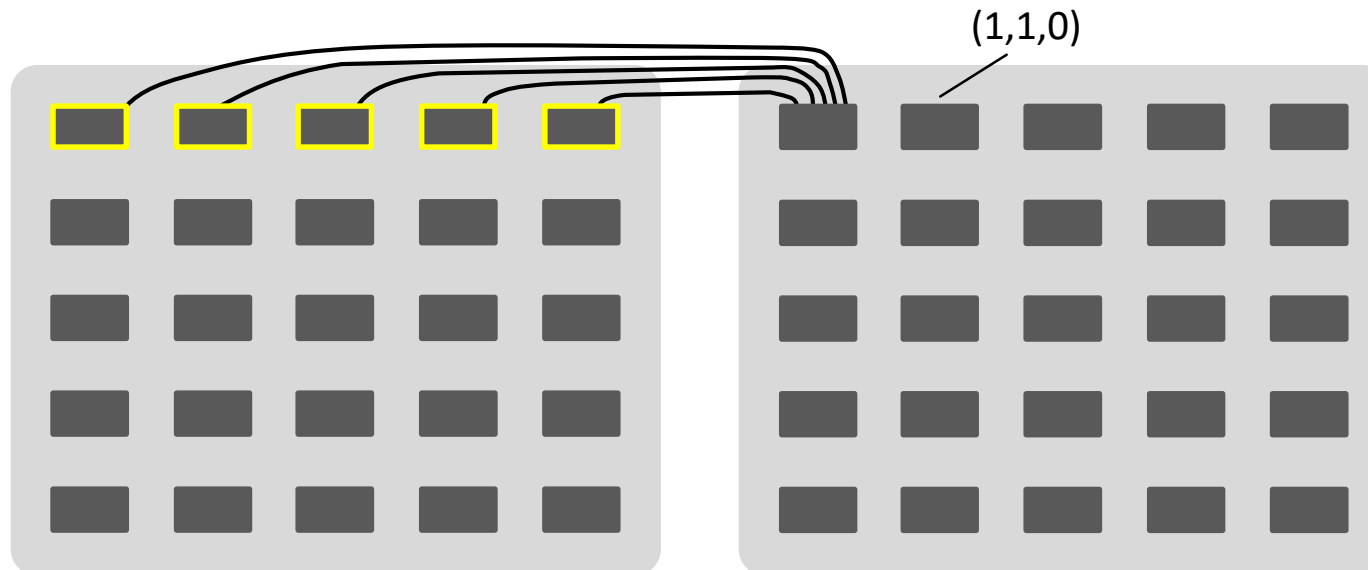
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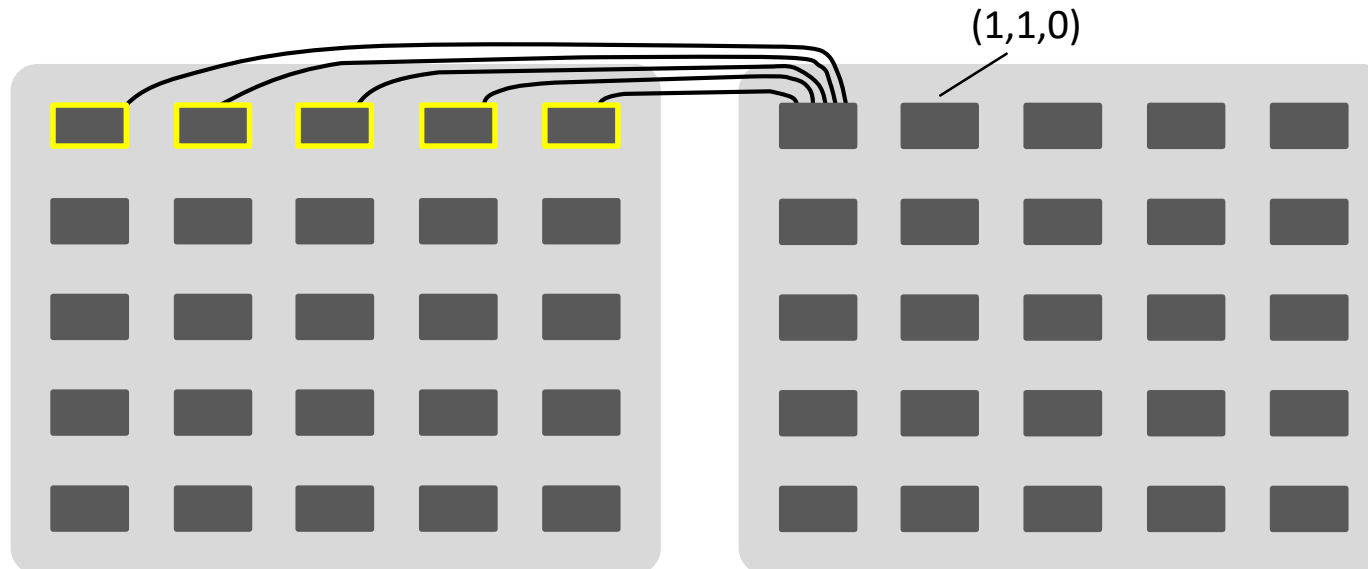
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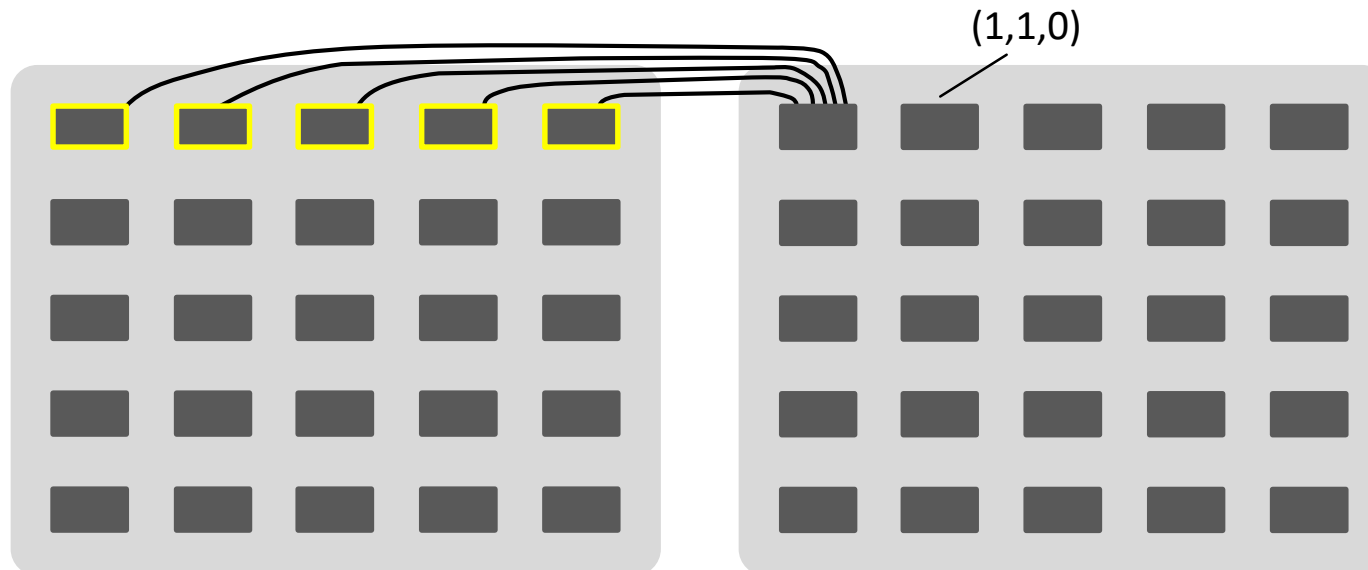
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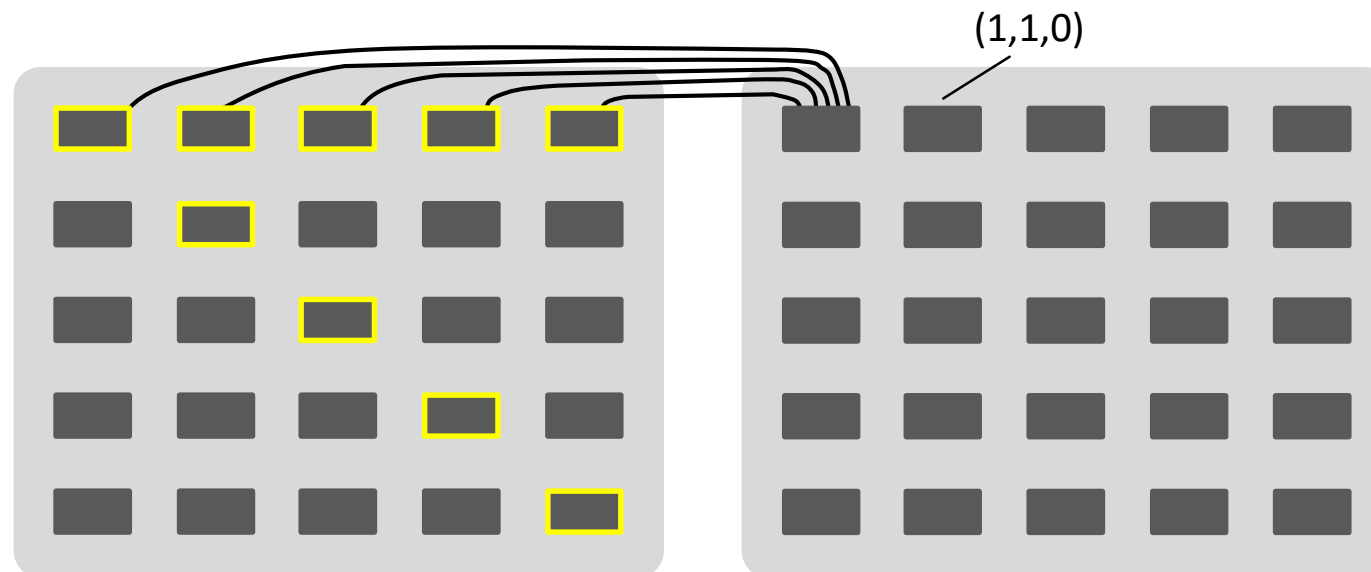
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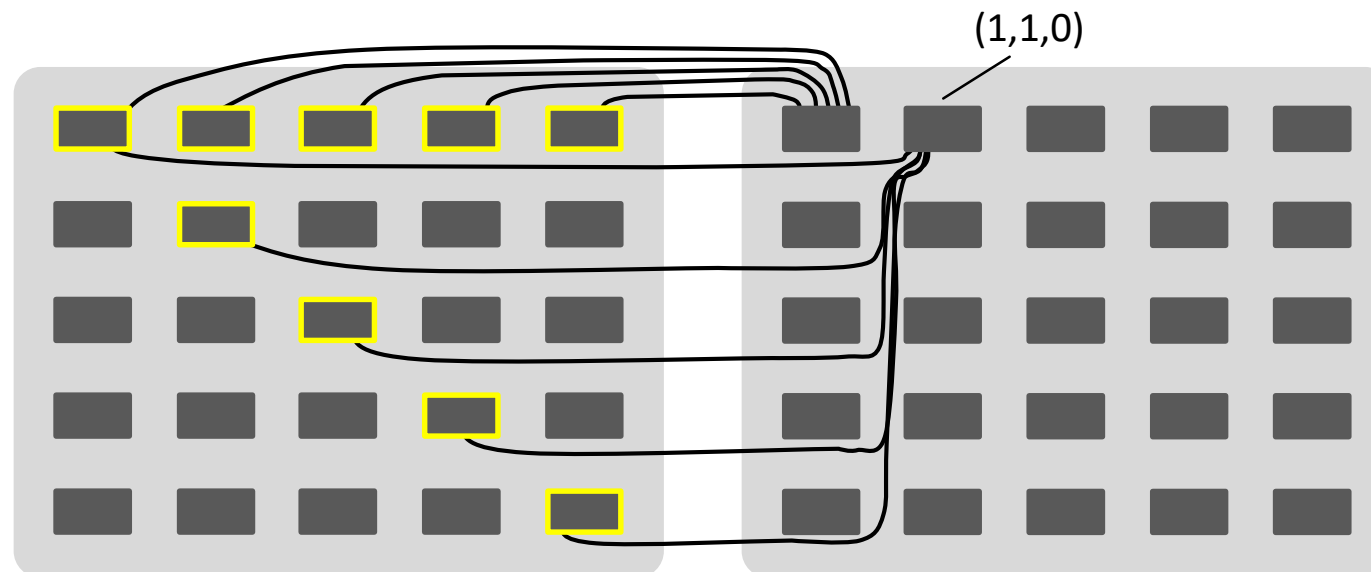
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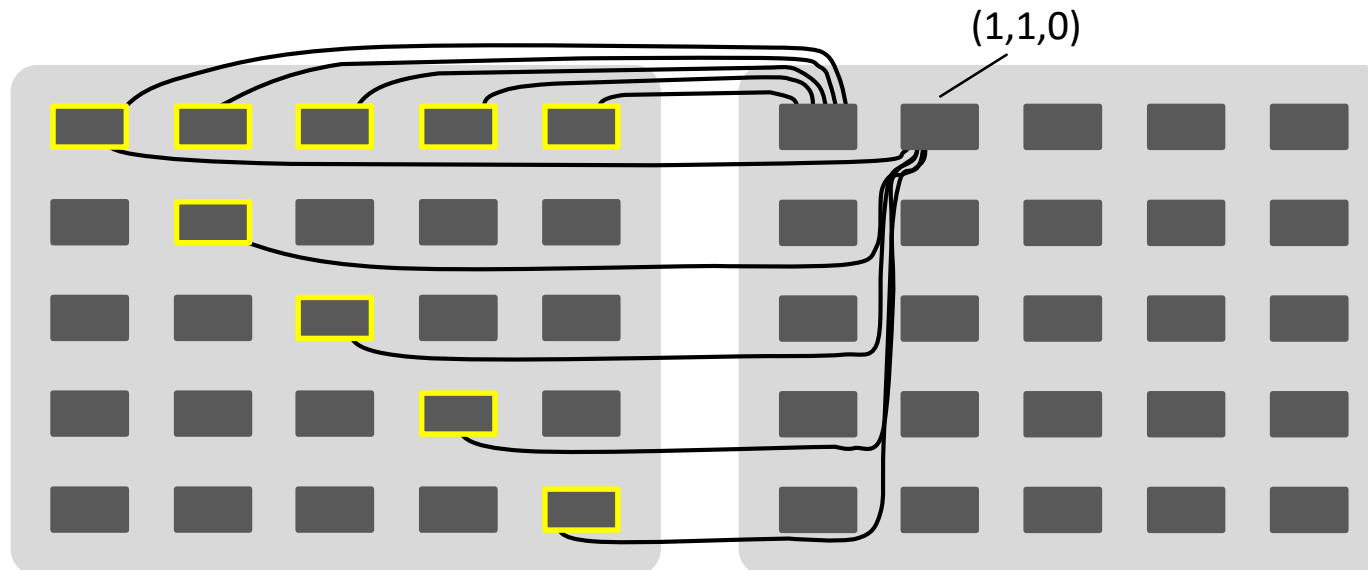
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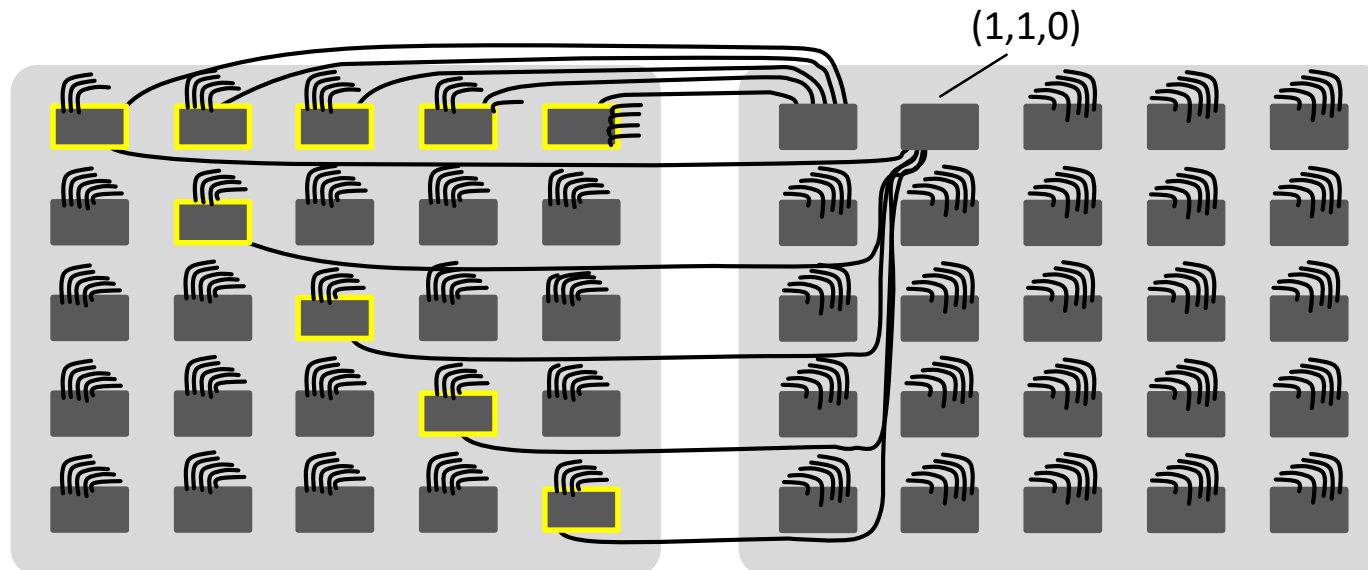
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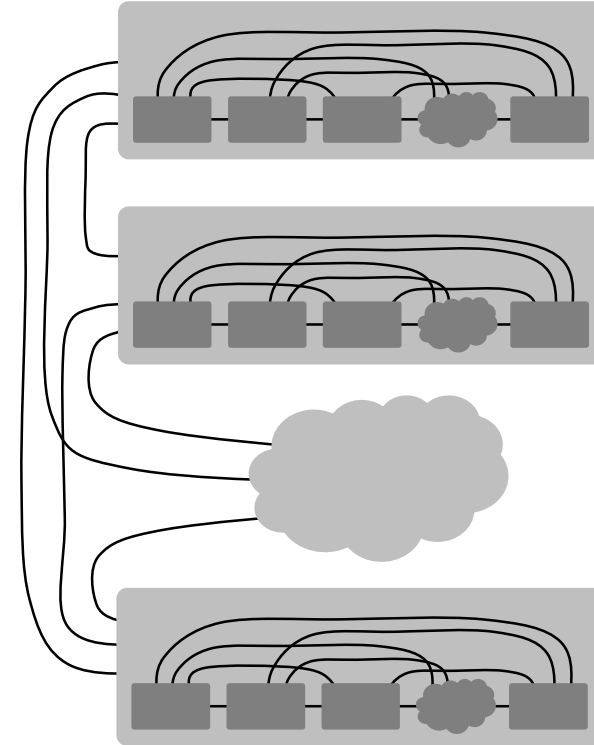
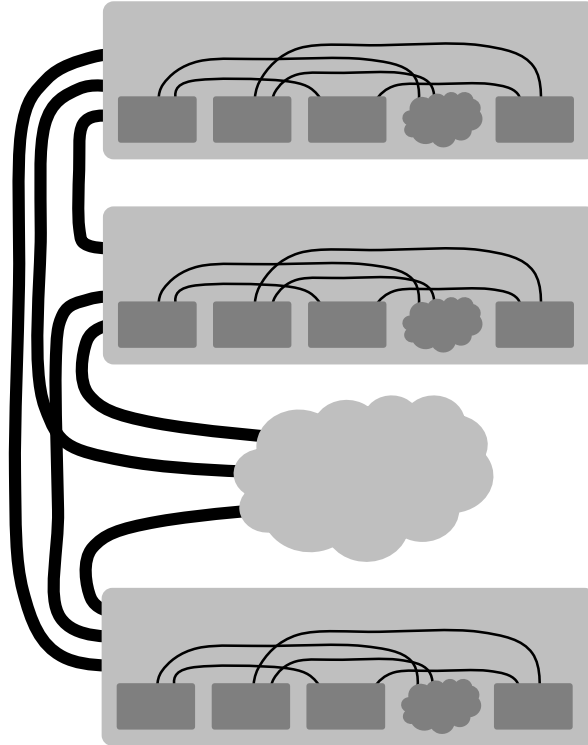
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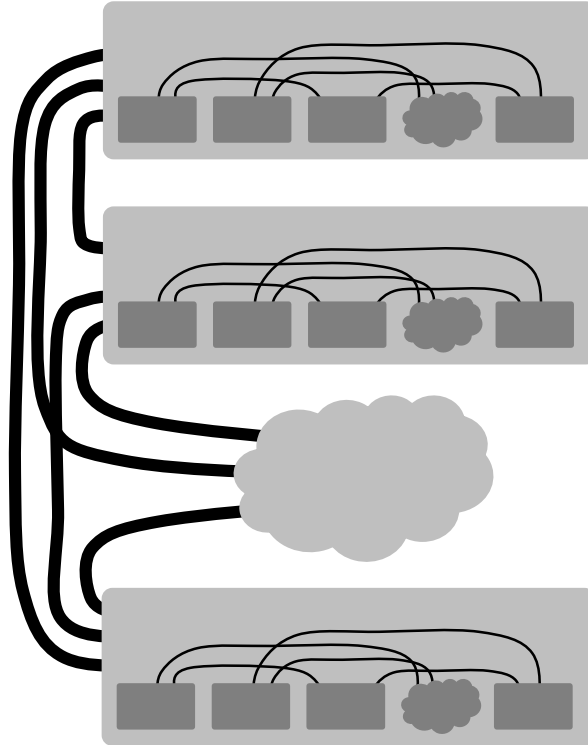


# DESIGN INTUITION

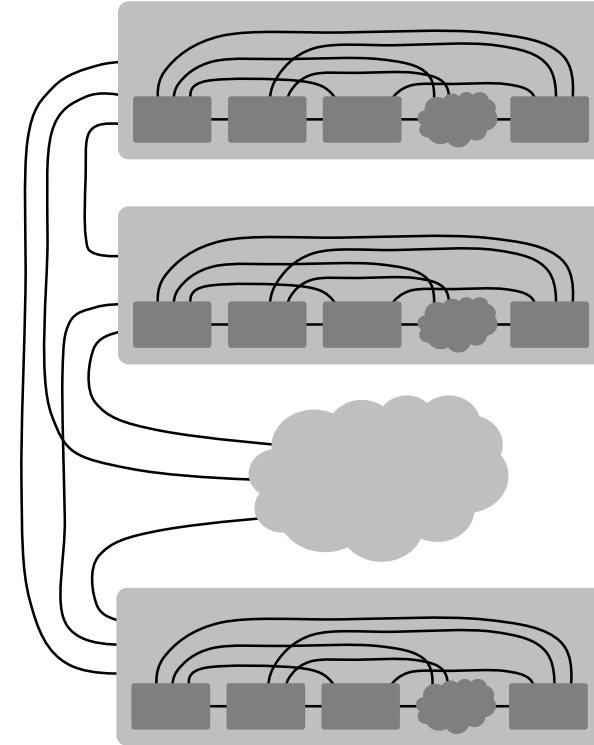


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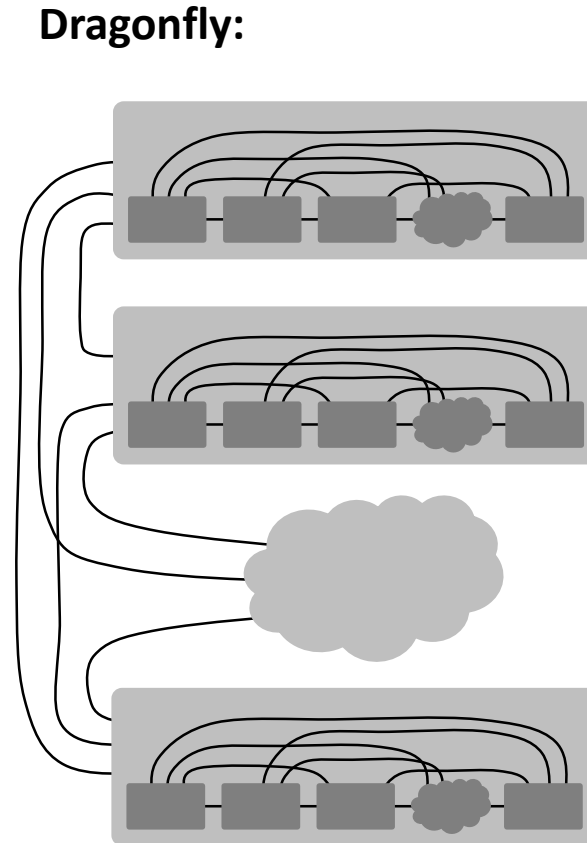
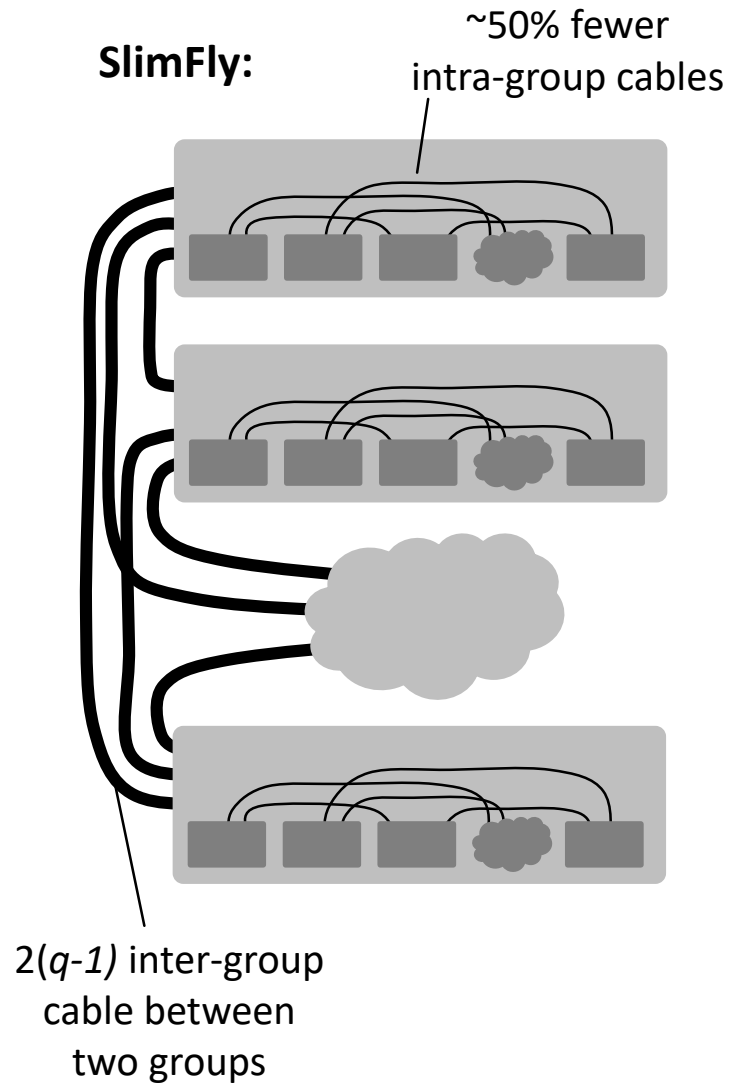
SlimFly:



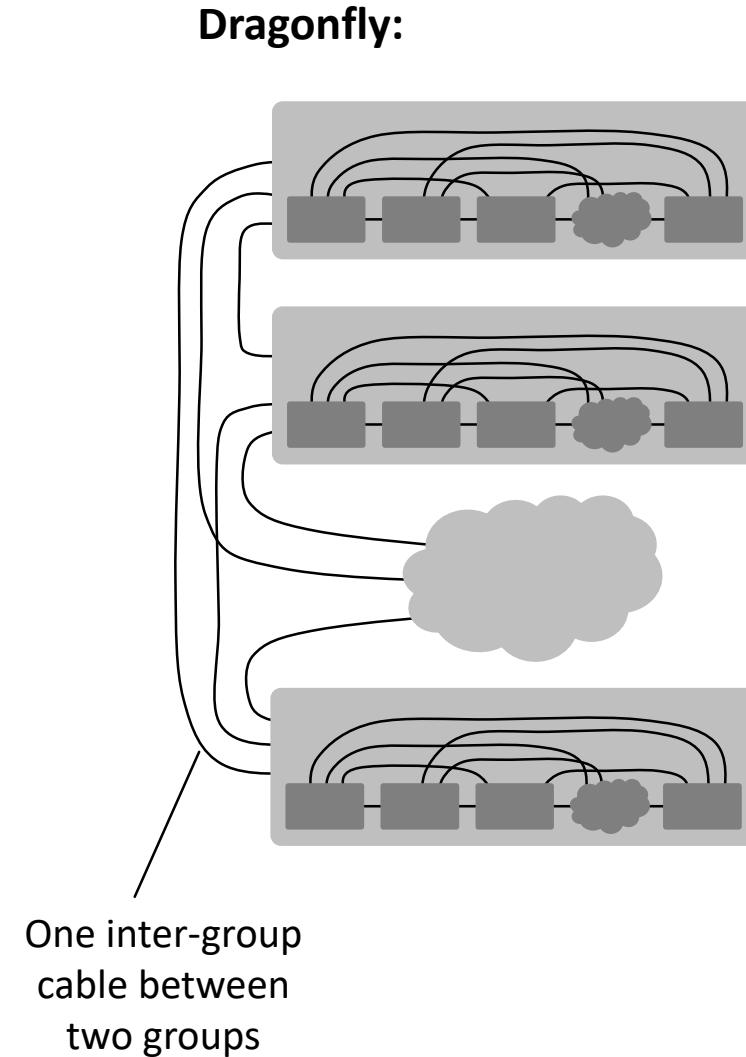
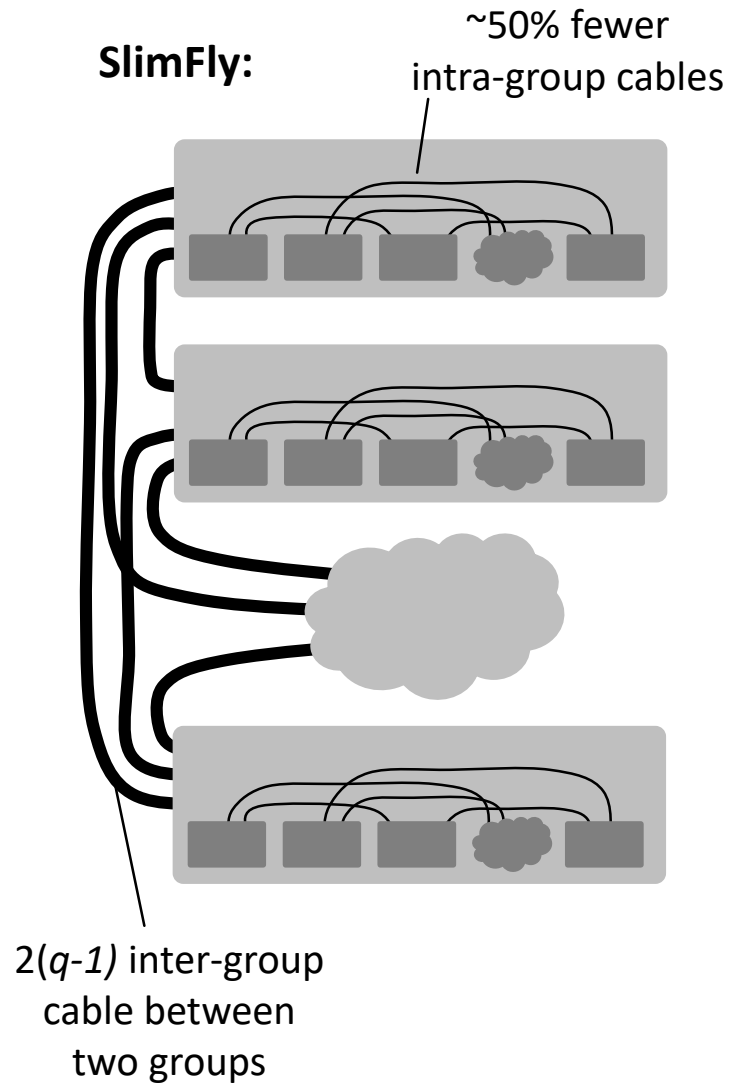
Dragonfly:



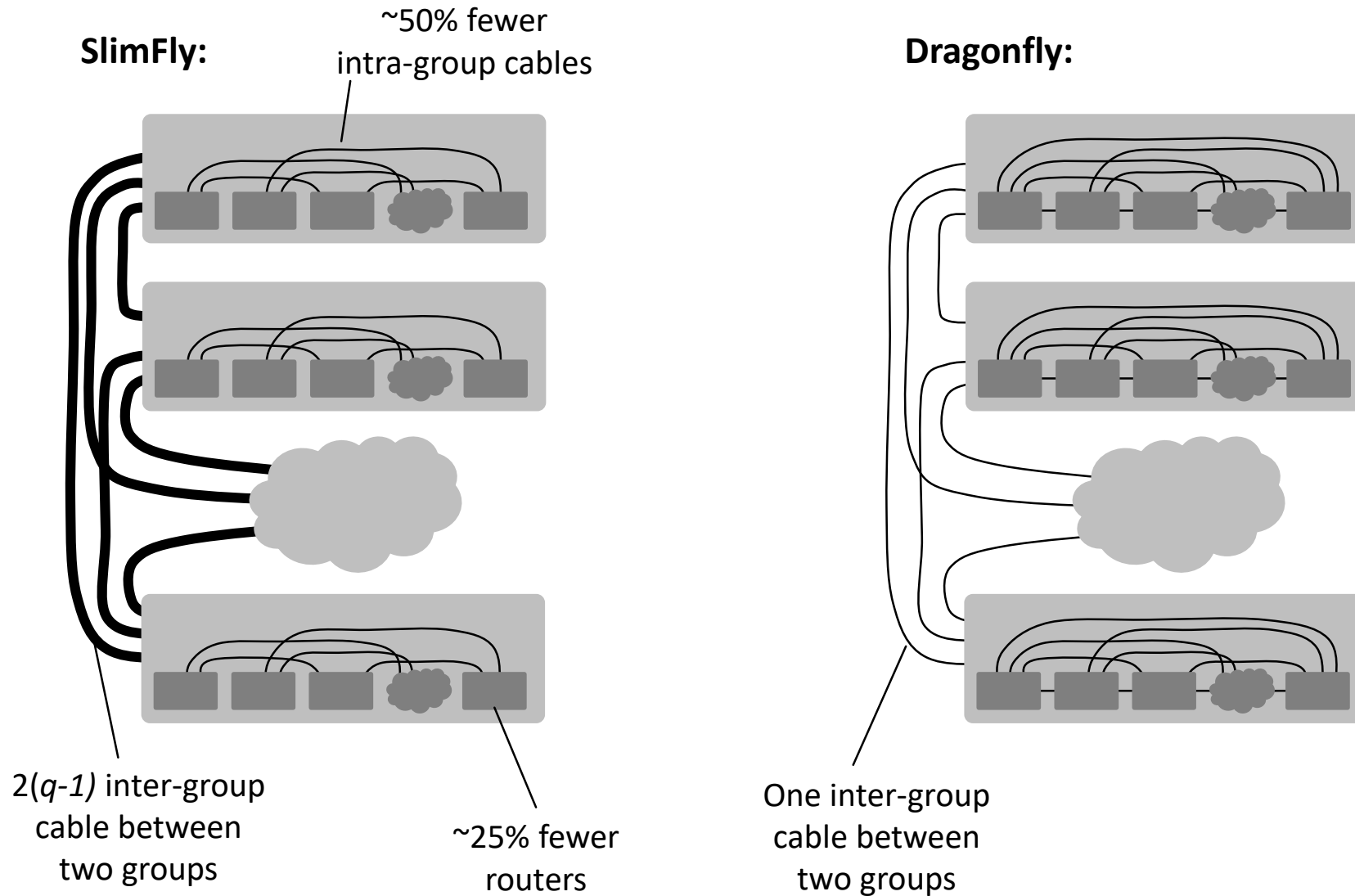
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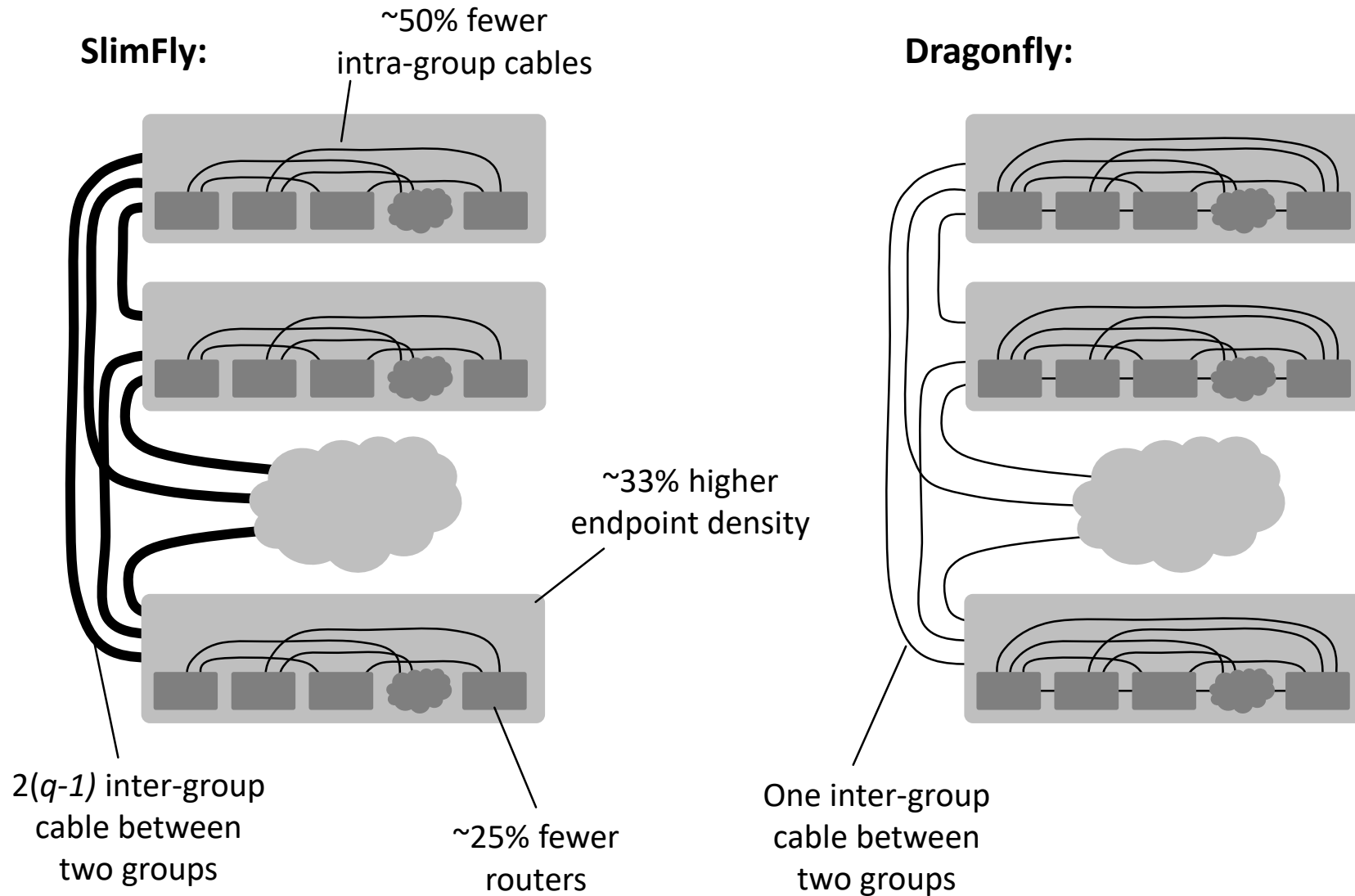


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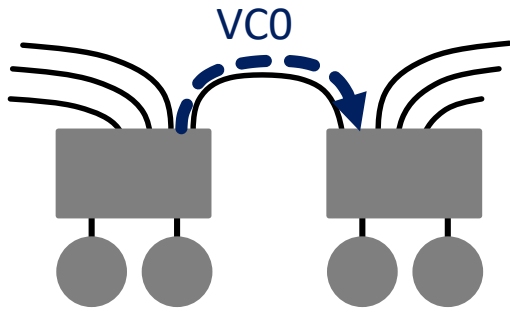
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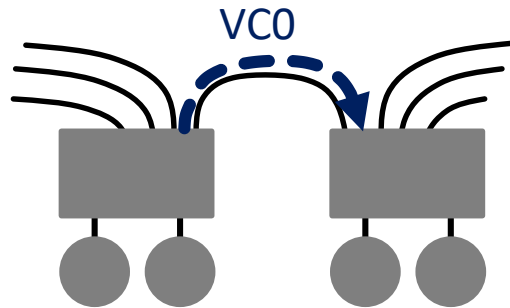
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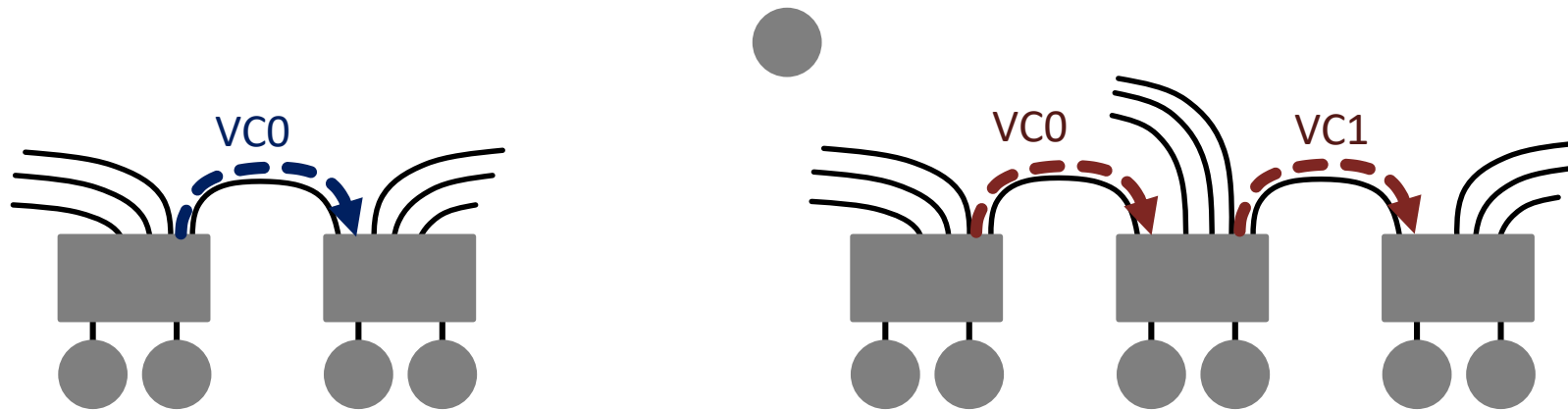
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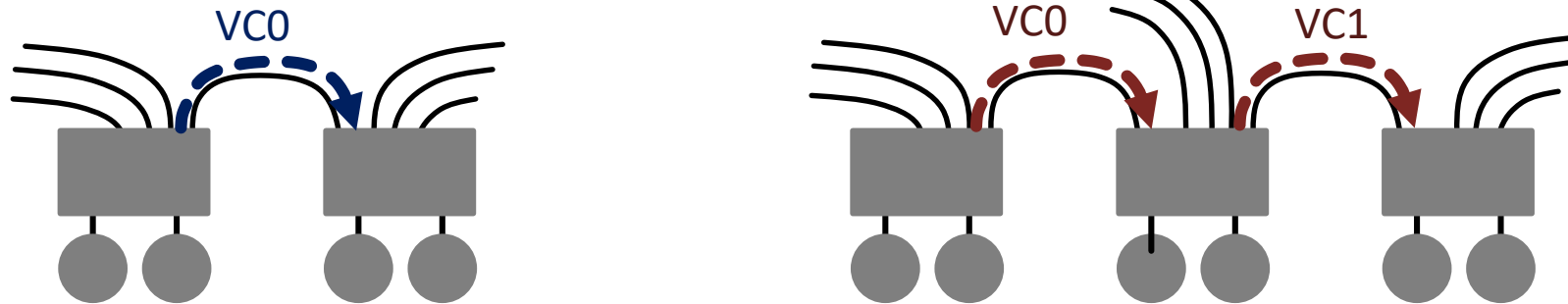
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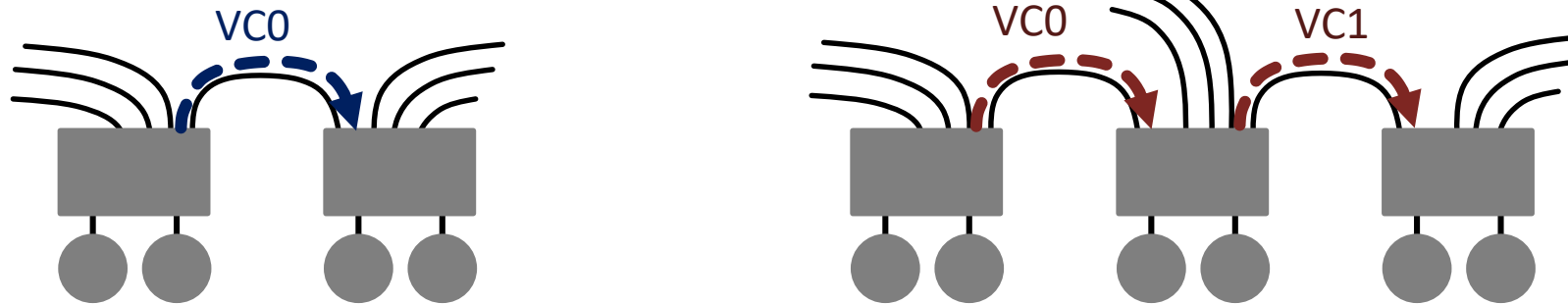




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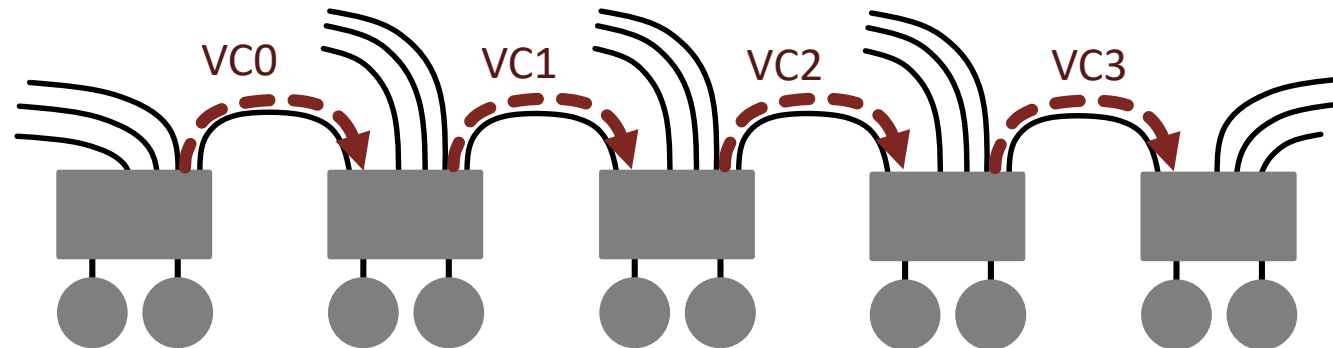
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# PERFORMANCE

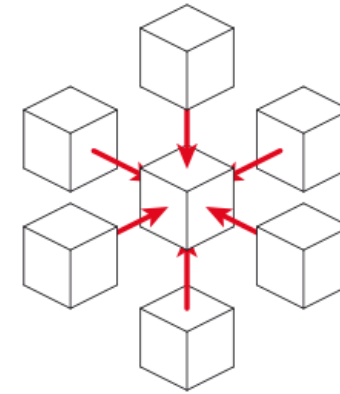
# PERFORMANCE

- Bit permutation traffic



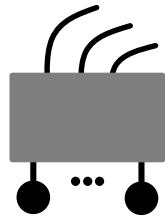
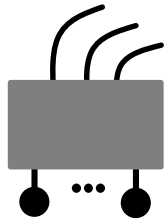
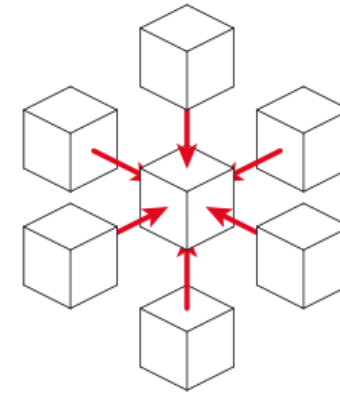
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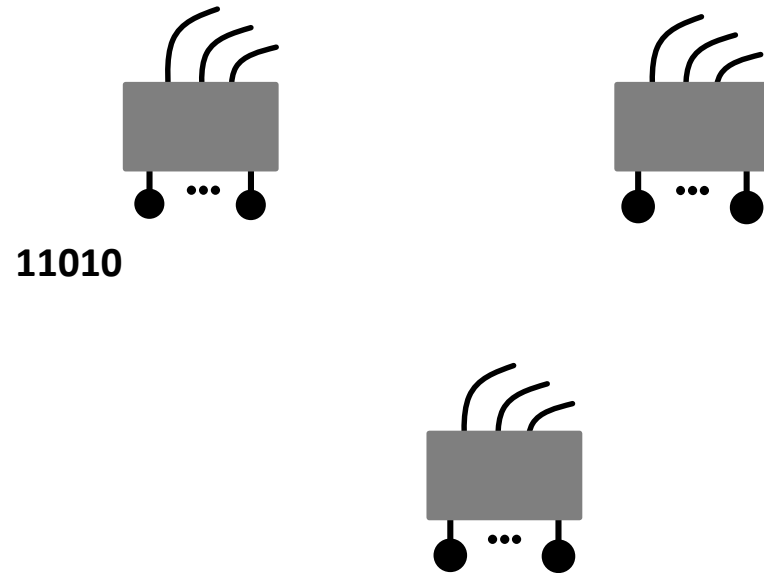
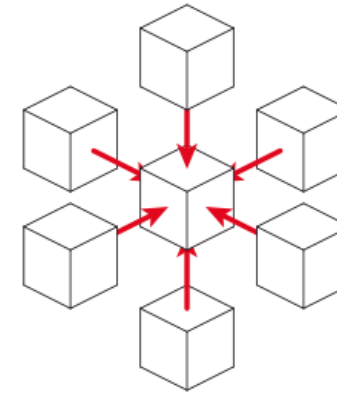
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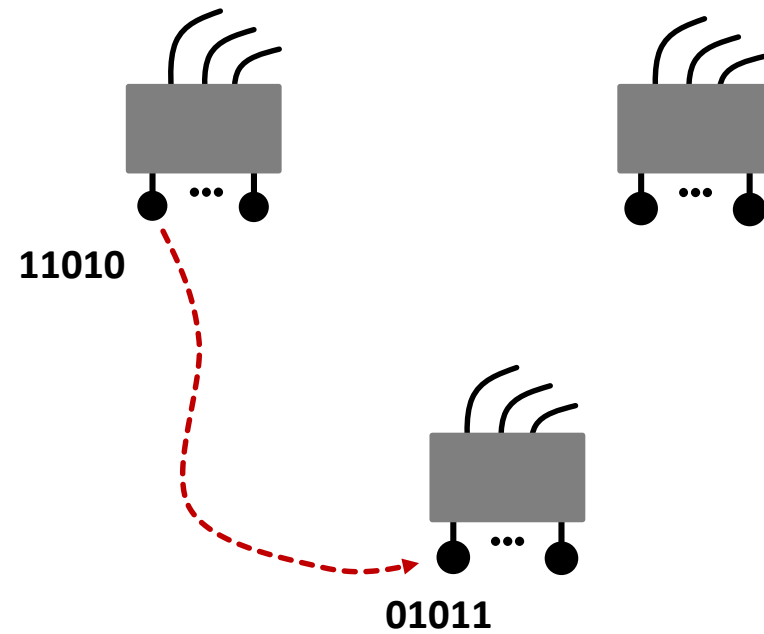
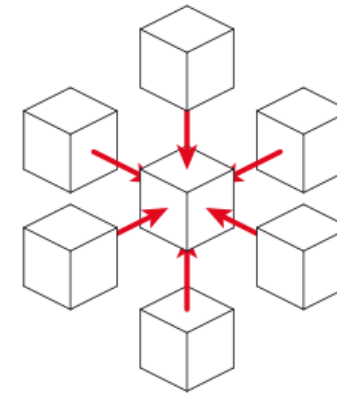
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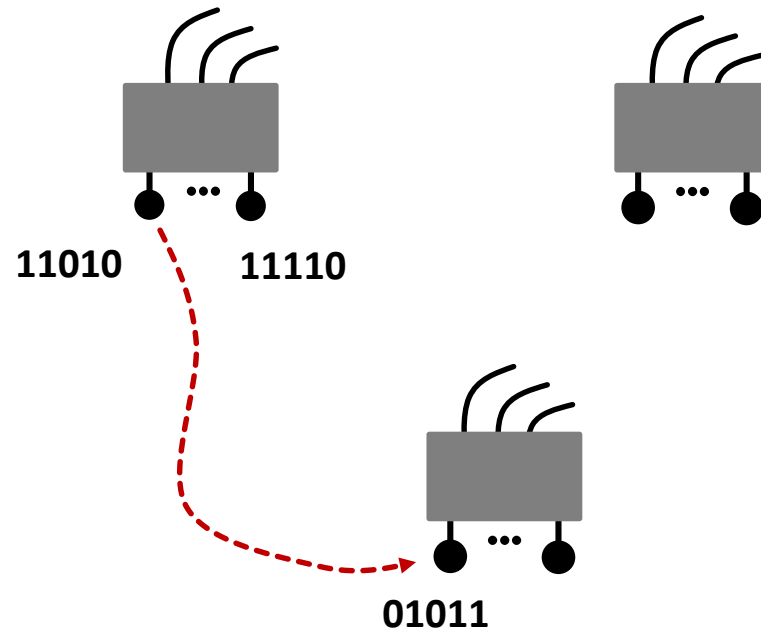
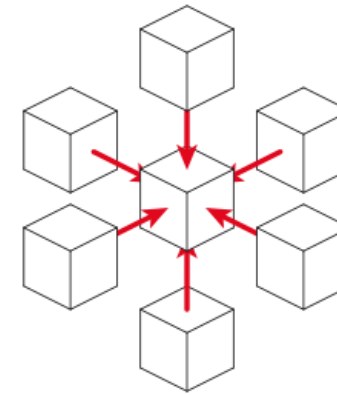
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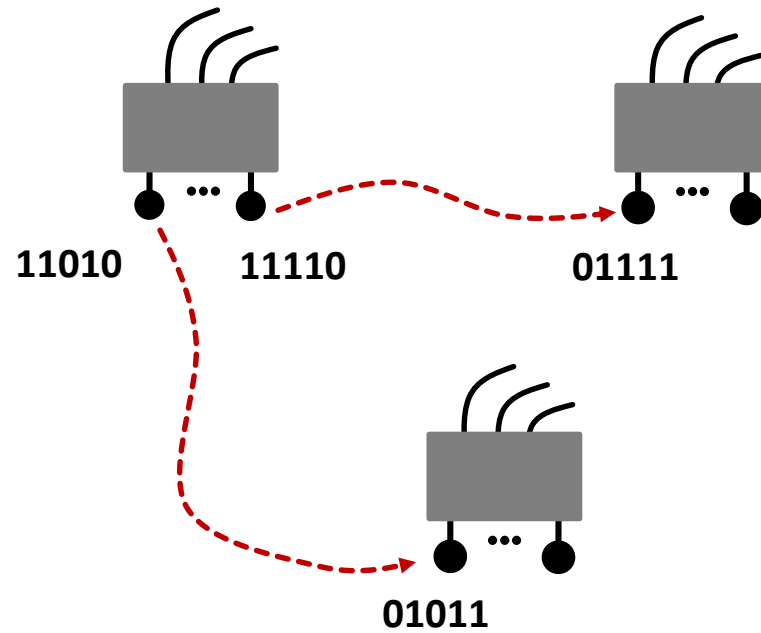
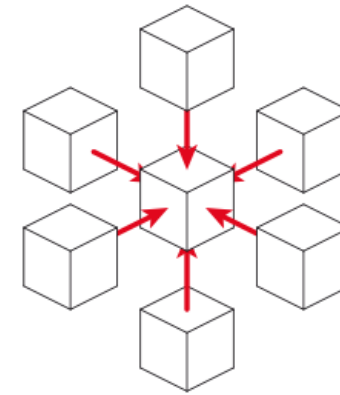
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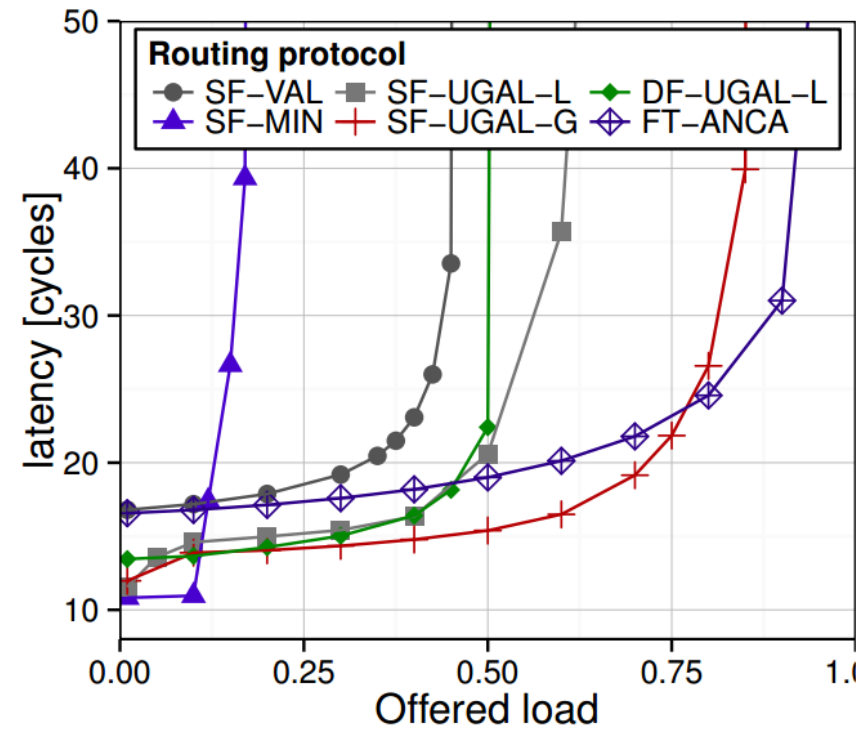
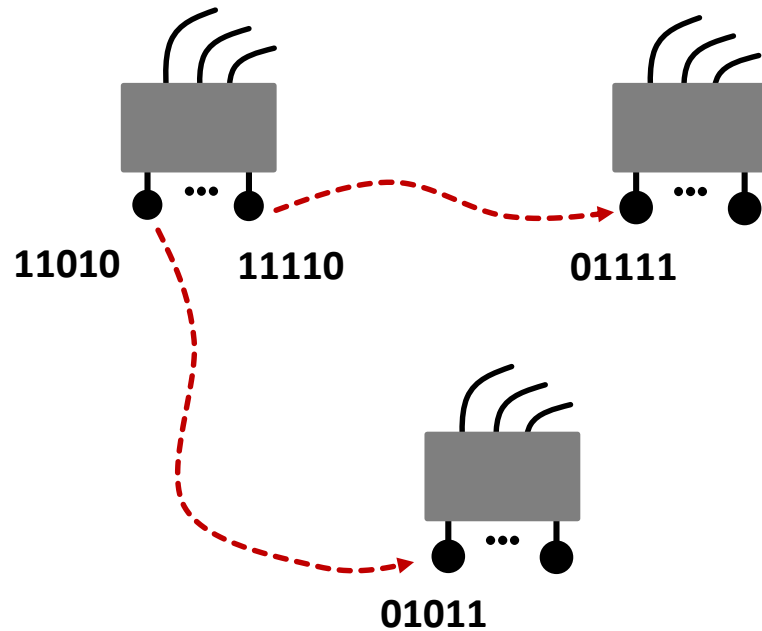
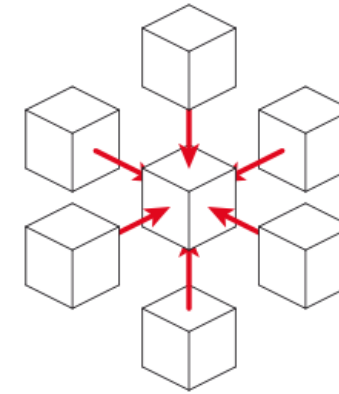
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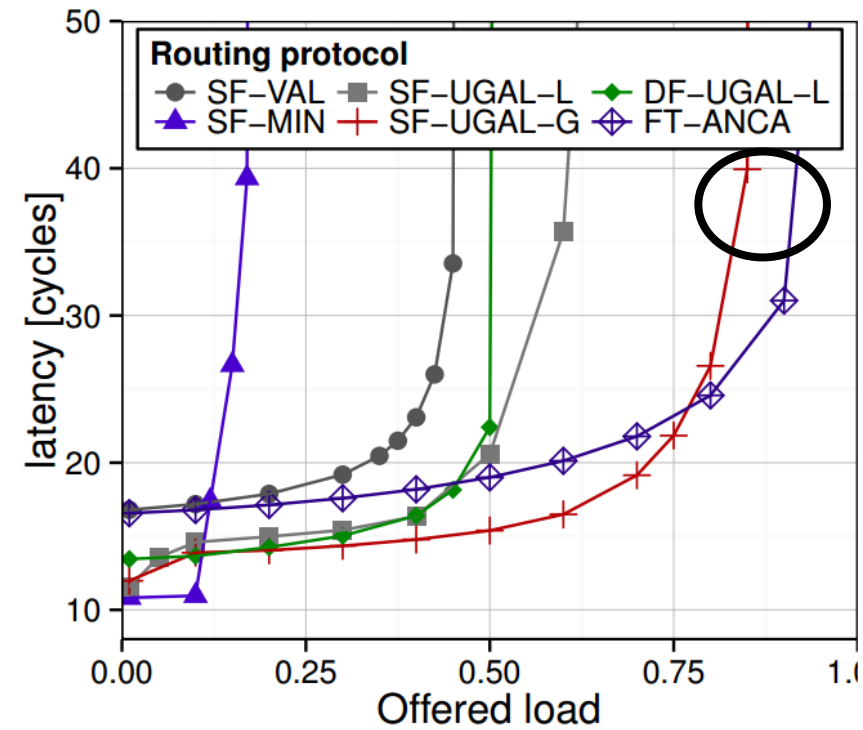
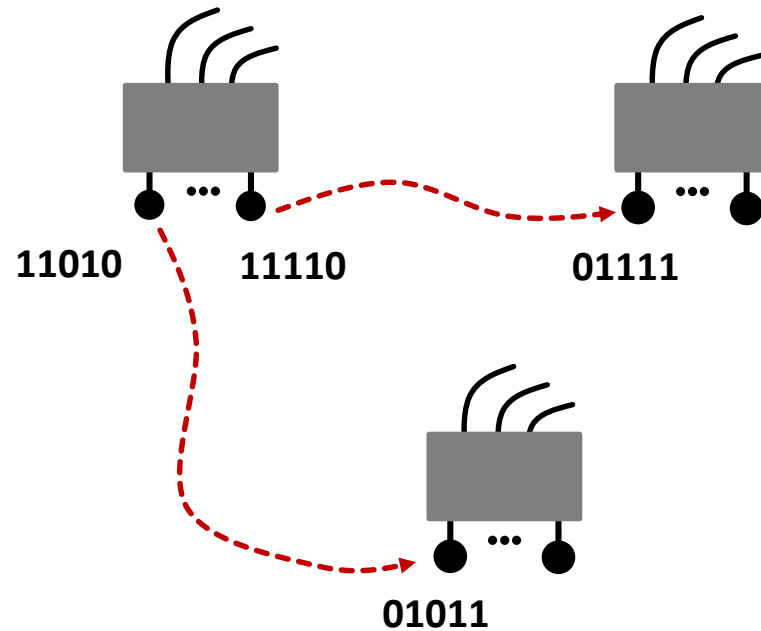
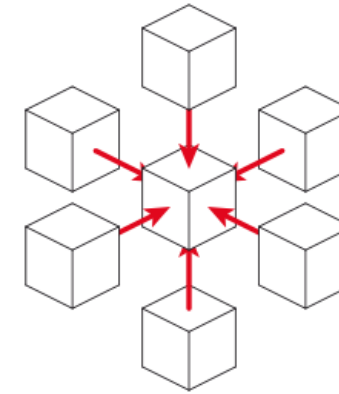
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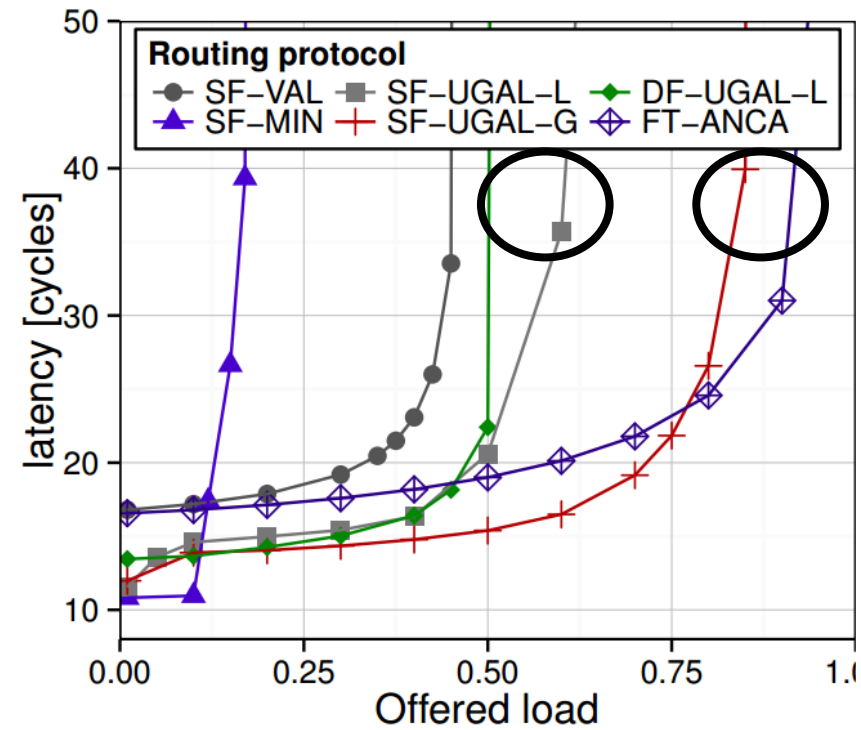
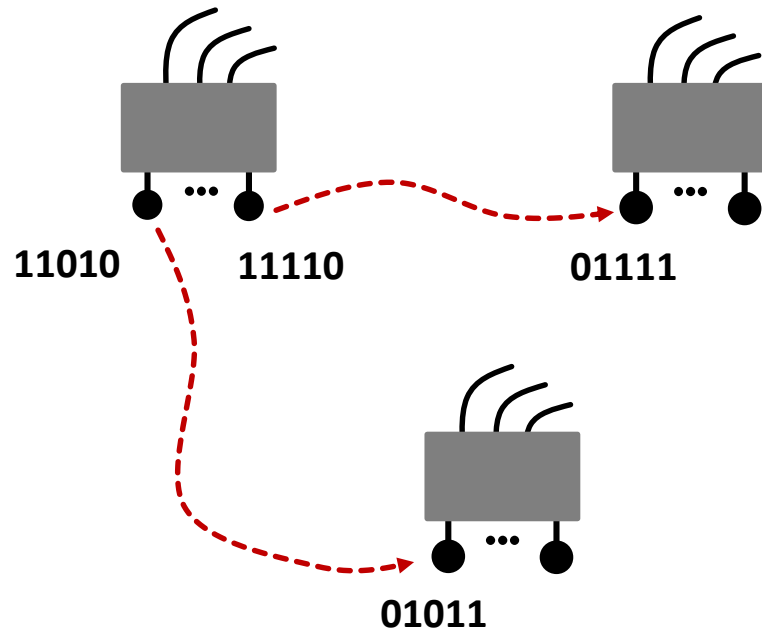
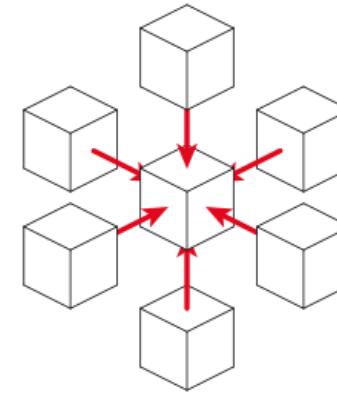
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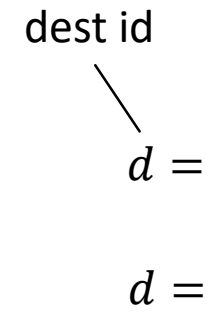
# PERFORMANCE

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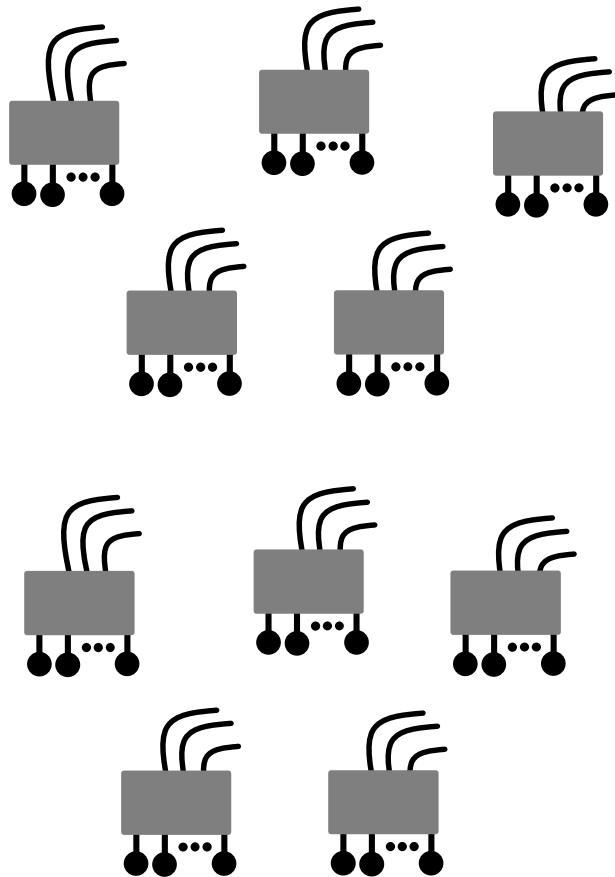
## PERFORMANCE

- Shift traffic

$$\begin{aligned} \text{dest id} & \quad \text{source id} \\ d &= \left( s \bmod \frac{N}{2} \right) + \frac{N}{2} \\ d &= s \bmod \frac{N}{2} \end{aligned}$$

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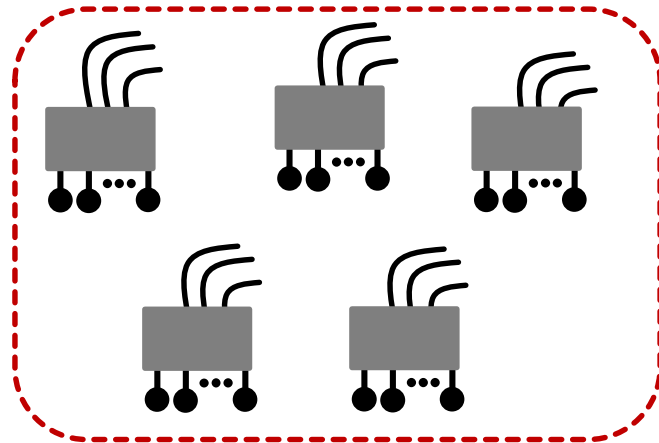
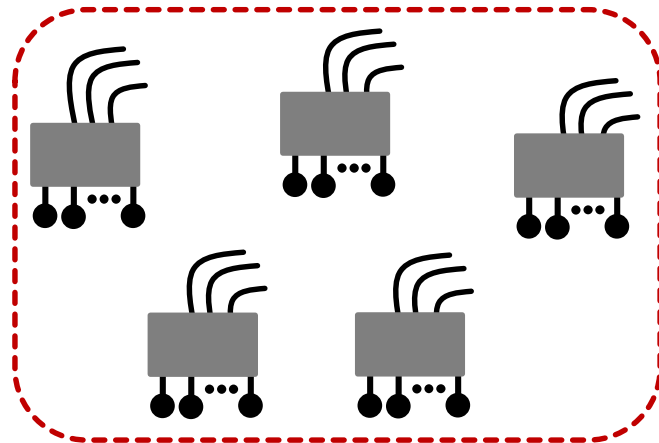
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Labels: dest id (pointing to d), source id (pointing to s)

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- Shift traffic



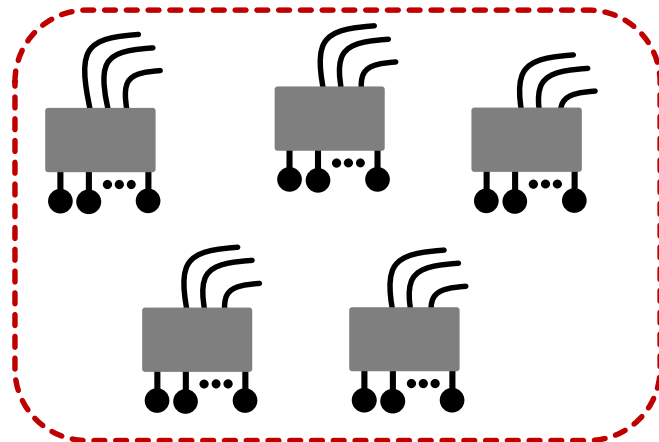
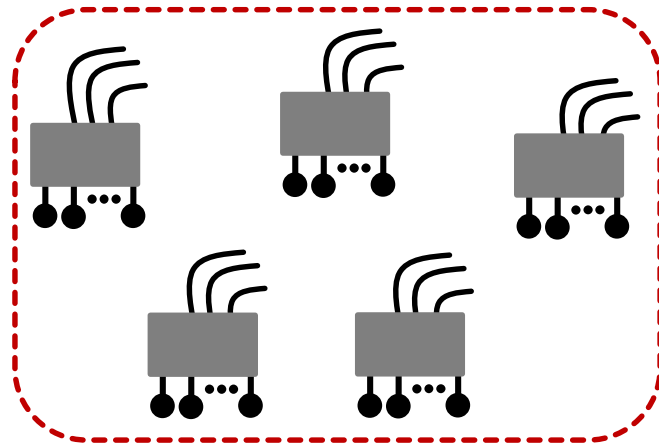
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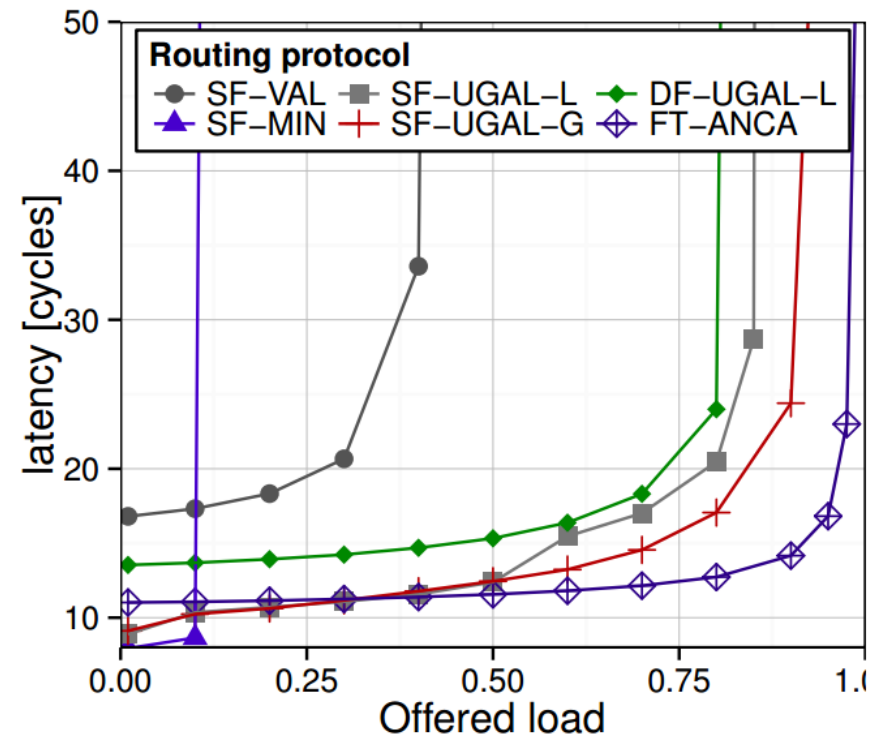
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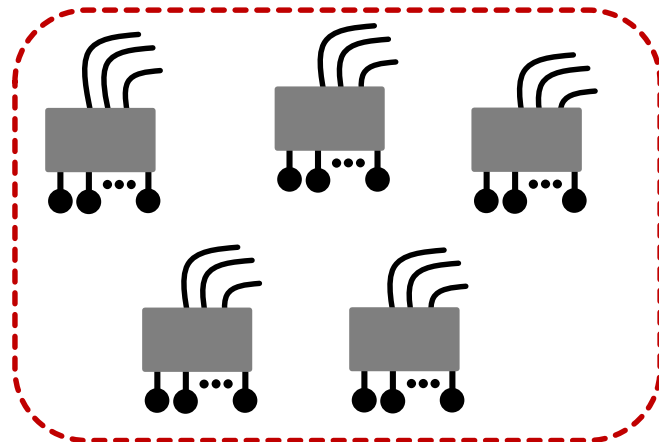
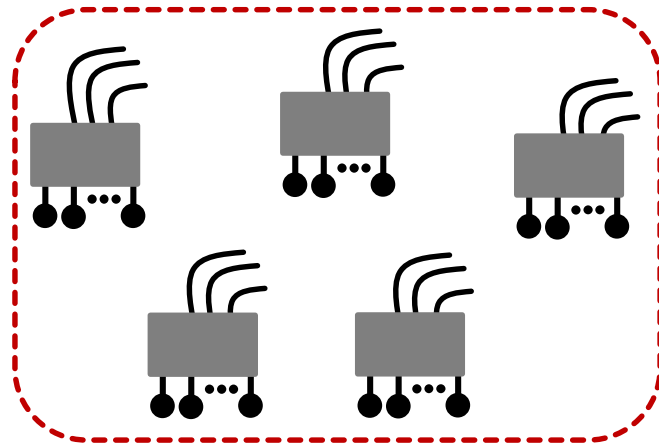
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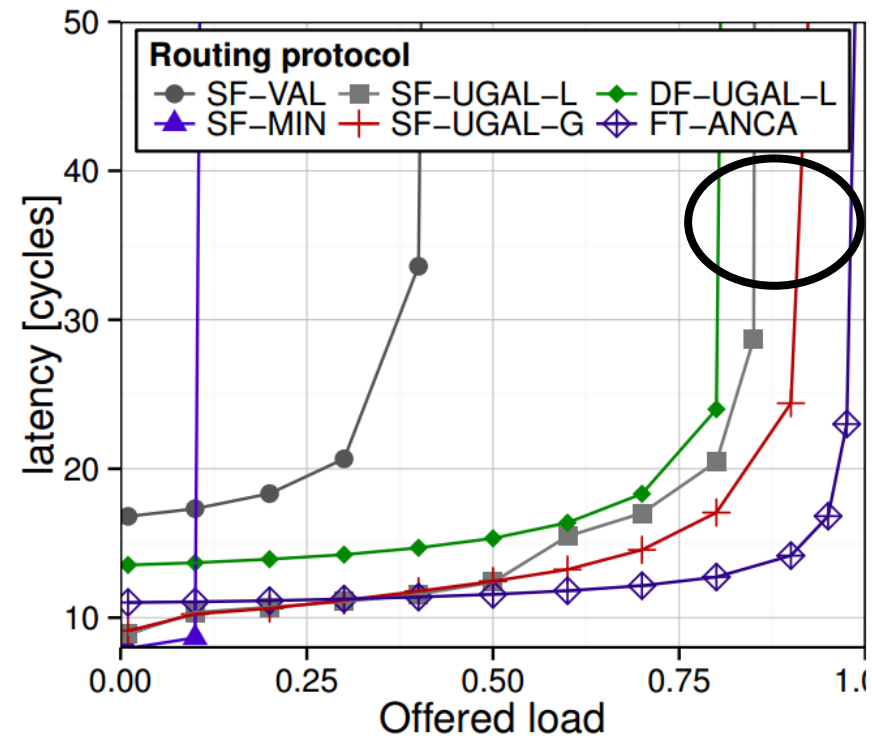
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$$d = \left( s \bmod \frac{N}{2} \right) + \frac{N}{2}$$

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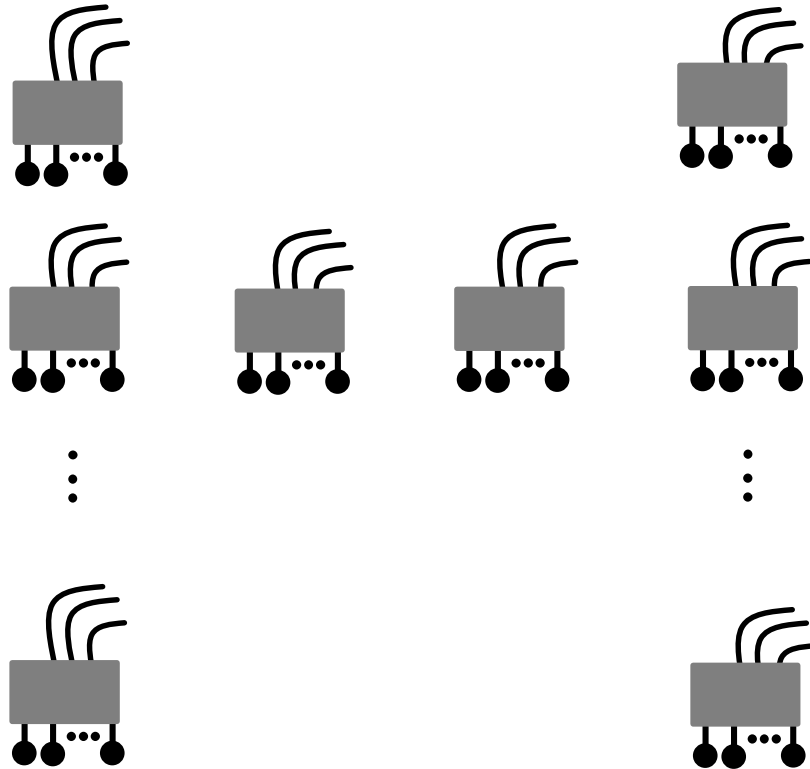
# PERFORMANCE

# PERFORMANCE

- Worst-case traffic

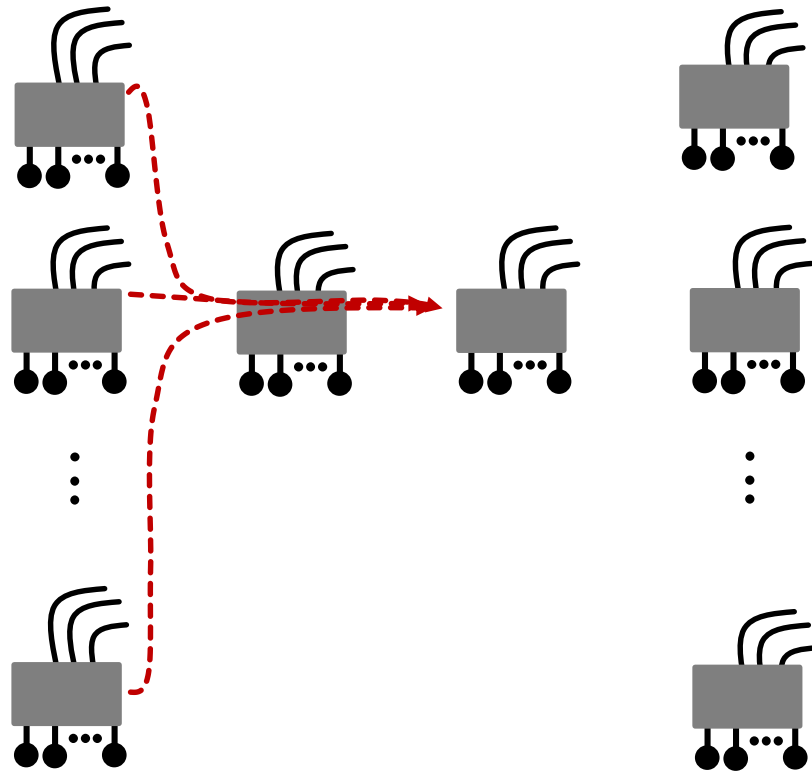
# PERFORMANCE

- Worst-case traffic



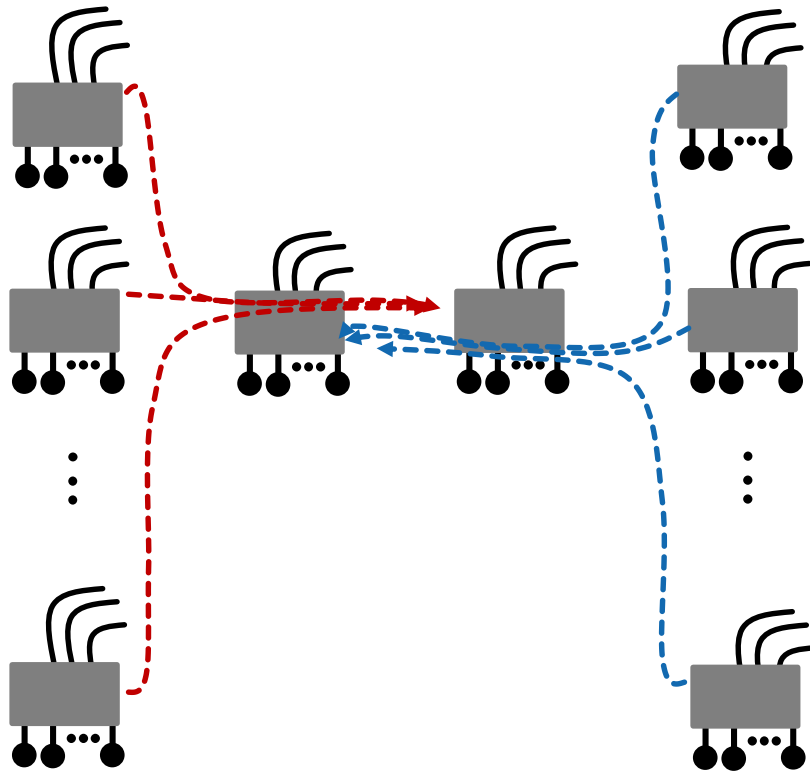
# PERFORMANCE

- Worst-case traffic



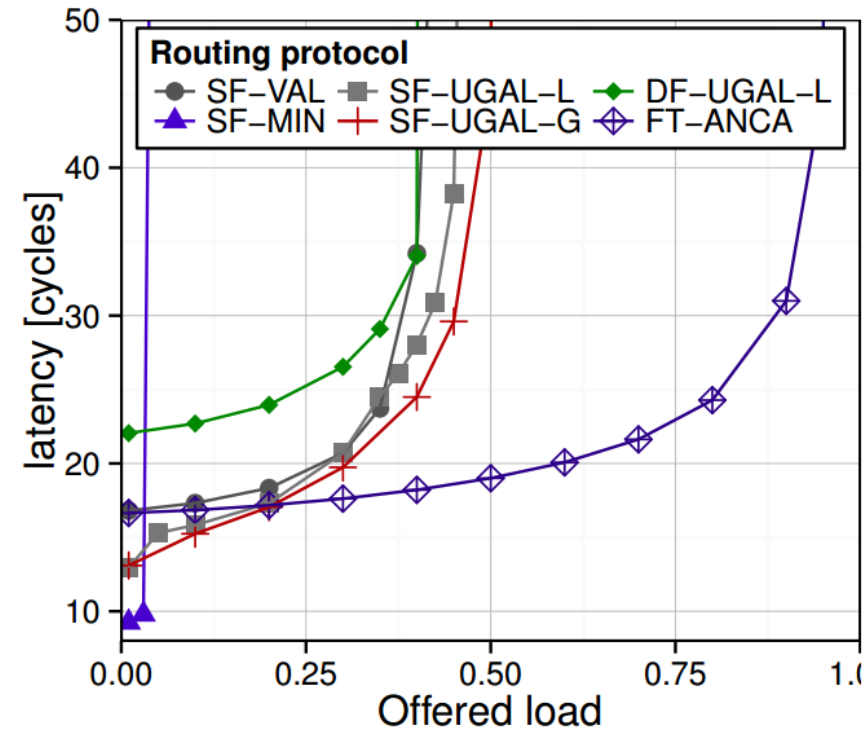
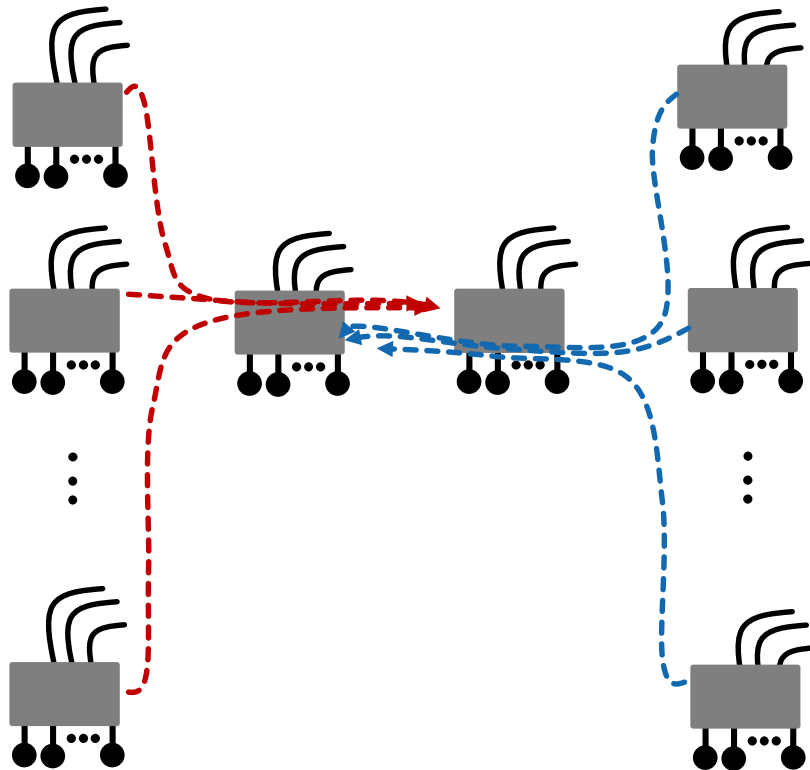
# PERFORMANCE

- Worst-case traffic



# PERFORMANCE

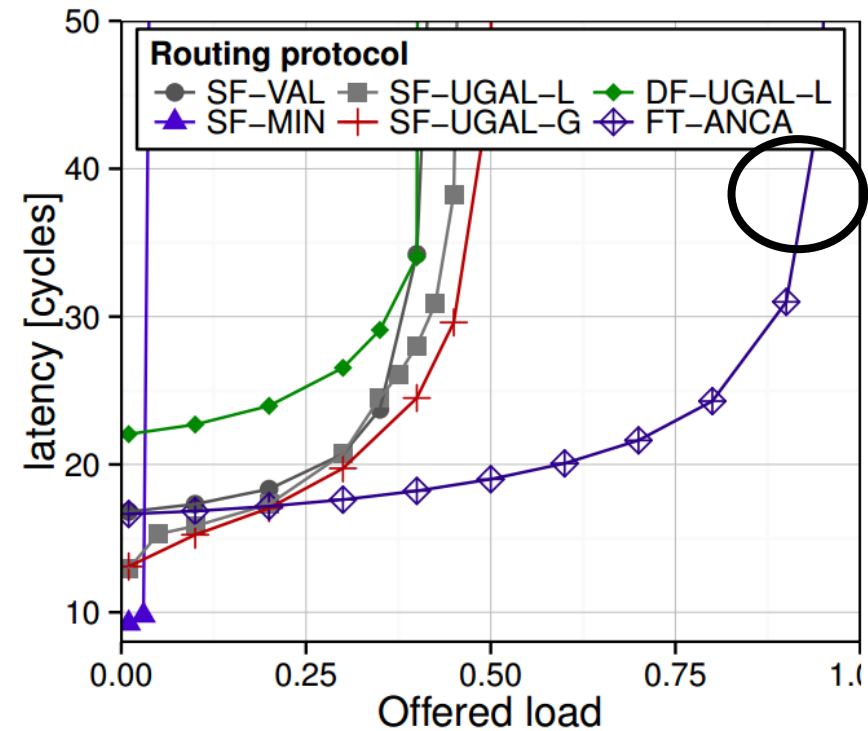
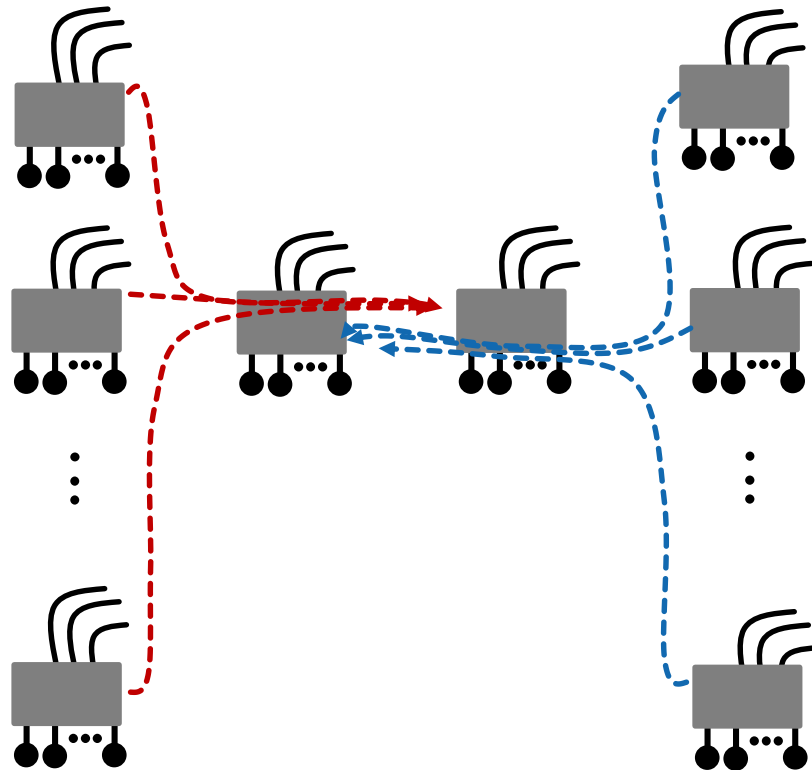
- Worst-case traffic





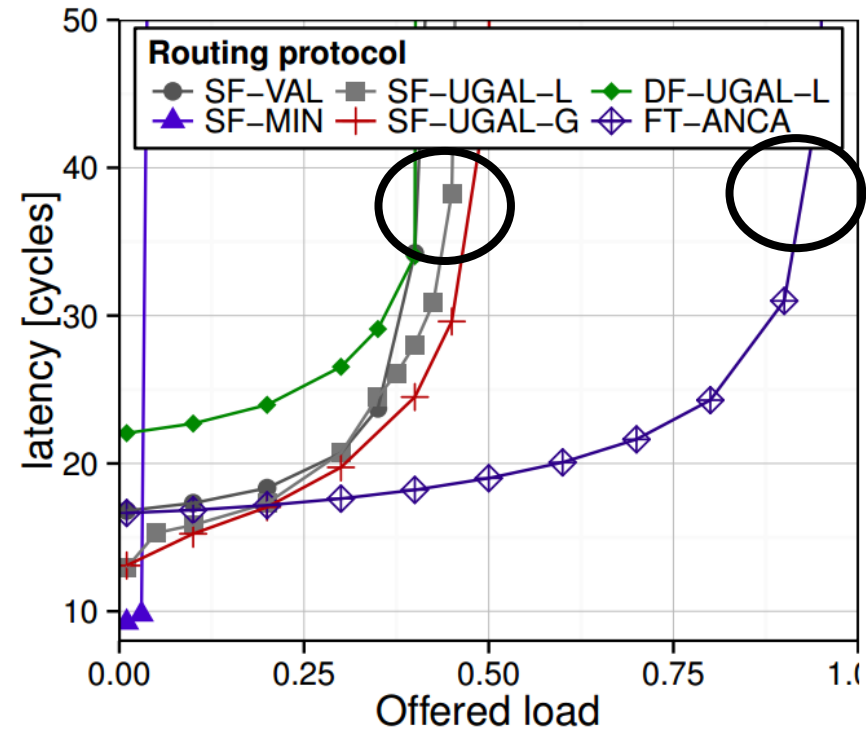
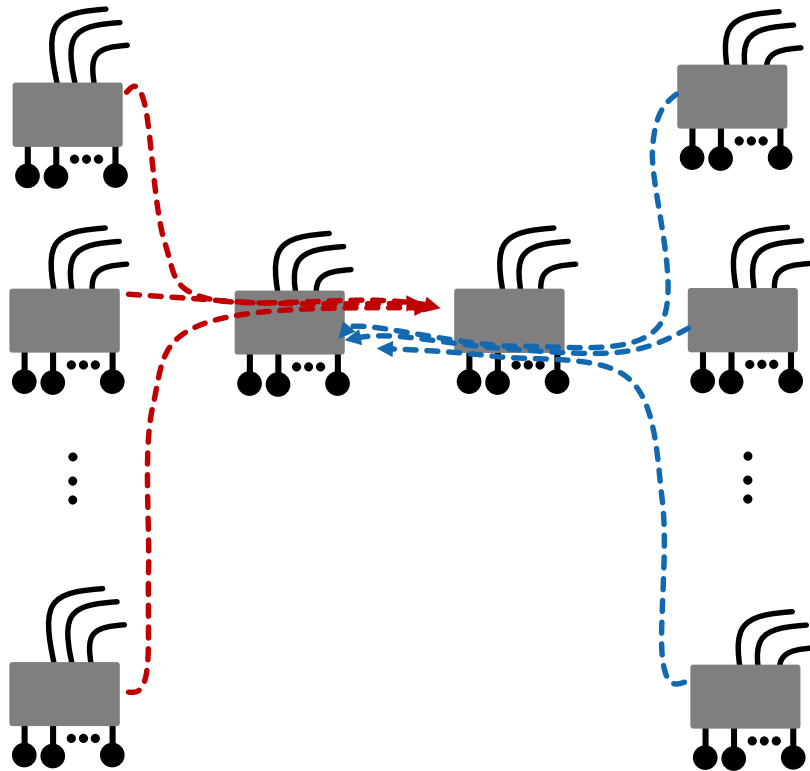
# PERFORMANCE

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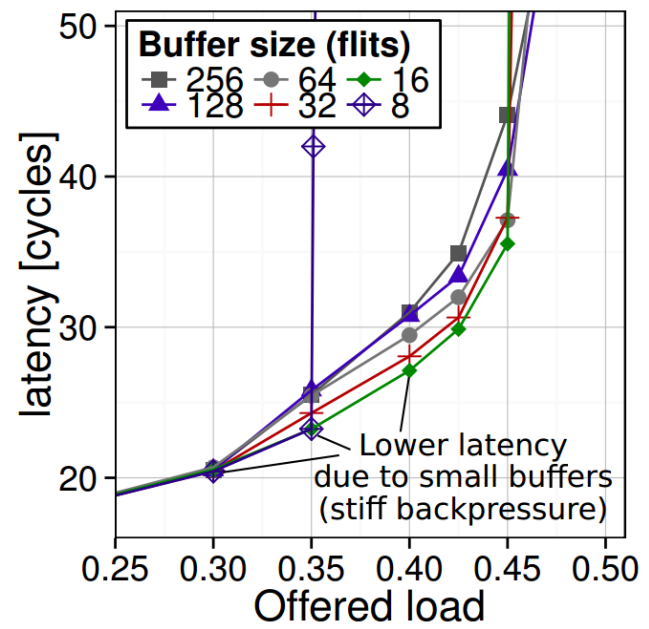
# PERFORMANCE

# PERFORMANCE

- Buffer sizes (UGAL-L, worst-case traffic)

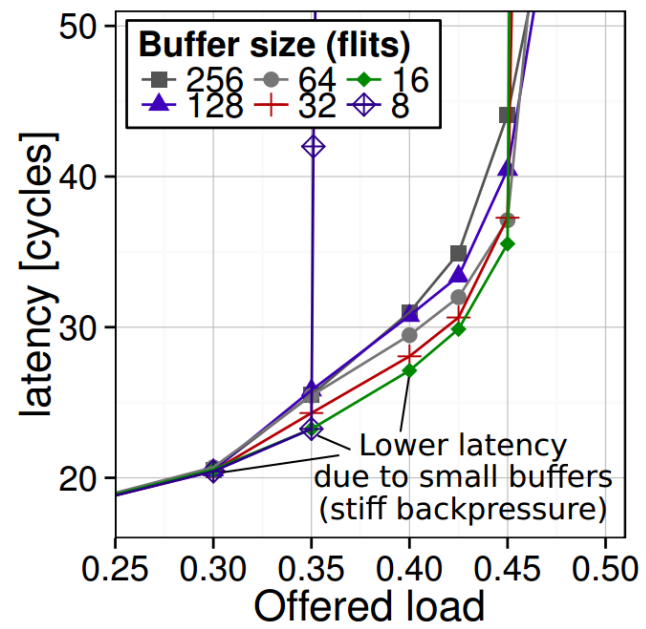
# PERFORMANCE

- Buffer sizes (UGAL-L, worst-case traffic)



# PERFORMANCE

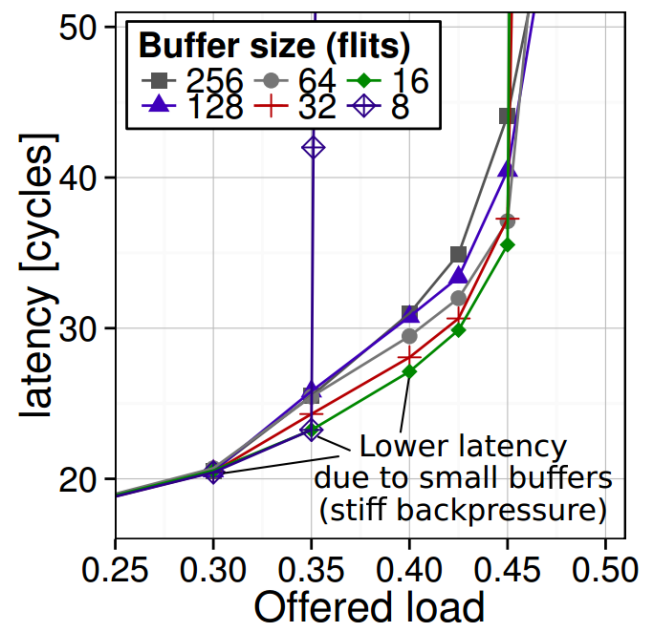
- Buffer sizes (UGAL-L, worst-case traffic)
- Oversubscription (64 flits)



# PERFORMANCE

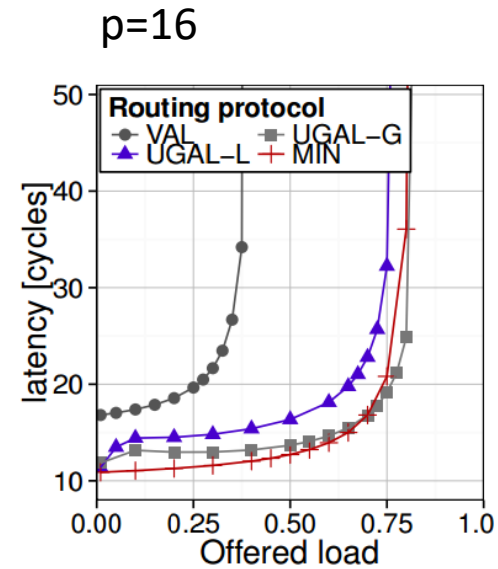
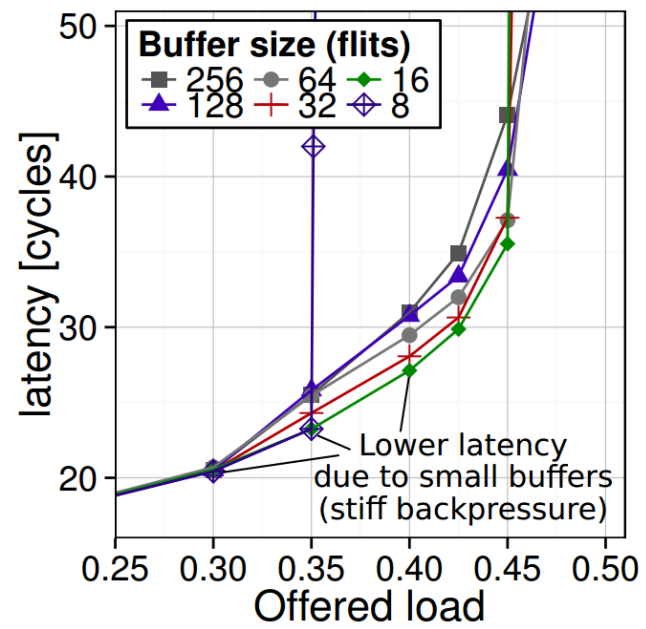
- Buffer sizes (UGAL-L, worst-case traffic)
- Oversubscription (64 flits)

$p=16$



# PERFORMANCE

- Buffer sizes (UGAL-L, worst-case traffic)
- Oversubscription (64 flits)

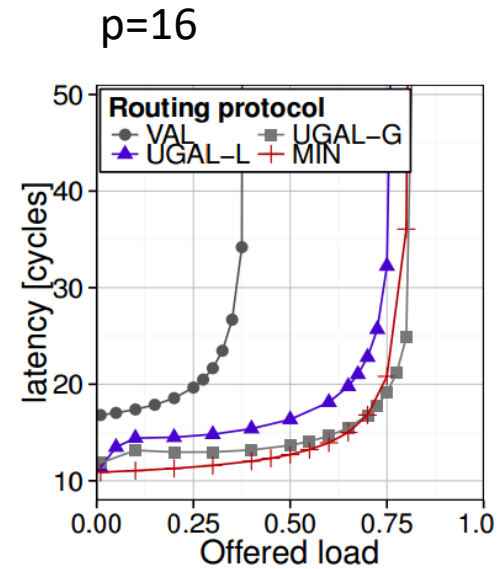
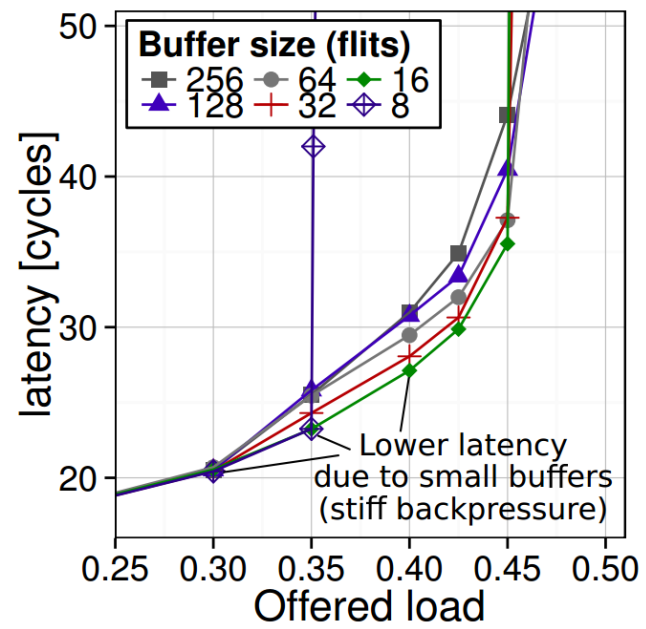


(b) Random traffic,  $p = 16$ .



# PERFORMANCE

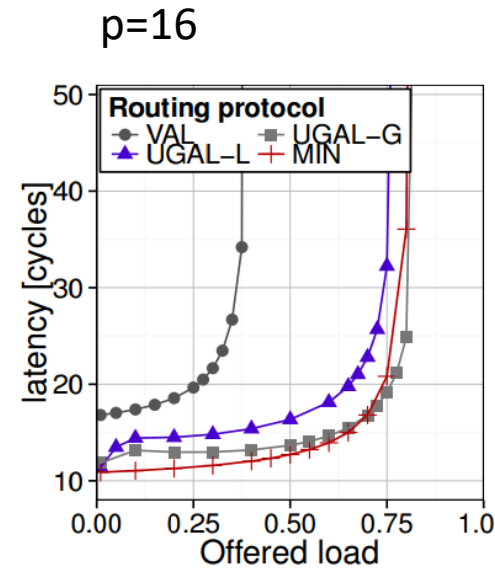
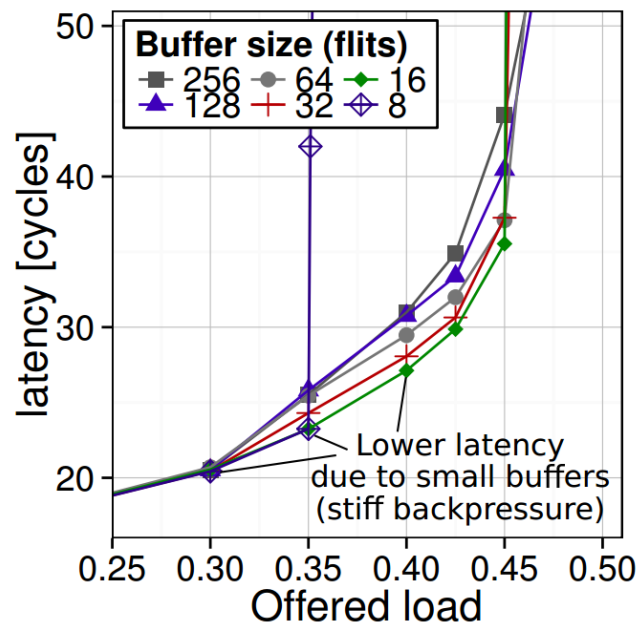
- Buffer sizes (UGAL-L, worst-case traffic)
- Oversubscription (64 flits)



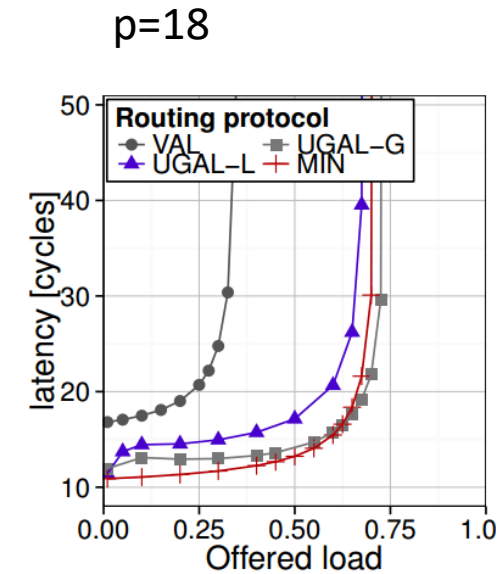
(b) Random traffic,  $p = 16$ .

# PERFORMANCE

- Buffer sizes (UGAL-L, worst-case traffic)
- Oversubscription (64 flits)



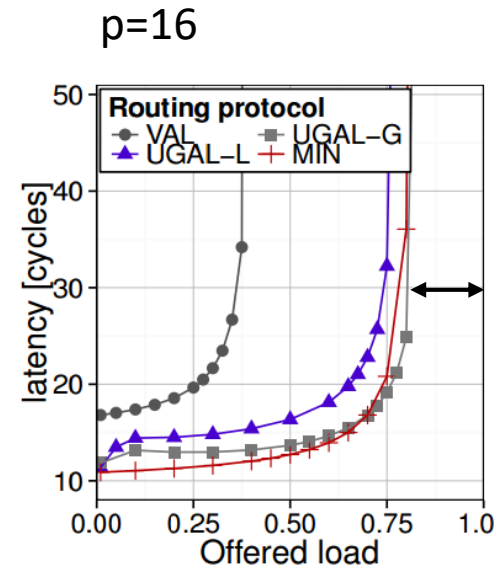
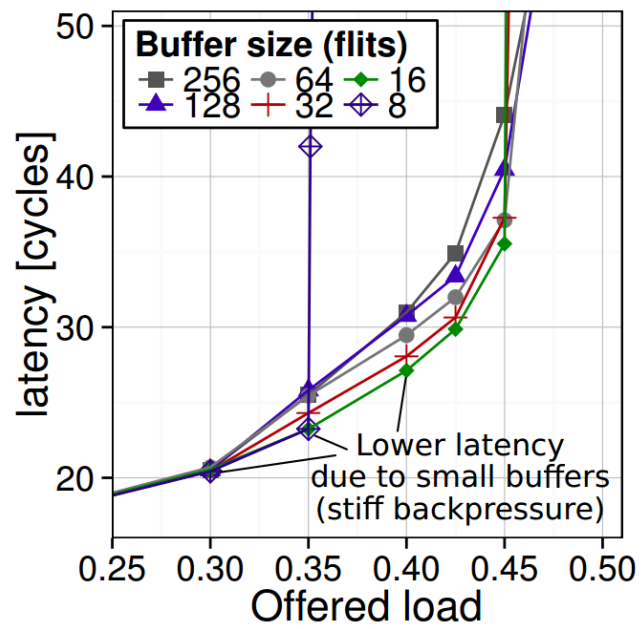
(b) Random traffic,  $p = 16$ .



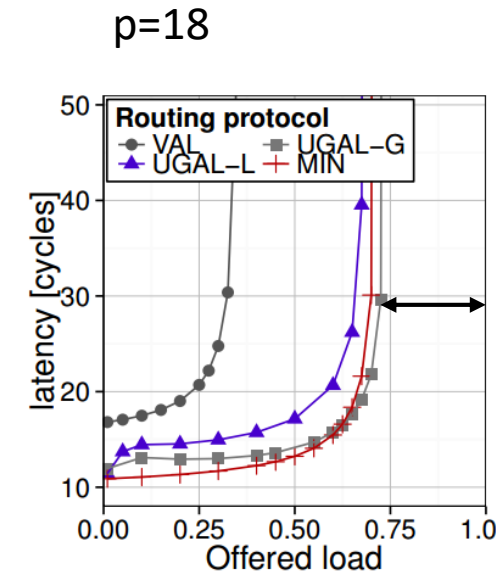
(d) Random traffic,  $p = 18$ .

# PERFORMANCE

- Buffer sizes (UGAL-L, worst-case traffic)
- Oversubscription (64 flits)



(b) Random traffic,  $p = 16$ .



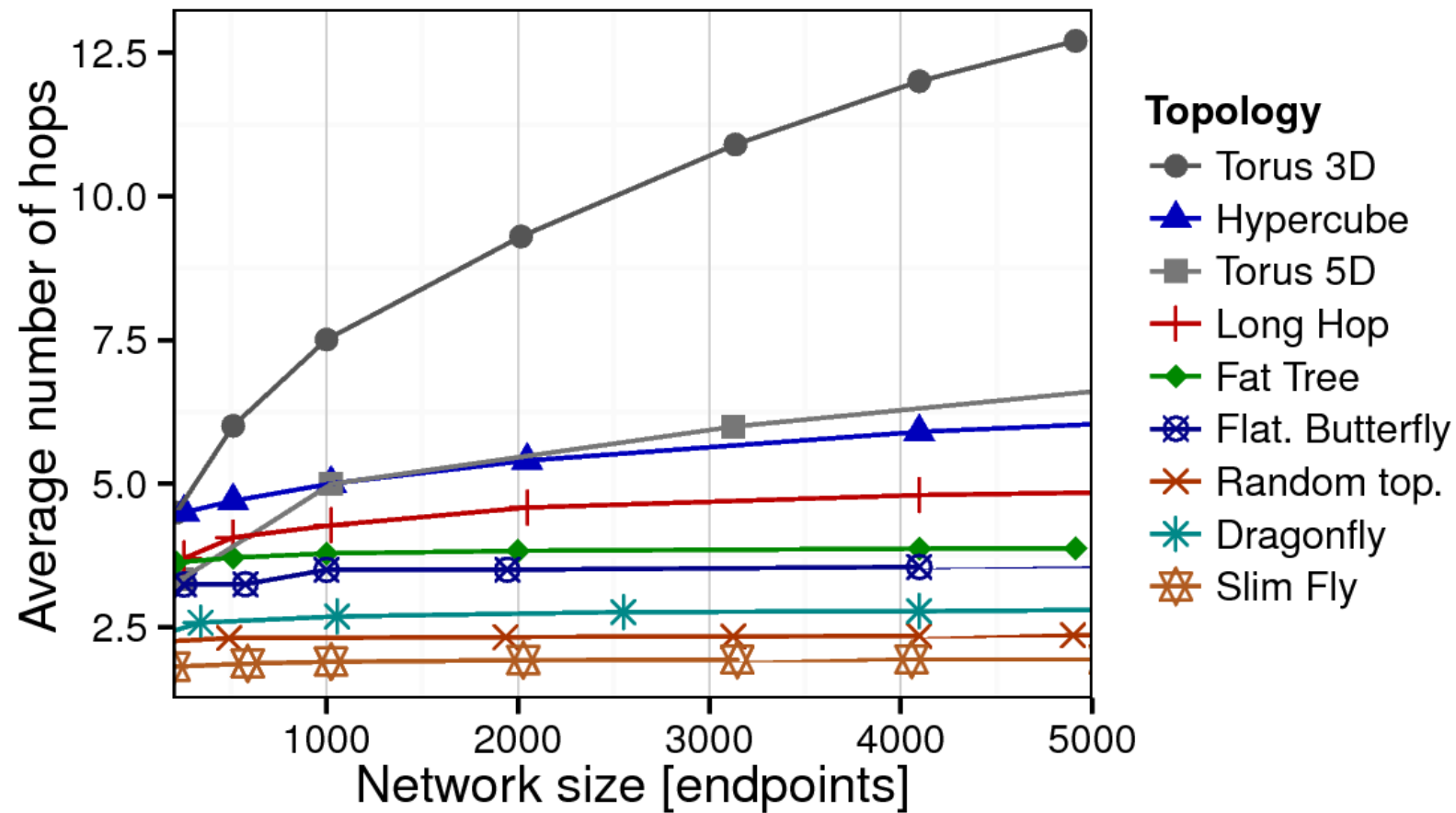
(d) Random traffic,  $p = 18$ .

# STRUCTURE ANALYSIS

## AVERAGE DISTANCE

# STRUCTURE ANALYSIS

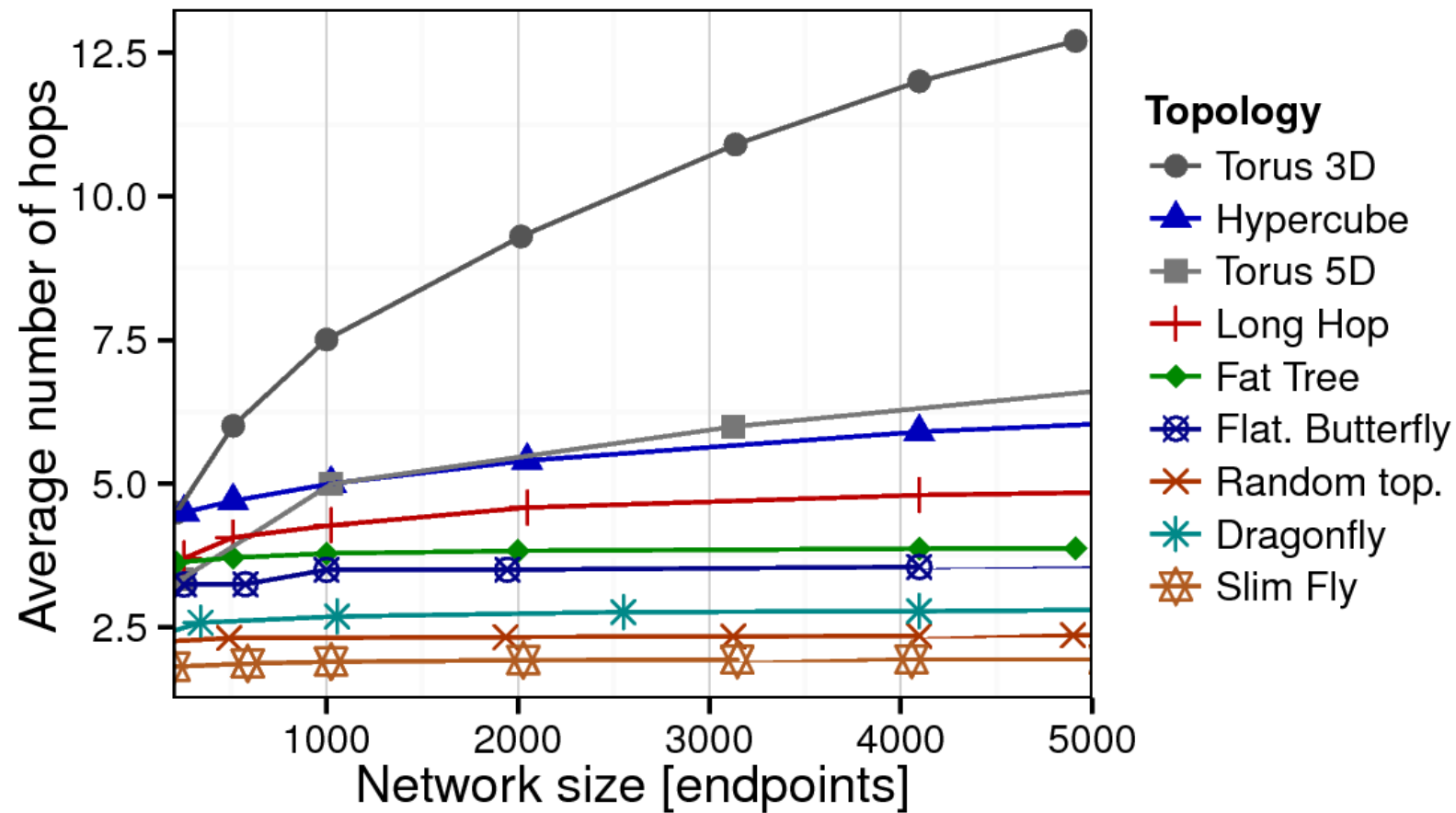
## AVERAGE DISTANCE



# STRUCTURE ANALYSIS

## AVERAGE DISTANCE

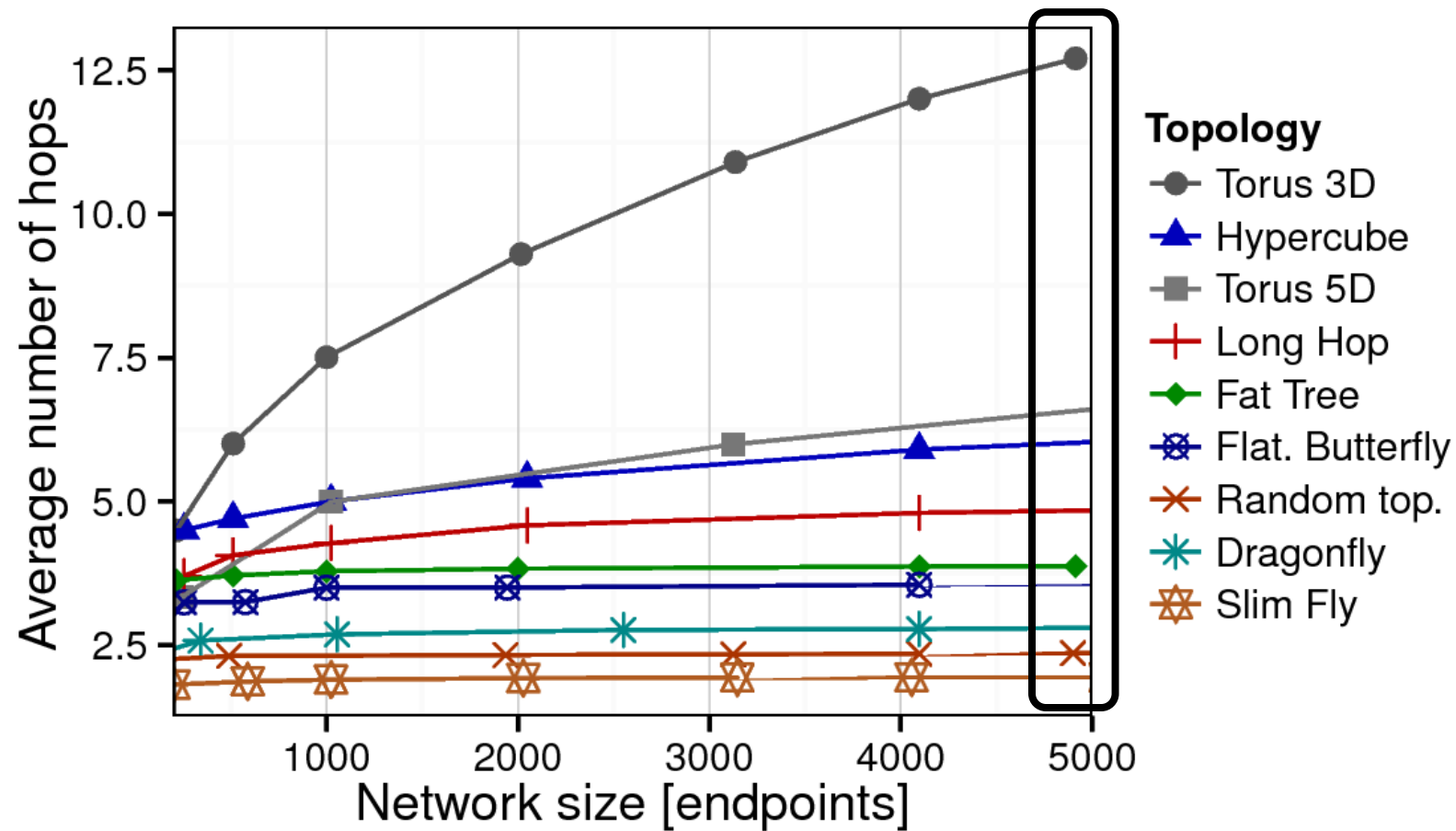
Random uniform traffic  
using minimum path routing



# STRUCTURE ANALYSIS

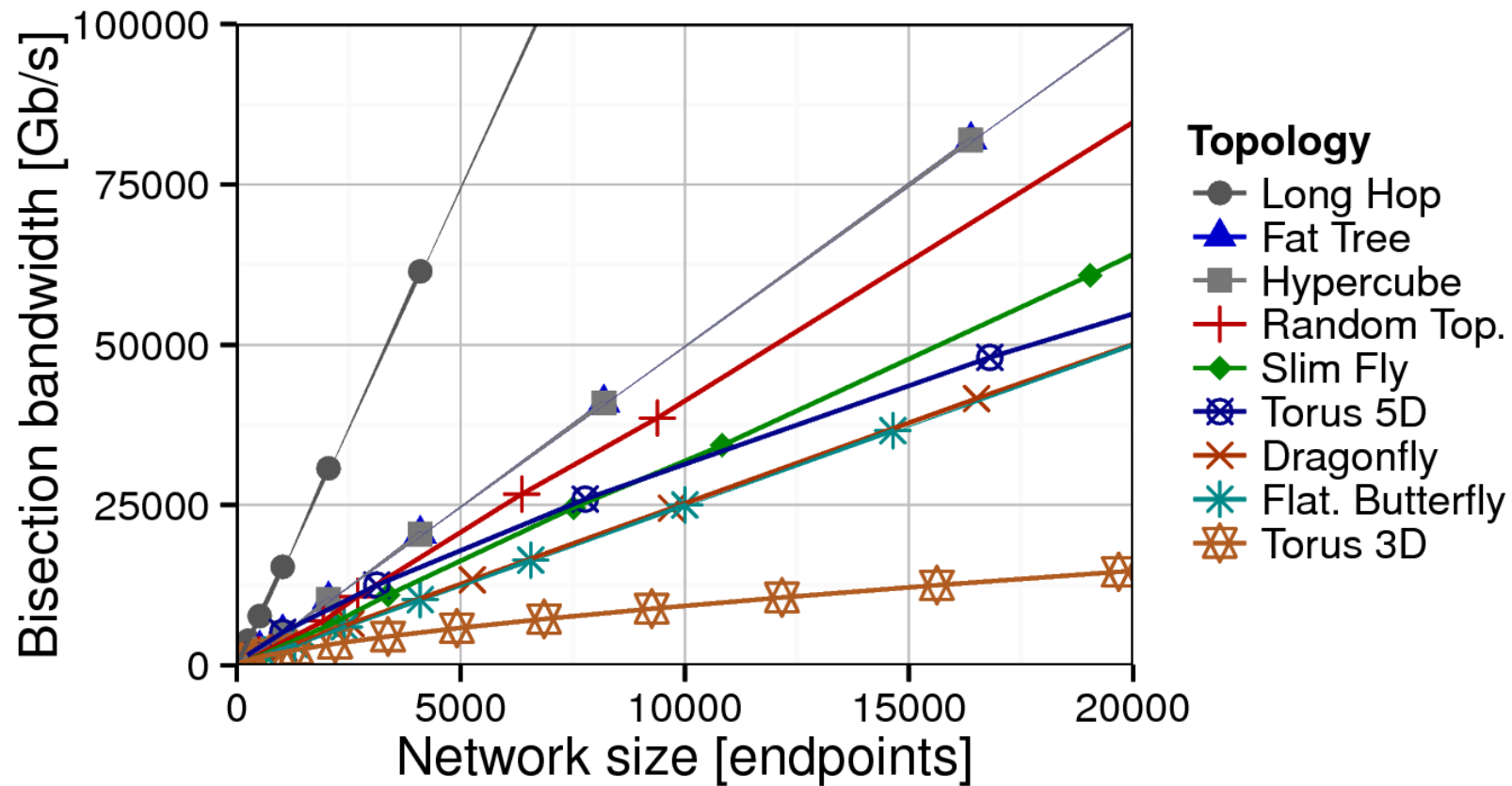
## AVERAGE DISTANCE

Random uniform traffic  
using minimum path routing



# STRUCTURE ANALYSIS

## BISECTION BANDWIDTH (BB)

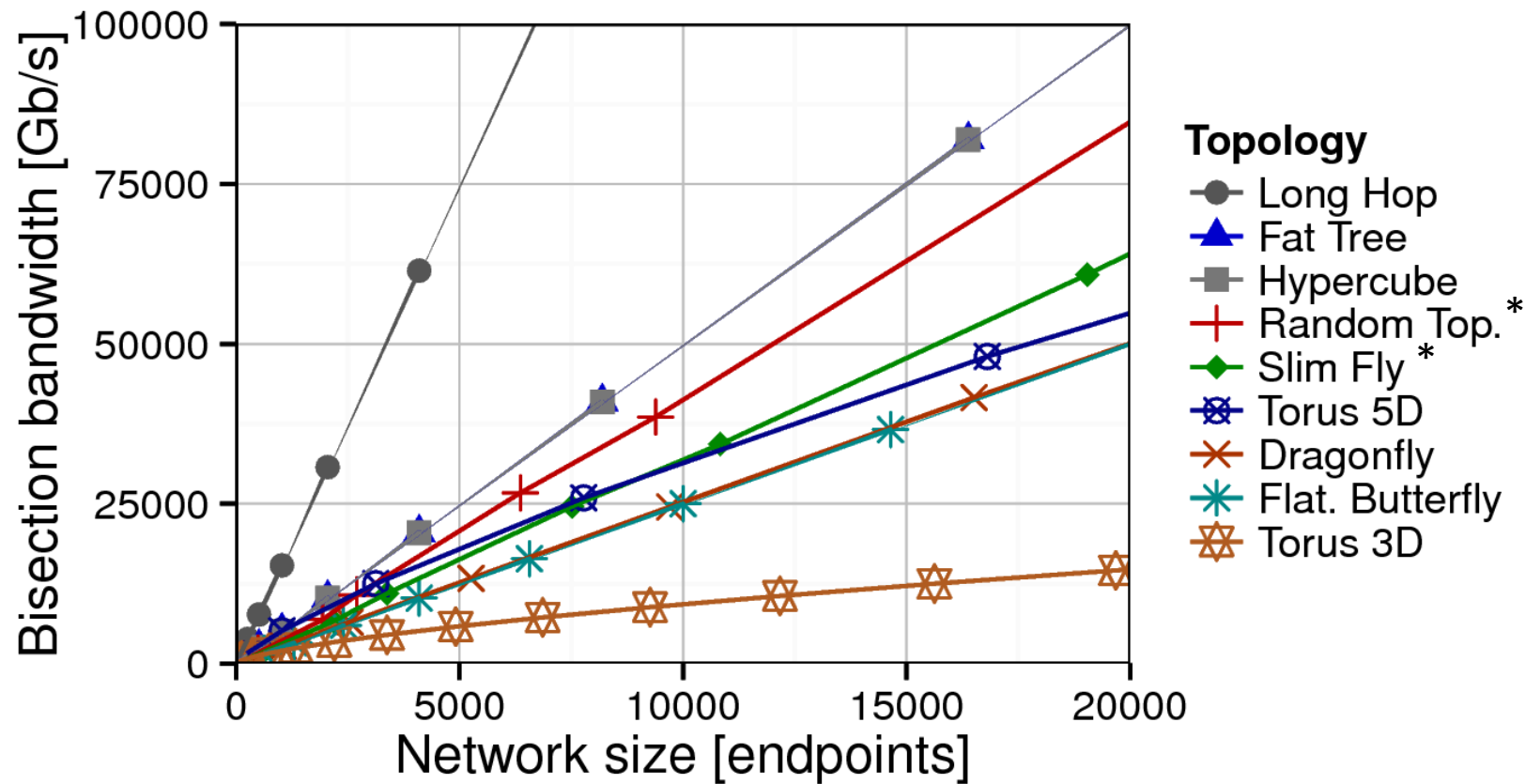




# STRUCTURE ANALYSIS

## BISECTION BANDWIDTH (BB)

\*BB approximated with the Metis partitioner [1]

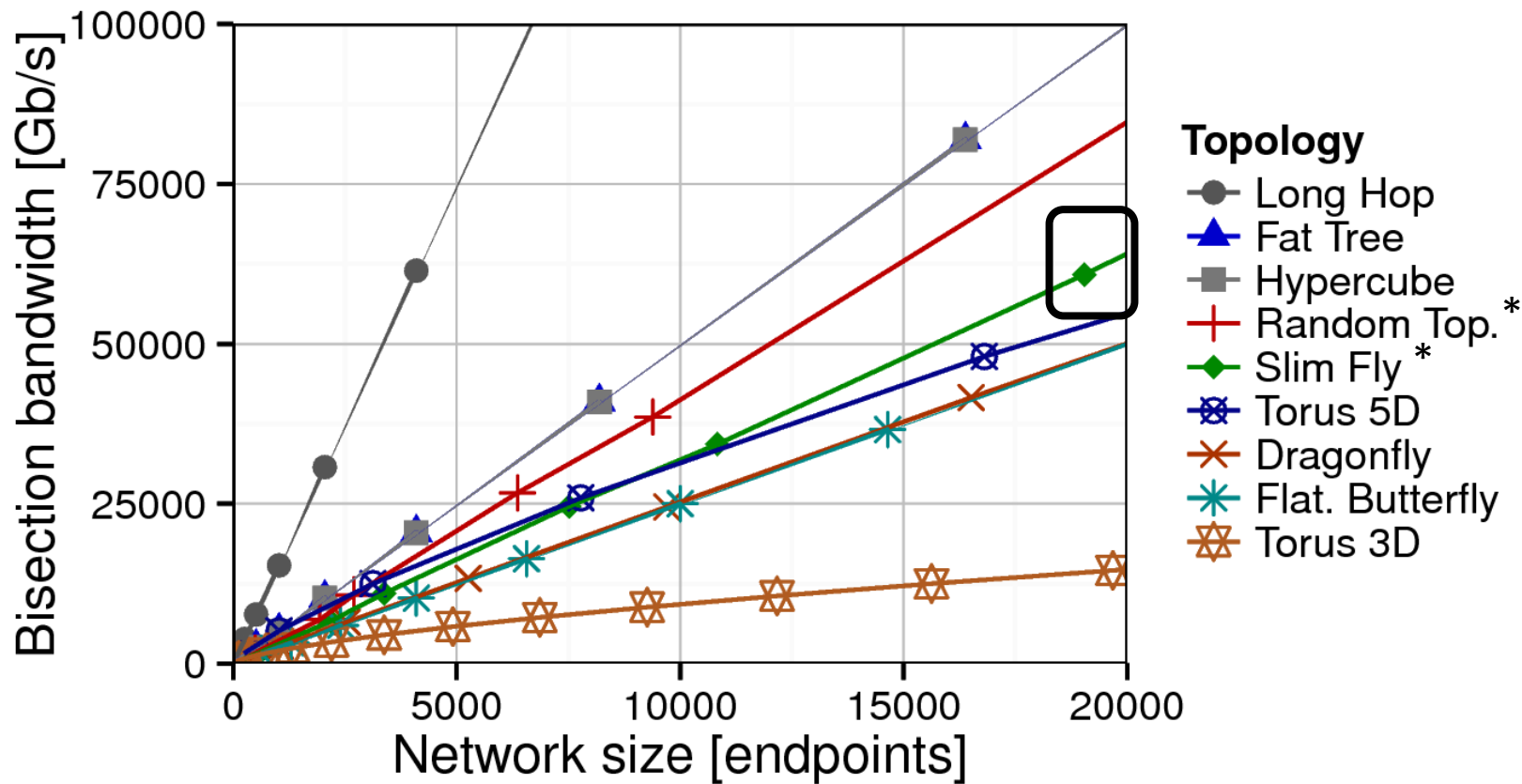


[1] G. Karypis, V. Kumar. A fast and high quality multilevel scheme for partitioning irregular graphs. ICPP'95

# STRUCTURE ANALYSIS

## BISECTION BANDWIDTH (BB)

\*BB approximated with the Metis partitioner [1]



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